Solving Model of Reaction-Diffusion System in Ecological by Symmetry Lie Group Methods

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Abstract

We proposed and analyzed methods for solving the fundamental element of a reaction- diffusion model in ecology. The model denotes an interaction between two species that describes ecological predations and is mathematically defined as a system of partial differential equations with initial and boundary conditions. Symmetry Lie group methods are used to transform this model into a system of ordinary differential equations, and then this system is solved by the generalized tanh function method when there exists a small parameter λ appear in one of the equation.

Keywords:- Reaction-diffusion system, symmetry Lie group methods. tanh method.

1.Introduction

Reaction-diffusion models for the interaction of the species have been studied widely, it is well known that eminent scientists A.J.Lotka and V.Volterra developed the mathematical foundation for ecology [3][6]. The equations that simulate the conflict between two species (predators and prey) bear the names of tow scientists: Lotka and Volterra. They were from various nations and followed separate career and life paths, but their interest in and accomplishments in mathematical modelling brought them together. Elementary ecology texts tell us that organisms interact in three fundamental ways, generally given the names competition, predation, and mutualism. The only interaction from which one species get benefit is predation. Mathematically, thus, predation can be defined, in brief, as a -/+ interaction, while competition and mutualism are respectively -/-, +/+. The Lie group method for establishing the transformations leaves a system of partial differential equations (PDEs) [11]. We mean a continuous group of transformations acting on the expanded space of variables which includes the equation parameters in addition to independent and dependent variables. We consider the transformations that can be found using the lie infinitesimal criterion with the properly expanded infinitesimal group generators [9]. The lie point symmetries will be analyzed, then the reduction forms will be found, and then the invariant solutions of the original system of reduced ordinary differential equations (ODE's) by using the generalized tanh function method [8]. The technique is outlined for the computation of closed-form solutions for nonlinear partial differential equations and ordinary differential equations. which is the analysis of scientific publications [5]. Volterra proposed the classical model

$$\frac{du}{dt} = u(a - bv)$$
$$\frac{dv}{dt} = v(-c + du) \qquad (1)$$

The cyclical levels of specific fish capture in the Adriatic are explained by the predation of one species by another [12]. In (1), the functions $u(t)$ and $v(t)$ describe, respectively, the time development of the numbers of predators and prey, and represent the growth rates of the two populations over time when the derivative is taken with respect to t, and a, b, c and d are real positive parameters that talk about the interaction of two species [1][2]. In words, one may formulate system (1) as follows

[Rate of changes of u]

= [net rate of growth of u without predation] − [net rate of loss of u due to predation],

[Rate of changes of v] = − [net rate of loss of v without prey] +
[net rate of growth of v due to predation]. [11]

To explain chemical reactions that have periodic behaviour in chemical concentrations Lokta suggested the same model. As a result, system (1) is referred to be the Lokta – Volterra model. [11]. The most crucial method for solving nonlinear issues is a symmetry group analysis based on the transformation group, now known as lie groups. The symmetries of the differential equations are analyzed using a thin method. For the nonlinear reaction-diffusion system, we analyze the symmetry group and look at similar solutions in this work. To present the most general Lie group of point transformations, we also consider a scalar $k - th$ order PDE represented by

$$\Delta(x, u, u_1, \dots u_k) = 0, \qquad (2)$$

Where $x = (x_1, x_2,\ x_3, \dots, x_n)$ denotes n independent variables, $u = (u_1, u_2, u_3, \dots, u_m)$ u denotes the set of variables that correspond to all of u's jth-order partial derivatives with respect to variable $x.$ indicates the set of m dependent (differential) variables, the Lie group of transformations infinitesimal generator for one parameter

$$X = \sum_{i=1}^{n} \xi_i(x, u) \frac{\partial}{\partial x_i} + \sum_{\alpha=1}^{m} \eta^{\alpha}(x, u) \frac{\partial}{\partial u^{\alpha}} \qquad (3)$$

Where $\xi_i(x,u), \eta^\alpha(x,u)$ are infinitesimals (3), and the kth prolongation of the infinitesimal generator (3)

$$pr^k X = X + \eta_i{}^{(1)\alpha}(x,u,u_1)\frac{\partial}{\partial u_i{}^\alpha} + \cdots + \eta^{(k)\alpha}{}_{i_1 i_2 \dots i_k}(x,u,u_1,\dots,u_k)\frac{\partial}{\partial u^\alpha{}_{i_1 i_2 \dots i_k}} \quad , \qquad (4)$$

,where $\eta_i{}^{(1)\alpha} = D_i\eta^\alpha - (D_i\xi_j)u_j{}^\alpha$

for $i,j = 1,2,\dots,n; \alpha = 1,2,\dots,m$

and

$$\eta^{(k)\alpha}{}_{i_1 i_2 \dots i_k} = D_{i_k}\eta^{(k-1)\alpha}{}_{i_1 i_2 \dots i_{k-1}} - (D_i\xi_j)u^\alpha{}_{i_1 i_2 \dots i_{k-1}}, \qquad (5)$$

,where $i = 1,2,3,\dots,n; i = 1,2,3,\dots,k \; and \; k = 2,3,4,\dots$,

Where D is the total derivative operator defined

$$D_i = \frac{\partial}{\partial x_i} + u_i{}^\alpha\frac{\partial}{\partial u^\alpha} + u_{ij}{}^\alpha\frac{\partial}{\partial u^\alpha{}_j} + \cdots + u^\alpha{}_{i_1,i_2,\dots,i_n}\frac{\partial}{\partial u^\alpha{}_{i_1,i_2,\dots,i_n}}, \qquad (6)$$

$$u_i = \frac{\partial u}{\partial x_i} \quad and \quad i = 1,2,3,\dots,n \;,$$

with summation over a repeated index [1][8].
The paper is organized as follows: In section two we investigate a reaction-diffusion model for a system of two species that displays interactions between predation and population. In section three, we describe the model as a non-dimensional reaction-diffusion system. In Section four, we used the basic terms used to obtain the infinitesimals of the equation (8), to solve reduced nonlinear ODE's and find the analytical solution by using the tanh method, and the conclusion is discussed in section 5.

2. Reaction-diffusion system

We study a reaction-diffusion model for a system of two species which exhibits predation population interactions, the model we will study here is

$$\frac{\partial u}{\partial t} = D_u\frac{\partial^2 u}{\partial x^2} + k_u u(-a + bM_u u^2 + S_u v)$$
$$\frac{\partial v}{\partial t} = D_v\frac{\partial^2 v}{\partial x^2} + k_v v(c - dM_v v^2 - S_v u)\,. \qquad (7)$$

Where D_u and D_v are diffusion coefficients, $k_u u(-a + bM_u u^2)$ is net rate of loss of u without predation and $k_w v(c - dM_v v^2)$ is the generalized logistic the net rate of growth of w without predation, [11]. In this model, intra- specific cooperation has the cooperative parameters $S_u v$ and $S_v u$. A Lotka-Voltera model can simply be constructed from this model when $n_u = n_v = 0$ in (7).

3. Dimensionless variables

Numerous dimensionless variables that apply to both a full-size object and its scaled counterpart can be found in the theory of scale modelling. As long as all of the dimensionless variables are the same for the full-scale object and the measured model, the physical behaviour of the measured model closely resembles the behaviour of the prototype. In actuality, maintaining equality between all the non-dimensional characteristics of the full-scale object and the measured model can often present challenges.[7]

We define dimensionless variables

$$u = U\bar{u} \qquad x = \left(\frac{D_u}{P_u}\right)^{\frac{1}{2}}\bar{x}$$
$$v = V\bar{v} \quad \text{and} \quad t = \frac{\bar{t}}{P_u}$$

Then

$$\frac{\partial u}{\partial t} = \frac{\partial(U\bar{u})}{\partial t} = U\frac{\partial \bar{u}}{\partial \bar{t}}\frac{\partial \bar{t}}{\partial t} = k_u U\frac{\partial \bar{u}}{\partial \bar{t}},$$
$$\frac{\partial u}{\partial x} = \frac{\partial(U\bar{u})}{\partial x} = U\frac{\partial \bar{u}}{\partial \bar{x}}\frac{\partial \bar{x}}{\partial x} = \left(\frac{k_u}{D_u}\right)^{\frac{1}{2}} U\frac{\partial \bar{u}}{\partial \bar{x}},$$
$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2(U\bar{u})}{\partial x^2} = \frac{k_u}{D_u}U\frac{\partial^2 \bar{u}}{\partial \bar{x}^2} \quad ,$$

and

$$\frac{\partial v}{\partial t} = \frac{\partial(V\bar{v})}{\partial t} = V\frac{\partial \bar{v}}{\partial \bar{t}}\frac{\partial \bar{t}}{\partial t} = k_u V\frac{\partial \bar{v}}{\partial \bar{t}},$$

$$\frac{\partial v}{\partial x} = \frac{\partial(V\bar{v})}{\partial x} = V\frac{\partial \bar{v}}{\partial \bar{x}}\frac{\partial \bar{x}}{\partial x} = \left(\frac{k_u}{D_u}\right)^{\frac{1}{2}} V\frac{\partial \bar{v}}{\partial \bar{x}} \ ,$$

$$\frac{\partial^2 v}{\partial x^2} = \frac{\partial^2(V\bar{v})}{\partial x^2} = \frac{k_u}{D_u}V\frac{\partial^2 \bar{v}}{\partial \bar{x}^2} \ ,$$

Substituted these values in equation (7), we get

$$\frac{\partial \bar{u}}{\partial \bar{t}} = \frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + \bar{u}(-a + bk_u\bar{u}^2U^2 + S_u\bar{v}V)$$
$$\frac{\partial \bar{v}}{\partial \bar{t}} = (\frac{D_v}{D_u})\frac{\partial^2 \bar{v}}{\partial \bar{x}^2} + \frac{P_v}{P_u}\bar{v}(c - dk_v\bar{v}^2V^2 - S_v\bar{u}U). \qquad (8)$$

The dimensionless parameters are $\gamma_1 = S_uV$, $\gamma_2 = S_vU, \beta_1 = bk_uU^2$, $\lambda = \frac{P_v}{P_u}$, $\frac{D}{\lambda} = \frac{D_v}{D_u}$

& $\beta_2 = dk_vV^2$. Substituted these values in equation (8), we get

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + u(-a + \beta_1 u^2 + \gamma_1 v)$$
$$\frac{\partial v}{\partial t} = \frac{D}{\lambda}\frac{\partial^2 v}{\partial x^2} + \lambda v(c - \beta_2 v^2 - \gamma_2 u) \qquad (9)$$

4. Symmetry Li group analysis of the system

This section describes the basic terms used to obtain the infinitesimals of the equation (8). We consider a single-parameter Li set of infinite transformations that operate on independent and dependent variables, we consider the following Lie group of changes with dependent variables u, v and independent variables x, t.

$$\bar{x} = \bar{x}(x,t,u;\varepsilon), \qquad \bar{t} = \bar{t}(x,t,u;\varepsilon),$$
$$\bar{u} = \bar{u}(x,t,u;\varepsilon), \qquad \bar{v} = \bar{v}(x,t,u;\varepsilon) \qquad (10)$$

Which ε a sign of the group is the parameter. The infinitesimal Lie group generator (4) in formula (3) can be extracted as follows

$$X = \xi^x\frac{\partial}{\partial x} + \xi^t\frac{\partial}{\partial t} + \eta^u\frac{\partial}{\partial u} + \eta^v\frac{\partial}{\partial w} \quad , \qquad (11)$$

in which ξ^x, ξ^t, η^u *and* η^v of group variables dependent and independent are called infinitesimal functions.

$$\xi^x = \xi^x(x,t)\,, \xi^t = \xi^t(t)\,, \eta^u = u\eta_1(x,t)\,, \eta^v = v\eta_2(x,t)$$

Further, Lie symmetry of eq. (8) will be generated by eq. (11), since the governing equations involve partial derivatives of the second order at most. Because of formula (4), the infinitesimal generator's prolongation, which includes the associated terms, is given by the following form :

$$pr^2X = X + \eta_x^u \frac{\partial}{\partial u_x} + \eta_t^u \frac{\partial}{\partial u_t} + \eta_{xx}^u \frac{\partial}{\partial u_{xx}} + \eta_x^v \frac{\partial}{\partial w_x} + \eta_t^v \frac{\partial}{\partial w_t} + \eta_{xx}^v \frac{\partial}{\partial w_{xx}} \quad . \qquad (12)$$

From equation (7), we get

$$(\eta_t^u - (-a + 3\beta_1 u^2 + \gamma_1 v)\eta^u - \gamma_1 u \eta^v - \eta_{xx}^u)|_{eq(*)} = 0$$

$$(\eta_t^v - \lambda(c - 3\beta_2 v^2 - \gamma_2 u)\eta^v + \gamma_2 v \eta^u - \frac{D}{\lambda}\eta_{xx}^v)|_{eq(**)} = 0 \qquad (13)$$

,where $\xi^x = \xi^x(x,t), \xi^t = \xi^t(t)$, $\eta^u = u\eta_1(x,t)$, $\eta^v = v\eta_2(x,t)$

by equation (5) ,we get

$\eta_x^u = D_x(\eta^u) - u_x D_x(\xi^x) - u_t D_x(\xi^t),$

$\eta_t^u = D_t(\eta^u) - u_x D_t(\xi^x) - u_t D_t(\xi^t),$

$\eta_{xx}^u = D_x(\eta_x^u) - u_{xx} D_x(\xi^x) - u_{tx} D_x(\xi^t),$

$\eta_x^v = D_x(\eta^v) - u_x D_x(\xi^x) - u_t D_x(\xi^t),$

$\eta_t^v = D_t(\eta^v) - u_x D_t(\xi^x) - u_t D_t(\xi^t),$

$\eta_{xx}^v = D_x(\eta_x^v) - u_{xx} D_x(\xi^x) - u_{tx} D_x(\xi^t),$

Where, $D_x = \frac{\partial}{\partial x} + u_x \frac{\partial}{\partial u}$ *and* $D_t = \frac{\partial}{\partial t}$,

This system substitute of equation (13), we get

$$u_t\eta_1 + u{\eta_1}_t - u_x\xi_t^x - u_t(\xi^t)' - (-a + 3\beta_1 u^2 + \gamma_1 v)u\eta_1 - \gamma_1 uv\eta_2 - (2u_{xx}\eta_1 + 3u_x{\eta_1}_x + u{\eta_1}_{xx} - 2u_{xx}\xi_x^x - u_x\xi_{xx}^x) = 0 \, ,$$

$$\left.\begin{array}{l} \eta_1 - (\xi^t)' = 0 \\ -\xi_t^x - 3\eta_{1_x} + \xi_{xx}^x = 0 \\ -2\eta_1 + 2\xi_x^x = 0 \\ \eta_{1_t} + a\eta_1 - \eta_{1_{xx}} = 0 \\ -3\beta_1\eta_1 = 0 \\ -\gamma_1\eta_1 - \gamma_1\eta_2 = 0 \end{array}\right\} \qquad (14)$$

and

$$v_t\eta_2 + v\eta_{2_t} - v_x\xi_t^x - v_t(\xi^t)' - \lambda(c - 3\beta_2 v^2 + \gamma_2 u)v\eta_2 - \gamma_2 vu\eta_1 - \frac{D}{\lambda}(2v_{xx}\eta_2 + 3v_x\eta_{2_x} + v\eta_{2_{xx}} - 2v_{xx}\xi_x^x - v_x\xi_{xx}^x) = 0$$

$$\left.\begin{array}{l} \eta_2 - (\xi^t)' = 0 \\ -\xi_t^x - \frac{3D}{\lambda}\eta_{2_x} + \frac{D}{\lambda}\xi_{xx}^x = 0 \\ -\frac{2D}{\lambda}\eta_2 + \frac{2D}{\lambda}\xi_x^x = 0 \\ \eta_{2_t} - \lambda c\eta_2 - \frac{D}{\lambda}\eta_{2_{xx}} = 0 \\ 3\lambda\beta_2\eta_2 = 0 \\ -\lambda\gamma_2\eta_2 - \gamma_2\eta_1 = 0 \end{array}\right\} \qquad (15)$$

By system (14) and (15), we obtain

$\eta_1 = 0$ *and* $\eta_2 = 0$, $\eta^u = 0$ *and* $\eta^v = 0$

Then $\xi_x^x = 0$and by integration $\xi^x = c_1$ and $(\xi^t)' = 0$ then by integration $\xi^t = c_2$

Therefore the system of nonlinear Reaction-diffusion equation has only a two parameter symmetry lie group,[4]

$$X = c_1\frac{\partial}{\partial x} + c_2\frac{\partial}{\partial t}\ , \qquad (16)$$

A Characteristic equation based on the symmetry groups must be written to construct the reduced form of the PDE system (16)

$$\frac{dx}{c_1} = \frac{dt}{c_2} = \frac{du}{0} = \frac{dv}{0} \qquad (17)$$

Then $d\xi = c_2 dx - c_1 dt$ or $d\xi = 0$

From the solution of the characteristic equation we have

$$\xi = c_2 x - c_1 t$$

$$u(x,t) = U(\xi) \ \ and \qquad v(x,t) = V(\xi)$$

$$\frac{\partial u}{\partial t} = -c_1 \frac{dU}{d\xi}, \quad \frac{\partial v}{\partial t} = -c_1 \frac{dV}{d\xi} \quad , \frac{\partial^2 u}{\partial x^2} = c_2^2 \frac{d^2 U}{d\xi^2} \ \ and \ \ \frac{\partial^2 v}{\partial x^2} = c_2^2 \frac{d^2 V}{d\xi^2} \qquad (18)$$

In which ξ is the similarity independent variable, $U(\xi)$ and $V\ (\xi)$ are the similarity dependent variables.

Substitute equation (18) in equation (9), and we get

$$c_2^2 \frac{d^2 U}{d\xi^2} + c_1 \frac{dU}{d\xi} + U(-a + \beta_1 U^2 + \gamma_1 V) = 0$$

$$\frac{Dc_2^2}{\lambda} \frac{d^2 V}{d\xi^2} + c_1 V' + \lambda V(c - \beta_2 V^2 - \gamma_2 U) = 0 \qquad (19)$$

Let's take into consideration the solutions of a system of ordinary differential equations (19) in the following forms in order to solve the reduced nonlinear ODE's (19) by the generalized tanh function method :

$$U(\xi) = F(Y) = \sum_{i=0}^{n} a_i Y \ \ and \ V(\xi) = G(Y) = \sum_{i=0}^{m} b_i Y \qquad (20)$$

Where a_i, b_i are parameters and n and m are integers to be determined and F is the solution of the equation

$$\left.\begin{aligned} \frac{d}{d\xi} &= (1 - Y^2)\frac{d}{dY} \\ \frac{d^2}{d\xi^2} &= -2Y(1 - Y^2)\frac{d}{dY} + (1 - Y^2)^2 \frac{d^2}{dY^2} \\ \frac{d^3}{d\xi^3} &= 2(1 - Y^2)(3Y^2 - 1)\frac{d}{dY} \\ &-6Y(1 - Y^2)^2 \frac{d^2}{dY^2} + (1 - Y^2)^3 \frac{d^3}{dY^3} \end{aligned}\right\} \qquad (21)$$

We introduce $Y = \tanh(\xi)$ and substitute equation (21) by

$$\left.\begin{array}{c}-2c_2^2Y(1-Y^2)\dfrac{dF(Y)}{dY}+(1-Y^2)^2\dfrac{d^2F(Y)}{dY^2}\\ +c_1(1-Y^2)\dfrac{dF(Y)}{dY}+F(Y)(-a+\beta_1(F(Y))^2+\gamma_1G(Y))=0\\ -\dfrac{2Dc_2^2}{\lambda}Y(1-Y^2)\dfrac{dG(Y)}{dY}+(1-Y^2)^2\dfrac{d^2G(Y)}{dY^2}\\ +c_1(1-Y^2)\dfrac{dG(Y)}{dY}+\lambda G(Y)(c-\beta_2(G(Y))^2-\gamma_2F(Y))=0\end{array}\right\} \quad (22)$$

Now, to determine the parameters n and m, we balance the highest power terms in $F(\xi)$to determine the parameter $(n)$and (m), we balance The linear term of highest–order onlinear terms of (22), in the equation (22) The balance $\frac{d^2U}{d\xi^2}=\frac{d^2F(\xi)}{d(\xi)^2}$ and $\frac{d^2V}{d\xi^2}=\frac{d^2G(\xi)}{d(\xi)^2}$, we get

$n+2=3n\Rightarrow n=1$ and $m+2=3m\Rightarrow m=1$

Now, the Tanh method admits the use of the finite expansion for:

$$\left.\begin{array}{c}u(x,t)=F(Y)=a_0+a_1Y\\ v(x,t)=G(Y)=b_0+b_1Y\end{array}\right\} \quad (23)$$

where $a_1\neq 0$ and $b_1\neq 0$, Find this term by equation (22) and (23), we get

$$\left.\begin{array}{c}-2c_2^2Y(1-Y^2)a_1+(1-Y^2)^2(0)\\ +c_1(1-Y^2)a_1+(a_0+a_1Y)\\ (-a+\beta_1(a_0+a_1Y)^2+\gamma_1(b_0+b_1Y))=0\\ -\dfrac{2Dc_2^2}{\lambda}b_1Y(1-Y^2)+(1-Y^2)^2(0)\\ +b_1c_1(1-Y^2)+\lambda(b_0+b_1Y)\\ (c-\beta_2(b_0+b_1Y)^2-\gamma_2(a_0+a_1Y))=0\end{array}\right\} \quad (24)$$

$$(c_1a_1-aa_0-a_0^3\beta_1+a_0\gamma_1b_0)Y^0+(2a_1c_2^2-2a_0^2a_1\beta_1-aa_1-a_0\gamma_1b_1+\gamma_1a_1b_0)Y^1 \\ +(\gamma_1a_1b_1-3a_0a_1^2\beta_1-c_1a_1)Y^2+(2a_1c_2^2-a_1^3\beta_1)Y^3=0 \quad (25)$$

$$(c_1 b_1 + c b_0 - b_0^3 \beta_2 + b_0 \gamma_2 a_0) Y^0 + \left(\frac{2 b_1 c_2^2 D}{\lambda} - 2 b_0^2 b_1 \beta_2 + c b_1 - b_0 \gamma_2 a_1 + \gamma_2 b_1 a_0\right) Y^1 + (\gamma_2 a_1 b_1 - 3 b_0 b_1^2 \beta_2 - c_1 b_1) Y^2 + \left(\frac{2 b_1 c_2^2 D}{\lambda} - b_1^3 \beta_2\right) Y^3 = 0 \qquad (26)$$

Then this leads to:

$$\left.\begin{array}{l} Y^0: (c_1 a_1 - a a_0 - a_0^3 \beta_1 + a_0 \gamma_1 b_0) = 0 \\ Y^1: (2 a_1 c_2^2 - 2 a_0^2 a_1 \beta_1 - a a_1 - a_0 \gamma_1 b_1 + \gamma_1 a_1 b_0) = 0 \\ Y^2: (\gamma_1 a_1 b_1 - 3 a_0 a_1^2 \beta_1 - c_1 a_1) = 0 \\ Y^3: (2 a_1 c_2^2 - a_1^3 \beta_1) = 0 \end{array}\right\} \qquad (27)$$

and

$$\left.\begin{array}{l} Y^0: (c_1 b_1 + c b_0 - b_0^3 \beta_2 + b_0 \gamma_2 a_0) = 0 \\ Y^1: \left(\frac{2 b_1 c_2^2 D}{\lambda} - 2 b_0^2 b_1 \beta_2 + c b_1 - b_0 \gamma_2 a_1 + \gamma_2 b_1 a_0\right) = 0 \\ Y^2: (\gamma_2 a_1 b_1 - 3 b_0 b_1^2 \beta_2 - c_1 b_1) = 0 \\ Y^3: \left(\frac{2 b_1 c_2^2 D}{\lambda} - b_1^3 \beta_2\right) = 0 \end{array}\right\} \qquad (28)$$

From solving the system of algebraic equation (27) and (28), we obtain

$$\left.\begin{array}{l} a_1 = \pm\sqrt{\frac{2}{\beta_1}}|c_2| \\ b_1 = \pm\sqrt{\frac{2D}{\beta_2 \lambda}}|c_2| \\ a_0 = \pm\frac{2\gamma_1 c_2^2 \sqrt{\frac{D}{\beta_1 \beta_2 \lambda}} + c_1 \sqrt{\frac{2}{\beta_1}}|c_2|}{6 c_2^2} \\ b_0 = \pm\frac{2\gamma_2 c_2^2 \sqrt{\frac{D}{\beta_1 \beta_2 \lambda}} + c_1 \sqrt{\frac{2D}{\beta_2 \lambda}}|c_2|}{6 \frac{D}{\lambda} c_2^2} \end{array}\right\} \qquad (29)$$

Finally, we find the solution in the form

$$u(x,t) = \pm \frac{2\gamma_1 c_2^2 \sqrt{\frac{D}{\beta_1 \beta_2 \lambda}} + c_1 \sqrt{\frac{2}{\beta_1}} |c_2|}{6c_2^2} \pm \sqrt{\frac{2}{\beta_1}} |c_2| \ \tanh(c_2 x - c_1 t)$$

$$v(x,t) = \pm \frac{2\gamma_2 c_2^2 \sqrt{\frac{D}{\beta_1 \beta_2 \lambda}} + c_1 \sqrt{\frac{2D}{\beta_2 \lambda}} |c_2|}{6\frac{D}{\lambda} c_2^2} \pm \sqrt{\frac{2D}{\beta_2 \lambda}} |c_2| \ \tanh(c_2 x - c_1 t)$$

5. Conclusions:

In this paper, we study a reaction-diffusion system (7), this model represents an interaction between two species in Ecology, which can be defined by predations. Symmetry Lie group solutions for this model are found using Tanh methods. The focus on the nonlinearity of global logistic growth was in the second order, where the system’s first request goes back to the Lotka-Volterra model. Analytical methods are sufficient to find the Symmetry Lie group for this model.

6. References

[1] Donald J. , Victor G., Ropp D., Shadid J., and Tavener S. (2008).An a posteriori - a priori analysis of multiscale operator splitting, SIAM J Numer Anal, Vol.46, pp.1116–1146.

[2] J. Billingham. (2001). Phase plane analysis of onedimensional reaction diffusion waves with degenerate reaction terms. Dynamics and Stability of Systems, vol. 15, 23-33.

[3] J.F. AL-Omari and S. A. Gourley.(2003).Stability And traveling Fronts In Lotka-Voltera Competition Models With Stage Structure}. J. Appl. Math, vol. 63, 2063-2086.

[4] K. Hardler and F. Rothe. (1975). Travelling fronts in nonlinear diffusion equations. Math. Biol., vol. 2, 251-263.

[5] Mira-Cristiana Anisiu. (2014). LOTKA, VOLTERRA AND THEIR MODEL, DIDACTICA MATHEMATICA, Vol. 32, pp. 9–17.

[6] N. F. Britton. (1990). Spatial structures and Periodic travelling waves in an integro-differential reaction-diffusion population model}. Siam Journal on Applied Mathematics, vol.50, No.6, 1663-1688.

[7] Peter O. Cervenka and Lou Massa.(1995). Applications of Dimensionless Variables to Scaling in the Infrared, CARDIVNSWC-TR-95/002.

[8] Qing. N., Frederic YM. W., Yong- Tau Z., and Xin- Feng L.. (2008). Compact integration factor methods in high spatial dimensions, J. Computer Phys, Vol. 227, pp.5238–5255.

[9] S. A. Gourley. (2005). Two-Species Competition With High Dispersal: The Winning Strategy. Mathematical Biosciences And Engineering, vol.2, No.2, 345-362.

[10] Shaker M. Rasheed and Hajar F. Ismail. (2014). Travelling wave solutions of a Reaction-Diffusion System: Slow Reaction and Slow Diffusion, IOSR Journal of Engineering, Vol. 04, PP 40-45.

[11] Vasyl' Davydovych.(2018).Lie Symmetry of the Diffusive Lotka–Volterra System with Time-Dependent Coefficients, National Academy of Sciences of Ukraine.

[12] Y. Hosono. (2003). Traveling Waves For A Diffusive Lotka-VolteraCompettion Model I: Singular Perturbations. Discrete And Continous Dynamical System s-Series B, vol. 3, 97-95.

Article submitted 1 March 2023. Accepted at 29 March

Published at 30 Jun 2023.