γpg^{**}- Closed Set in Topological Spaces

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Abstract
In this paper we investigate the definitions of g-closed sets, gp-closed sets, pg-closed sets, gsp-closed sets, gα-closed sets, αg-closed sets. we introduced a new class of set called γpg^{**}-closed sets which is settled properly in between the class of semi-closed and the class of g^{**}-closed sets.

Keywords: gp-closed sets, pg-closed sets, γpg^{*}-closed set**,** γpg^{**}-closed set.

1. Introduction

Levine [1] introduced the class of g-closed set in 1970. H. Maki.K. Balachandran [2] and R. Devi defined gs-closed set in 1996. Which were used for characterizing s-normal space and Dontchev [3] introduced gsp-closed set respectively. We introduce a new class of set called γpg^{**}-closed sets. Which properly placed in between the class of closed sets and the class of g^{*}-closed sets. We also showed that this new class is properly contained in the class of gs-closed sets, gsp-closed sets, γpg^{*}-closed set.

Levine [1]. In 2021, Ali. Al kazaragy, Faik. Mayah and Ali Khala Hussain Al-Hachami [4] introduced semi-open sets and pre-open sets respectively. Marwah Munther Hassan and Ali Khalaf Hussain [5] called semi pre –generalized closed sets as semi-preopen sets. H. Maki [2] and p.Bhattacharya [6] and B.Lahiri introduced and studiedgs-closed sets. M.

Paulin Mary Helen. Ponnuthaiselvarani and Veronica vijayan [7] introduced and studied g^{**}-closed sets .

2. Preliminaries

Definition 2.1

A subsets A of a space (X, τ) is referred to as
1) pre – open [8] if A ⊆ int (cl(A))
2) Semi – open [9] if A ⊆ cl (int (A))
3) Semi – preopen [5] if A ⊆ cl (int(cl(A))
Preclosed [8] (resp - semiclosed [9]) semipreclosed [5] is a space (X, τ) preopen (resp. semiopen, semipreopen) sets complement.

Definition 2.2

Let (X, τ) be a topological space. A subset B of the space X is called
1) A generalized closed set (brifly, g - closed) [1] if cl(B) ⊆ V, every time B ⊆ V and V is open in (X, τ).
2) A semi – generalized closed set (brifly, sg – closed) [6] if scl(B) ⊆ V, every time B ⊆ V and V is semi – open in (X, τ) .
3) A generalized semi- closed set (brifly, gs – closed) [10] if Scl(B) ⊆ V, every time B ⊆ V and V is open in (X, τ).
4) generalized α – closed (brifly, gα – closed) [11] if $\alpha -$ cl(B) ⊆ V, every time B ⊆ V and V are α – open . or equvelent, if B was g – closed in relation to α(X).
5) A generalized semi - preclosed set (brifly, gsp – closed) [3] if Spcl (B) ⊆ V, every time B ⊆ V and V is open in (X, τ) .
6)*A* α - generalized closed set (brifly, αg – closed) [3] if α – cl(B) ⊆ V, every time B ⊆ V and V is open in (X, τ) .
7) A regular generalized closed set (brifly, rg–closed) [12] if cl(B) ⊆ V, every time B ⊆ V and V is regular open in (X, τ) .
8)A γ- generalized closed set (brifly , γg – closed) [13] if γ– cl(B) ⊆ V, every time B ⊆ V and V is open in (X, τ) .

9)A γ- generalized regular closed set (brifly, γgr – closed) [13] if γ– cl(B) ⊆ V, every time B ⊆ V and V is regular open in (X, τ) .

3. Basic properties of γpg^{**} - closed sets

We introduce the following defintions

Definition 3.1[11]

A subset A of a topological space (X, τ) is called generalized pre-closed set (briefly: gp-closed) if $pcl(A) \subseteq V$ whenever $A \subseteq V$ and V is open set in X .

Example 3.2

Let X= $\{1,2,3\}$, $\tau = \{\emptyset, X, \{1\}, \{1,2\}\}$ be a topology in X
And let A $= \{1,3\}$, it is clear $pcl(A) = X$
The only open sets containing A is X, and It also contains $pcl(A)$
So A $= \{1,3\}$ is a gp-closed set
Hence $A^c = \{2\}$ is a gp-open set.

Proposition 3.3[14]

Each closed set is a gp-closed set
But the convers of (**proposition 3.3**) is false. In gerenal as on the next as an example.

Example 3.4

Let X= $\{1,2,3\}$, $\tau = \{\emptyset, X, \{1\}, \{1,2\}\}$ be a topology define in X
And let $A = \{1,3\}$, $B = \{2\}$
So A is a gp-closed set but it is not closed set
And B is a gp-open set but it is not open set.

Proposition 3.5[14]

Each g-closed set is a gp-closed set.
But the convers of (**proposition 3.5**) is false. In general as on the next as an example.

Example 3.6

Let X $= \{1,2,3\}$, $\tau = \{X, \emptyset, \{1,3\}\}$ be a topology defined in X
Let A $= \{1\}$, it is clear that $pcl(A) = \{1\}$, the open sets that contain A are : X, $\{1,3\}$ and it is also contain $pcl(A)$

So B is a gp-closed set but it is not g-closed set because $\overline{(B)} = X$

Hence that $A \subseteq \{1,3\}$ but $\overline{(A)} = X \nsubseteq \{1,3\}$.

Definition 3.7[11]

A subset A of a topological space (X, τ) is called a pre generalized-closed set (briefly: pg-open) if $pcl(A) \subseteq V$ whenever $A \subseteq V$ and V is a pre-open set in X.

Example 3.8

Let X $= \{1,2,3\}$, $\tau = \{\emptyset, X, \{1\}, \{2,3\}\}$ be a topology define in X

And let $A = \{3\}$, it is clear that $pcl(A) = \{3\}$

The pre-open sets containing A are $X, \{2,3\}, \{1,3\}, \{3\}$, and it also contains a $pcl(A)$.

So $A = \{3\}$ is a pg-closed set

Hence $A^c = \{1,2\}$ is a pg-open set.

Proposition 3.9[14]

Every closed set is a pg-closed set.

But the convers of (**proposition 3.9**) is false. In general, as on the next as an example.

Example 3.10

Let X = $\{1,2,3,4\}$, $\tau = \{X, \emptyset, \{2\}, \{1,2,3\}\}$ be a topology defined X

And let $A = \{3\}$, so $pcl(A) = \{3\}$

The p-open sets containing A is: $X, \{1,2,3\}, \{2,3\}, \{2,3,4\}$, and it is also contained $pcl(A)$

So A is a pg-closed set but it is not closed set.

Remark 3.11[13]

The union of two pg-closed sets is not necessary to be a pg-closed set, as shown by the following example .

Example 3.12

Let $X = \{1,2,3,4\}$, $\tau = \{ X, \emptyset, \{1,2,3\}\}$ be a topology defined in X

And let $A = \{1,2\}$, $B = \{2,3\}$

So $X, \{1,2,3\}$ is a pre-open set which contain each of A and B and contain $pcl(A)$ and $pcl(B)$

Hence that each of A and B are a pg-closed set

And since $A \cup B = \{1,2,3\}$, so $pcl(A \cup B) = X$

Hence $A \cup B \subseteq \{1,2,3\}$ and $pcl(\ A \cup B) \nsubseteq \{1,2,3\}$
So $A \cup B$ is not pg-closed set.

Definition 3.13

A subset A of a topological space (X, τ) is said to be a γpg*-closed set if γpcl(A) ⊆ V whenever A ⊆ V and V is g-open set in (X, τ).

Definition 3.14

A subset A of a topological space (X, τ) is said to be a γpg**-closed set if γpcl(A) ⊆ V whenever A ⊆ V and V is g*-open set in (X, τ) .

Theorem 3.15

Each closed set is γpg**-closed set.

Proof:

Let A be any closed and G be any g*-open set containing A in
(X, τ).
Since A is closed.
Cl(A) = A, so cl(A) ⊆ V.
Hence A is γpg**-closed in (X, τ).
the convers of (**Theorem 3.15**) is false.In gerenal as on the next as an example shows .

Example 3.16

Let X = {1, 2, 3}, $\tau = \{X, \emptyset, \{1\}, \{1, 2\}\}$.
Then the set A ={1, 3} is γpg**-closed set but not closed set in (X, τ) .

Proposition 3.17[15]

Each pg-closed set is a gp-closed set.
But the convers of (**Proposition 3.17**) is false. In general as on the next as an example.

Example 3.18

Let X= {1,2,3,4,5} and let $\tau = \{X, \emptyset, \{1\}, \{3,4\}, \{1,3,4\}\}$ be a topology defined on
And let $A = \{2\}$, so $pcl(A) = \{2,5\}$
The only p-open sets containing A is X and it is also contained $pcl(A)$

So A is a gp-closed set but it is not pg-closed set since the p-open set containing A are: $\{1,2,4\},\{1,2,4,5\},\{1,2,3,4\},\{1,2,3,5\},\{1,2\},\{1,2,3\}$ and it is not contained $pcl(A)$.

Proposition 3.19[17]

Each gp-closed set is gsp-closed.

the convers of (**Theorem 3.19**) is false. In general, as on the next as an example shows

Example 3.20

Let X= $\{1,2,3\}$, $\tau = \{X, \emptyset, \{1,2\}\}$ be a topology defined in **X**

And let $A = \{2\}$, so $spcl(A) = \{2\}$

The only open sets containing A is $\{1,2\}$ and it is also contained $spcl(A)$

So A is a gsp-closed set but it is not gp-closed set since $A \subseteq \{1,2\}$

But $\overline{(A)} = X \not\subset \{1,2\}$

Proposition 3.21

Every γpg**-closed set is gsp-closed but not conversely.

Example 3.22

Let X= $\{1,2,3\}$, $\tau = \{X, \emptyset, \{1\}, \{2\}, \{1,2\}\}$ be a topology defined in X.

And let A= $\{1, 2\}$ is a gsp - closed set but it is not γpg**-closed set of (X, τ)

Remark 3.23[18]

Every gp-closed set is gpr-closed but not conversely.

Proposition 3.24

Each γpg**- closed set is gpr- closed but not conversel .

Example 3.25

Let X= $\{1,2,3\}$, $\tau = \{X, \emptyset, \{1\}\}$ be a topology defined in X.

And let A= $\{1\}$ is gpr- closed set but it is not γpg**-closed set

Proposition 3.26

Each γpg**- closed set is gp- closed but not conversely

Theorem 3.27

Every $\alpha - closed\ set$ is γpg**-closed set.

Proof:

Let A be $\alpha - closed$ and G be g*-open set in (X,τ) $such\ that$
$A \subseteq G$.
Since A is $\alpha - closed,\ \alpha cl(A) = A$.
But γpcl(A) ⊆ $\alpha cl(A)\ is\ always\ true$.
Thuse γpcl(A) ⊆ G .
Hence A is γpg**- closed set in (X, τ) .
the convers of (**Theorem 3.27**) is false. In general, as on the next as an example shows.

Example 3.28

Let X = {1, 2, 3}, $\tau = \{X, \emptyset, \{1\}, \{1, 2\}\}$.
Then the set A = {2} is γpg**- closed set but not $\alpha -$ closed set in (X, τ).

Proposition 3.29

If A is a γpg** - closed set of (X, τ) then γpcl(A) / A does not contain any non – empty γpg*- closed set.

Proof:

Let M be a g* - closed set of (X, τ) such that $M \subseteq \gamma p$cl(A) / A.
Then $A \subseteq$ X / M.
Since A is γpg** - closed and X / M is γpg*- open, γpcl(A) ⊆ X / M.
This implies M ⊆ X / γpCl(A).
So, M ⊆ ($\mathcal{X}$/ γpCl(A)) ∩ (γpCl(A) /A) ⊆ (X /γpCl(A)) ∩ γpCl(A) = ∅ .
Therefore M = ∅ .
Hence γpcl(A) /A does not contain any non - empty γpg* - closed set.

Proposition 3.30

If A is γpg**- closed set of (X, τ) such that A ⊆ B ⊆ γpcl(A), Then B is also a γpg**- closed set of (X, τ) .

Proof:

Let V be γpg*- open set of (X, τ) such that B ⊆ V.
Then A ⊆ V where V is γpg*-open.
Since A is γpg**- closed, γpcl(A) ⊆ V.
Then γpcl(B) ⊆ γpcl (γpcl(A)) = γpcl(A) ⊆ V.
Thus, B would be γpg** - closed.

Theorem 3.31

Let A⊆ $Y \subseteq X$ and assume that Is the a γpg**-closed set of (X, τ). Then, A is γpg**-closed in relative to Y.

Proof:

Let A ⊆ $Y \cap G$, where G is g*- open.

Therefore A ⊆ G as a result, γpcl (A) ⊆ G .

This suggests that $Y \cap \gamma pcl(A) \subseteq Y \cap G$.

A is thus γpg**-closed relative to Y.

Theorem 3.32

If A be γpg**-closed in (X, τ). Then A is preclosed if and only if γpcl(A) - A is g*-closed.

Proof:

Assume A is preclosed.

As a result, γpcl(A) = A, and γpcl(A) - A = ∅, which is g*-closed.

Conversely:

Let's say that γpcl(A) - A is g*-closed.

Since A is γpg**- closed, γpcl(A) -A = ∅.

That would be, γpcl(A) = A or A is preclosed.

Definition 3.33

The subset A in X is referred to as γpg**-open in (X, τ) if X - A is γpg**-closed in (X, τ).

Theorem 3.34

The set A is γpg**- open in (X, τ) if and only when F ⊆ γpint(A) everytime F was g*-closed in (X, τ) when F ⊆ A.

Proof:

If F was g*-closed & F ⊆A, then let's say that F ⊆ γ pint(A).

Assume X- A⊆ G when G is g*-open to (X, τ).

Consequently G ⊆ X$-G$ and X – G ⊆ $\gamma pint(A)$.

And hence $\mathcal{X}$ – A is γpg**- closed in (X, τ) .

Therefore, A is γpg**- open in (X, τ).

Conversely:

Assume that A is γpg**-open, F ⊆ A, & F becomes g*-closed on (X, τ).

As a result, $\mathcal{X}$ - F is g*-open while X - A ⊆ X- F.

Consequently, γpcl(X- A) ⊆ X- F.

Nevertheless, γpcl(X - A) = X- pInt (A)

And therefore, F ⊆ γpint(A)

Theorem 3.35

A subset A is γpg** - open in (X, τ) if and only if G = X everytime G is g* - open when γpint(A) ∪ (X- G) ⊆ G.

Proof:

Assume that A is γpg**-open, G is g*-open, and that γpInt(A) ∪ (X -A) ⊆ G.

The result is X - G ⊆ (X - γpint(A)) ∩ (X -A) = X- γpint(A) - (X- A) = γpcl(X-A)- (X-A).

because X -A is γpg**-closed and X-G is g*-closed.

In light of Theorem (3.34), it is evident that X– G = ∅ .

As a result, X = G.

Conversely:

Assume that F is g*-closed and that F ⊆ A.

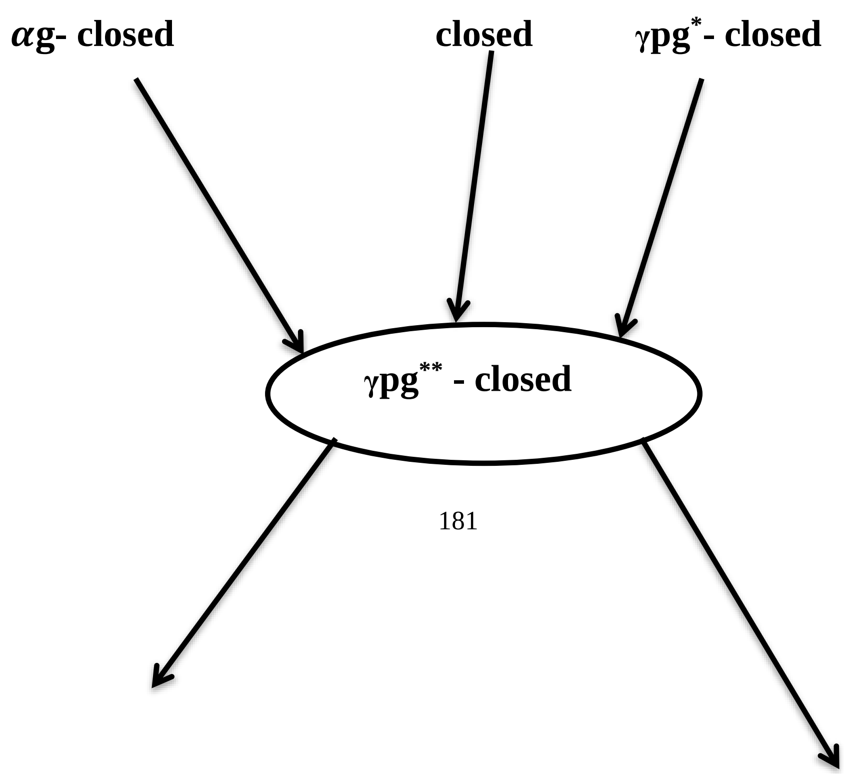
Since (X - A) ∪ γpInt(A) ⊆ γpint(A) ∪ (X-F).

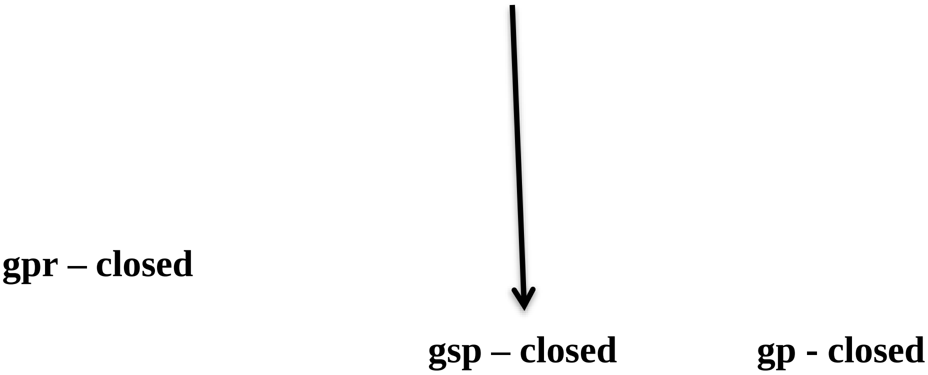
Consequently, γpint(A) ∪ (X - F) = X .

Therefore, F⊆int(A).

Hence, A is γpg**-open in (X, τ).

The above results can be represented in the following figure .





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