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R-Compatible Ideals of topological spaces

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Abstract. This paper's goal is to present the idea of regular-compatibility of ideals by replaced the open set is Regular-open set and with respect to topology, which is abbreviated as ($\tau \sim R\, I$), Using a brand-new local function termed the regular-local function A(*R) (I, T), it is also possible to examine the connection between this idea and the idea of the R-compatibility of an ideal with I. A few comparable circumstances are established. By employing the new operator is Regular -psi set short (ΨR(A)), we further analyze the characterizations of Regular -compatibility with the aid of this local function. And the relationship between them. Several notions of compatibility between a partially ordered set and a topology on its underlying set are comparable to separation axioms and other well-known ideas from general topology.

Keywords. and Phrases, Ideal topological space, local functions, R-local functions, Regular-open sets, Compatible ideals, R- Compatible ideals, ΨR(A)-operator.

1. Introduction.

Velicko [1] first presented the concepts of θ –open, closed, and closure subset in order to examine the significant class of H-closed space in terms of arbitrary filter bases. The definition of the I-open set notion by Tankovic and Hamlett [2, 3] in 1990 used Vaidyanathaswamy's local function like a starting point. In just an ideal topological space, Kuratowski [4] established the concept of the local function. A large number of mathematicians, including Hayashi [2], Three researchers Natakaniec [5], Modak, and Bandyopadhyay [6] have examined this area and demonstrated some novel findings.

As in ideal topological framework. Per-local function was first established by JeonGikang and Chang SnKim [7] in 2009. Both semi-Local and α-Local functions, . 2011 saw the introduction of the δ -local function by Shyamapada Modak [8]. 2013 saw the definition of the local closure by AI-Omari and Noiri [9].

Also introduced the g-local function, as did K. Bhavani [10, 11]. The idea of a regular-local function is introduced in this work, and some of its aspects are looked into. In this work, we define the new type of local function by employing well -known local function in the sense of Kuratowski. We look at this local function's fundamental characteristics and provide the definition of the R(A) operator, we obtain novel topologies using this operator and compare compatibility with R- compatible.

2. Preliminaries

The notation that is used is as follows. The family of open neigh bour hoods at the point x will be T(x) if (X,T) is a topological Space [12]. cl(A) is the set closure, and Int(A) is the set interior. A subset A of X is said to be Pre- open [7] (resp. Semi- open [13], 14],[22] and Alpha- open [23] if $A \subseteq cl(int(A))$ (resp. $A \subseteq intcl(A))$ and $A \subseteq cl(int (cl(A))))$. Pre- closed (Resp. Semi- closed and Alpha- closed) is the complement of These. If each point in a set A contains the closure of an open Neighborhood in A, then A is said to be θ- open [9]. (There is a $V \in T(x)$ such that $cl(v) \subseteq A$). It's fine knowledge that the collection of all θ-open subsets of (X,T) are topologies on X that we will designate by Tθ. What is immediately obvious according to the definitions, $T\theta \subset T$. Then $T\theta = T$, consequently, then every open set is θ-open and only when the space (X,T) is regular. If we have $cl(u) \cap A \neq \emptyset$ for any open Neighborhood u of X, then a point $x \in X$ is said to be in the θ-closure of a subset $A \subseteq X$ [16]. We'll use cl_θ to refer for θ-closure(A) . If $A = cl\theta$ then a subset $A \subseteq X$ is said to be θ-closed (A) . Generally speaking, a set need not be a θ-closed set for its θ- closure. However, it is constantly closed. A non - empty set collection of X subsets that fulfills the condition is an ideal I on a topological space (X,T), An ideal is a partly ordered collection of sets that is thought to be tiny or unimportant in the discipline of mathematics known as set theory. Any subset of an ideal element must also be an ideal element, (This formalizes the notion that an ideal is a tiny notion), and the union of any two elements of the ideal must also be in the ideal. given a set X, an ideal I on X is a nonempty subset [17] of the power set of X, such that :

1- $\emptyset \in I$.
2- $A \in I$, and $B \subseteq A$ imply $B \in I$.
3- $A \in I$ and $B \in I$ imply $A \cup B \in I$.

Some authors add a fourth stipulation that X on its alone does not belong in I. Ideals with extra attributes are said to be suitable ideals. The term ideal topological space and the symbol are used to refer to a topological space (X, T) having an ideal I on X. (X, T, I). The local function of A with regard to I and T is defined as $A^*(l) =\{x \in X: U \cap A \notin I$ for any $U \in T(x) \}$ for a subset $A \subseteq X$ [11]. In order to avoid any misunderstanding, we just write A*. For a topology T*(I) known as the T*-topology finer than T, $cl^* (A) = A \cup A^*$ is defined as a Kuratowski [4] closure operator. A subset A of an ideal space (X, T, I) is T∗-closed [18] (resp. ∗-dense in itself), and ∗-perfect [18] if $A^* \subset A$ (resp. $A^* \subset A$, $A = A^*$). There is no doubt that A is only ∗-perfectly formed if and only if it is internally T*-closed and ∗-dense. If there is an open set U of x such that $U \cap A \in I$, then $A \in I$, for every subset A of X and for every $x \in A$, then I is said to be compatible with [3], indicated by $I \sim \tau$, , given an ideal topological space (X, I). Compatible ideals are termed as super compact ideals in [19]. Further, Hamlett and Jankovic in 1990 [20], studied the properties of ideal topological spaces and introduced another operator called Ψ-operator, defined as follows; for every $A \in$

X, ΨR(A) ={ x ∈ X: there exists U ∈ T (X) such that U–A ∈ I}. An operator Ψ : P(X) → τ satisfying Ψ(A) = X - (X - A)* for each A ∈ P(X) and we discuss some characterizations this operator by using Regular-open set. If assume that A is a subset of an ideal topological space (X, T, I). and then after that.

Definition 2.1. Let (X, T) be a topological space. If A = Int(cl(A)) [, then a subset A of X is said to be Regular-open [8]. Regular-closed is the term used to describe a Regular -open set's counterpart [21]. The term RO(X) (or RC(X)) stands for the collection of all Regular-open (or Regular-closed) sets in X.

Remark 2.1. We denote all Regular-open sets forms a topology T^R or TR.

Definition 2.2. Let (X, T) be a topological space. Then the Regular-interior and the Regular-closure of A in X defined as Int_R (A) = ∪ { U : U ⊆ A, U ∈ T^R } and cl_R (A) = ∩ { F : A ⊆ F, X – F ∈ T^R }.From definition, Int_R (A) is a Regular-open set and cl_R (A) is a Regular-closed set.

Remark 2.2.

1. Every Regular-open is open.
2. Every Regular-closed is closed.

Theorem.2.2 If A is ClOPEN set(open and closed) then A is Regular-open set.

Proof :-

Let A is clopen that is mean A is open set then A = int(A).

A is closed set then A = cl(A).

Since int (A) = ∪{O ⊆ X, O ⊆ A, where O is open }.

CI(A) = ∩{V ⊆ X, A ⊆ V, where V is closed }.

Since A is open and closed.

Then O = V.

Then A = int(A)andA = CI(A).

Then A = int(cl(A)),

Then A is Regular-open.

clopen → R-open → open ← θ-open

Diagram 1. Relationship between open, clopen, R-open and θ-open.

3- Regular-local function of A with respect to I

Definition 3.1. Assume that (X, T, and I) is an ideal topological space. We define the operator A(*R) (I, T) = {x ∈ X: for a subset A of X, where A ∩ U ∉ I for every UT ∈ R (x)}. Just so there is no misunderstanding The Regular-local function of A with respect to I and T is indicated by A(*R) (I, T) and is abbreviated as A(*R). Also A(*R) = A^{*R} .

Remark 3.1. Assume that A ⊂ X and that (X, T, I) is an ideal topological space. Then, cl(*R)(A) = A ∪ A(*R) is a ∗R-closure operator.

Remark 3.2. If A ⊂ X and (X,T,I) is an ideal topological space, Then

$$T^{*R} = \{ X - A : cl^{*R}(A) = A\}.$$

Example 3.1. Assume that (X, T, I) is an ideal topological space and that A ⊂ X. with X = { 1, 2, 3 }, T = { X, {1, 2 }, {2}, {1}, ∅ }, with I = { ∅, {3} }.

A	A^{*R}	cl^{*R}
∅	∅	∅
X	X	X
{1}	{1, 3}	{1, 3}
{2}	{2, 3}	{2, 3}
{3}	{3}	{3}
{1, 2}	X	X
{1, 3}	{1, 3}	{1, 3}
{2, 3}	{2, 3}	{2, 3}

Then $T^{*R} = \{ \emptyset, X, \{1, 2\}, \{2\}, \{1\} \}$.

Lemma 3.1. An ideal topological space would be (X, T, and I). When every subset A of X is considered, A*(I, T) ⊂ A(*R) (I, T).

Proof. Let x ∈ A*, Our statement is that x ∈ A(*R). If not, a Regular-open set U x that contains x and is such that Ux ∩ A ∈ I exists. Because every Regular-open is open. Since, Ux is open, Ux ∩ A ∈ I is in conflict with the supposition x ∈ A*. Consequently, x ∈ A(*R). This shows that A* ⊂ A(*R). It is demonstrated in this case that converse is not generally true.

Example 3.2. consider an ideal topological space (X, T, I) where $X = \{1, 2, 3\}, T = \{\emptyset, X, \{2, 3\}, \{1, 3\}, \{1\}, \{3\}\}$, $I = \{\emptyset, \{c\}\}, RO(X) = \{X, \emptyset, \{a\}, \{b, c\}\}$. Let $A = \{1,3\}$, then $A^{*R} = X \not\subset \{3\} = A^{*}$.

Theorem 3.1. A and B should be subsets of X in an ideal topological space. Then, the Consequently, the following characteristics apply for R-local functions.

1- $(\emptyset)^{*R} = \emptyset$.

2- $A^{*R} \subset B^{*R}$, If $A \subset B$.

3- $(A \cup B)^{*R} = A^{*R} \cup B^{*R}$.

4- $(A \cap B)^{*R} \subset A^{*R} \cap B^{*R}$.

5- If $j \in J$, therefore $(j)^{*R} = \emptyset$.

Proof.(1) It is clear from the definition of an R-local function, that ∅ = ∅(*R).

2) Take x ∈ A(*R). So Ux ∩ A ∉ I for any Regular-open set Ux including x. Assuming A ⊂ B, then Ux ∩ A ⊂ Ux ∩ B∉ I follows. Consequently, Ux ∩ B ∉ I. The implication is that x ∈ B(*R). Consequently, A(*R) ⊂ B(*R).

3) Consider that A ⊂ A ∪ B and B ⊂ A ∪ B. Following that, through the (2) A(*R) ⊂ (A ∪ B)(*R) and B(*R) ⊂ (A ∪ B)(*R), Therefore, A(*R) ∪ B(*R) ⊂ (A ∪B)(*R). On the other hand, assume that x ∉ A(*R) ∪ B(*R). After the last, x ∉ A(*R) and x ∉ B(*R). There's a Regular-open set Ux that contains x and is such that Ux ∩ A ∈ I if x ∉ A(*R). Similar to how x ∉ B(*R),there is regular-open set Vx contains x and is such that Vx ∩ A ∈ I. So, through the inherited quality of ideal, once more by the ideal's limited additivity, respectively, A ∩ (Ux ∩ Vx) ∈ I and B ∩ (Ux ∩ Vx) ∈ I. (Ux ∩ Vx) ∩ (A ∪ B)∈ I, Consequently, x ∉ (A ∪B)(*R). Therefore, (A ∪ B)(*R) ⊂ A(*R) ∪ B(*R). Therefore A(*R) ∪ B (*R) = (A ∪ B)(*R).

4) Given that A ∩ B ⊂ A and A ∩ B ⊂ B, we may deduce via (2) that (A ∩ B)(*R) ⊂ A (*R) and (A ∩ B)(*R) ⊂ B (*R) Consequently, (A ∩ B)(*R) ⊂ A (*R) B ∩ (*R).

5)Have j ∈ J and x ∈ j (*R). hence j ∩ Ux ∉ J for every Ux ∈ TR(X). However, this is a contradiction ,since, for every case when Ux ∈T R(X),then, j ∈ J ,j ∩ Ux ∈ J, and therefore j(*R) = ∅.

4. R-compatible with I

Definition.4.1 an ideal topological space (X, τ, I), I is said to be Regular- compatible with τ, denoted by τ ~ R I, if for every subset A of X and for each x ∈ A, there exists an Regular- open set U of x such that U ∩ A ∈ I, then A ∈ I.

Example 4.1 Assume that (X, T, I) is an ideal topological space and that A ⊂ X. with X = { 1, 2, 3 }, T = { X, {1, 2 }, {1, 3 }, {2}, {1}, ∅ }, with I = { ∅, {1} }, and T^R = {X, {1, 3 }, {2}, ∅ }.

If $A = \{1\}$, then $A \in I$.

Then T is R- compatible with idea l for every $A \subset X$.

Theorem 4.1 . If τ is R- compatible with I, then τ is compatible with I.

Proof. Let A be a subset of X containing x and suppose that there exists a Regular -open set U containing x such that $(U) \cap A \in I$. Since every regular-open set is open by Remark **2.2 (1)** .then, there exists an open set containing x such that $U \cap A \in I$. Since, τ is R-compatible with I and hence $A \in I$. Therefore τ is compatible with I.

Example 4.2 Assume that (X, T, I) is an ideal topological space and that $A \subset X$. with $X = \{1, 2, 3\}$, $T = \{X, \{1, 2\}, \{1, 3\}, \{2\}, \{1\}, \emptyset\}$, with $I = \{\emptyset, \{1\}, \{1, 3\}, \{3\}\}$, and $T^{R} = \{X, \{1, 3\}, \{2\}, \emptyset\}$, Since τ is R-compatible with I, Then τ is compatible with I.

Theorem.4.3 Let (X, τ, I) be an ideal topological space and A a subset of X. The ideal I is said to be R- compatible with τ if and only if $A \setminus A^{*} \in I$, for every $A \subseteq X$.

Theorem.4.2 Let (X, τ, I) be an ideal topological space, the following properties are equivalent:

1. $\tau \sim R\ I$.

2. If a sub set A of X has a cover of Regular-open sets each of whose intersection with A is in I, then $A \in I$.

3. For every $A \subseteq X$, $A \cap A^{*R} = \emptyset$ implies that $A \in I$.

4. For every $A \subseteq X$, $A \setminus A^{*R} \in I$.

5. For every $A \subseteq X$, if A contains no nonempty subset B with $B \subseteq A^{*}R$, then $A \in I$.

Proof. (1) to(2): The proof is obvious.

(2) to (3): Let $A \subseteq X$ and $x \in A$. Then $x \setminus \in A^{*R}$, Since $A \cap A^{*R} = \varphi$ and there exists Regular-open set $R_x \in T^{R}$, such that $R_x \cap A \in I$. Therefore, we have $A \subseteq \cup \{R_x : x \in A\}$ and $R_x \in T^{R}$ and by (2) $A \in I$.

(3) to (4): For any $A \subseteq X$, $(A \setminus A^{*R}) \subseteq A$ and $(A \setminus A^{*R}) \cap (A \setminus A^{*R})^{*} R \subseteq (A \setminus A^{*R}) \cap A^{*R} = \emptyset$. By (3), $(A \setminus A^{*R})) \in I$.

(4) to (5): By (4), for every $A \subseteq X$, $(A \setminus A^{*R}) \in I$. Let $(A \setminus A^{*R}) = i \in I$, then $A = i \cup (A \cap A^{*R})$ and by Theorem **3.1 (3**) and **(5**), $A^{*R} = (i)^{*R} \cup (A \cap A^{*R})* R = (A \cap A^{*R})(*R)$. Therefore, we have $A \cap A^{*R} = A \cap (A \cap A^{*R})(*R) \subseteq (A \cap A^{*R})(*R)$ and $(A \cap A^{*R}) \subseteq A$. By the assumption $A \cap A^{*R} = \emptyset$ and hence $A = (A \setminus A^{*R}) \in I$.

(5) to (1): Let A ⊆ X and assume that for every x ∈ A, there exists Regular-open set Rx ∈ T^R such that A^{*R} ∩ A ∈ I. Then A ∩ A^{*R} = ∅. Suppose that A contains B such that B ⊆ B^{*R} Then B = B ∩ B^{*R} ⊆ A ∩ A^{*R} = ∅. Therefore, A contains no nonempty subset B with B ⊆ B^{*R}. Hence A ∈ I.

Example.4.3 If X = {a, b, c, }, with T ={ φ, x, {b, c}, {a, c }, {a }, {c}, and I = { φ, , {c} }, then RO(X) ={ φ, x, {b, c}, {a } }.

A	**A(*R)**
X	X
Φ	Φ
{a }	{a }
{b}	{b, c}
{c}	Φ
{a, b }	X
{a, c }	{a }
{b, c}	{b, c}

Then x\x =φ ∈I, φ\ φ = φ ∈I, {a } \ {a } =φ ∈ I, {b}\{b, c}= φ ∈ I, {c}\ φ = {c}∈ I, {a, b }\ x = φ ∈ I, {a, c }\{a }={c} ∈ I, {b, c}\{b, c} = φ ∈ I. Hence, R- compatible with I.

5 - Regular-open set operator ψ

Definition 5.1. Let (X, τ, I) be an ideal topological space. An operator Ψ:P(X) → τ is called Regular-open set operator ψ and defined as follows; for every A ∈ X, ΨR(A)={ x ∈ X: there exists U ∈ RO(X) such that U–A ∈ I}.We observe that ΨR(A) = X –(X – A)(*R) .

Example.5.1 If X = {a, b, c }, with T ={∅, x, {b, c}, {a, c}, {a}, {c}}, and I = { ∅, {c}}, then RO(X) ={ φ, x, {a}, {c, b} }.

A	**ΨR(A)**
∅	∅
X	X
{a}	{a}
{b}	{b, c}
{c}	∅
{a, b}	X
{a, c}	{a}
{b, c}	{b, c}

Theorem.5.1 Let (X, τ, I) be an ideal topological space and A a subset of X. The ideal I is said to be R- compatible with τ if and only if $\Psi R \setminus A \in I$, for every $A \subseteq X$.

Definition. 5.2[11] Let (X, τ, I) be an ideal topological space and A a subset of X. The ideal I is said to be:

(1) Weakly R- compatible with τ , if $A(*R) = \emptyset$ implies that $A \in I$.

(2) R- Compatible with τ , if $A(*R) \cap A = \emptyset$ implies that $A \in I$.

Example. 5.2 If $X = \{1, 2, 3, 4\}$, with $T = \{\varphi, x, \{4\}, \{1, 4\}, \{2, 4\}, \{1, 2, 4\}\}$, and $I = \{\varphi, , \{1\}, \{2\}, \{1, 2\}\}$, then $RO(X) = \{\varphi, x\}$.if $A = \{1\}$, $A(*R) = \varphi$, then $A \in I$

Theorem 5.2. [11] Let (X, τ, I) be an ideal topological space and A a subset of X. If I is an ideal compatible with τ , then I is an ideal weakly compatible with τ.

Theorem. 5.3 Let (X, τ, I) be an ideal topological space, and A a subset of X. If I is an ideal compatible with τ the following properties are holds.

1- $A^* \cap (\Psi(A) \setminus A) \in I$ if $\Psi(A) \setminus A \in I$.
2- $A^* \cap (\Psi(A) \setminus A) \in I$ if $A \setminus A^* \in I$.
3- $A^* \cap (A \setminus A^*) \in I$ if $A \setminus A^* \in I$.
4- $A^* \cap (A \setminus A^*) \in I$ if $\Psi(A) \setminus A \in I$.

Proof (1).Take I is an ideal compatible with τ, and A subset of X, then for $A \setminus A^* \in I$, $\Psi(A) \cap (A \setminus A^*) \in I$, Therefore, $A^* \cap (\Psi(A) \setminus A) \in I$, then from $\tau \sim I, \Psi(A) \setminus A \in I$.

Conversely suppose that $\Psi(A) \setminus A \in I$, for every $A \subseteq X$. Let $x \in A$. Also there is $U_x \in T(x) \in$ such that $U_x \cap A \in I$ for every $x \in A$. Then $x \notin A^*$, and hence $x \in X \setminus A^*$. Thus $A \subseteq X \setminus A^*$. then, $\Psi(X \setminus A^*) \setminus (X \setminus A^*) = [X \setminus (X \setminus (X \setminus A^*)^*)] \setminus (X \setminus A^*)$ Therefore, $A^* \cap (\Psi(A) \setminus A) \in I$.

Proof 2, 3 and 4 are the same proof.

Corollary.5.1 Let (X, τ, I) be an ideal topological space, and A a subset of X. If I is an ideal compatible with τ the following properties are holds.

1- $\Psi(A) \cap (\Psi(A) \setminus A) \in I$ if $\Psi(A) \setminus A \in I$ or $A \setminus A^* \in I$.

2- $\Psi(A) \cap (A \setminus A^*) \in I$ if $\Psi(A) \setminus A \in I$ or $A \setminus A^*$

Example.5.3 An ideal topological space (X, T, I) where $X = \{a, b, c\}$, with $T = \{\varphi, x, , \{a, b\}, \{a\}\}$, and $I = \{\varphi, , \{a\}\}$.

A	**A***	**A \ A***	**Ψ(A)**	**Ψ(A)\A**
X	{b, c}	{a}	X	∅

∅	∅	∅	∅	∅
{a}	∅	∅	{a}	∅
{b}	{c}	∅	{a, b }	{a}
{c}	{c}	∅	{a}	∅
{a, b }	{b, c}	∅	{a, b }	∅
{a, c }	{c}	{a}	∅	∅
{b, c}	{b, c}	∅	∅	∅

Corollary.5.2 Let (X, τ, I) be an ideal topological space, and A a subset of X. If I is an ideal R-compatible with τ then following properties are holds.

1-ΨR(A) ∩ (ΨR(A) \ A) ∈ I if ΨR(A) \ A ∈ I or A \ A(*R) ∈ I.

2-ΨR(A) ∩ (A\ A(*R)) ∈ I if ΨR(A) \ A ∈ I or A \ A(*R) ∈ I.

Theorem.5.4 Let (X, τ, I) be an ideal topological space, and A a subset of X. If I is an ideal compatible with τ the following properties are holds.

1-A(*R) ∩ (ΨR(A) \ A) ∈ I if ΨR(A) \ A ∈ I or A\ A(*R) ∈ I.

2-A(*R) ∩ (A \ A(*R)) ∈ I if ΨR(A) \ A ∈ I or A\ A(*R) ∈ I.

Example.5.4 An ideal topological space (X, T, I) is R − compatible with τ, where X = {a, b, c }, with T ={ φ, x, , { b, c }, { a, c }, {a }, { c } }, and I = { φ, , { c } } ,RO(X) = { φ, x, , { b, c }, {a } }. If A = {b}, A(*R) = { b, c } and ΨR(A) = { b, c } ,then all of Theorem 5.4 (1, 2) are holds.

Theorem.5.5 Let (X, τ, I) be an ideal topological space, and A a subset of X. If I is an ideal R-compatible with τ the following properties are holds.

1- ΨR(A) ∩ (ψ R (A) \ A) ∈ I, if ΨR(A) \ A ∈ I or A\ A(*R) ∈ I.
2- ΨR(A) ∩ (A \ A(*R)) ∈ I, if Ψ(A)R \ A ∈ I or A\ A(*R) ∈ I.

Theorem. 5.6 Let (X, τ, I) be an ideal topological space, and A a subset of X. If I is an ideal compatible with τ the following properties are holds.

1- (A \ A*) ∩ (Ψ(A) \ A) ∈ I, for Ψ(A) \ A ∈ I or A \ A* ∈ I.
2- (A \ A*) ∪ (Ψ(A) \ A) ∈ I, for Ψ(A) \ A ∈ I or A \ A* ∈ I.

Corollary5.3. Let (X, τ, I) be an ideal topological space, and A a subset of X. If I is an ideal R- compatible with τ the following properties are holds.

1-(A \ A(*R)) ∩ (ΨR(A) \ A) ∈ I, for Ψ(A) \ A ∈ I or A\ A(*R) ∈ I.

2- (A \ A(*R)) ∪ (ΨR(A) \ A) ∈ I, for ΨR(A) \ A ∈ I or A\ A(*R) ∈ I.

Theorem 5.7 Let (X, τ, I) be an ideal topological space, and A a subset of X. If I is an ideal compatible with τ the following properties are holds.

1- $A^* \cup (A \setminus A^*) \notin I$, if $A \setminus A^* \in I$ or $\Psi(A) \setminus A \in I$.
2- $A^* \cup (\Psi(A) \setminus A) \notin II$, if $A \setminus A^* \in I$ or $\Psi(A) \setminus A \in I$.
3- $\Psi(A) \cup (A \setminus A^*) \notin I$,if $A \setminus A^* \in I$ or $\Psi(A) \setminus A \in I$.
4- $\Psi(A) \cup(\Psi(A) \setminus A) \notin I$, if $A \setminus A^* \in I$ or $\Psi(A) \setminus A \in I$.

Corollary 5.5 Let (X, τ, I) be an ideal topological space, and A a subset of X. If I is an ideal R- compatible with τ then following properties are holds.

1- $A^{*R} \cup (A\setminus A(^{*R})) \notin I$, if $A \setminus A(^{*R}) \in I$ or $\Psi(A) \setminus A \in I$.

2- $A(^{*R}) \cup (\psi R(A) \setminus A) \notin I$, if $A \setminus A^{*R} \in I$ or $\Psi R(A) \setminus A \in I$.

3- $\Psi R(A) \cup (A \setminus A(^{*R})) \notin I$, if $A \setminus A(^{*R}) \in I$ or $\Psi R(A) \setminus A \in I$.

4- $\Psi R(A) \cup(\Psi R(A) \setminus A) \notin I$, if $A \setminus A(^{*R}) \in I$ or $\Psi R(A) \setminus A \in I$.

6 - Conclusions.

We defined the concepts of R -local function $A(^{*R})(I)$, by using idea of local function , also Regular-open set operator $\Psi R(A)$, R-compatibility ($\tau \sim R\, I$) , by replaced the open set is Regular-open set, And get some relationships, results and rules through the use of those definitions, And obtaining some relationships, results, and rules through the use of these definitions then developing and finding an opportunity to create new spaces similar or different in their composition in relation to the previous spaces. So, it appears that the idea of topology and partial order compatibility relies on the context.

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