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Total Italian Domination: Structural Results on Middle Graphs of Standard Families

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ABSTRACT: Let G=(V,E)G = (V, E)G=(V,E) be a graph. An Italian dominating function (IDF) is defined as a mapping f:V→{0,1,2}f: V \rightarrow \{0, 1, 2\}f:V→{0,1,2} satisfying the condition that, for any vertex mmm with f(m)=0f(m) = 0f(m)=0, there exists either at least one neighbor vvv with f(v)=2f(v) = 2f(v)=2 or at least two neighbors n,sn, sn,s with f(n)=f(s)=1f(n) = f(s) = 1f(n)=f(s)=1. The weight of an IDF is the sum of the function values over all vertices, and the minimum weight among all IDFs of GGG is denoted by γI(G)\gamma_I(G)γI(G).
A total Italian dominating function (TIDF) is an IDF in which every non-zero vertex is adjacent to at least one other non-zero vertex. The minimum weight of any TIDF is referred to as the total Italian domination number, denoted by γtI(G)\gamma_{tI}(G)γtI(G).
In this study, we investigate TIDFs for a graph GGG and its middle graph M(G)M(G)M(G), obtained by inserting a new vertex on each edge of GGG and connecting two new vertices if they share a common adjacent vertex in GGG. For selected standard families of graphs, we determine the exact values of these domination parameters.

Keywords: Italian domination, total Italian domination, Middle graph.

1. INTRODUCTION

The significant expansion of domination theory over the past three decades can be attributed to its diverse applications in both theoretical contexts and practical problems, including urban defense strategies, facility location problems, and more. Several domination-type parameters have been explored in graph theory, including Roman domination [5,11], total Roman domination [2,3], restrained Roman domination [10], Italian domination [4,7], and restrained Italian domination [8,12].

The concept of Roman domination originated from Ian Stewart's article "Defend the Roman Empire!" [11] And was further developed by Cockayne et al. [5]. In this context, the values 1 and 2 represent the number of Roman guards stationed on a particular vertex vvv. A neighboring vertex mmm is considered unsecured if it has no guard assigned. Such a vertex can be safeguarded by moving a guard from a neighboring vertex vvv, provided that doing so does not leave vvv itself unsecured—that is, only vertices with a value of 2 can spare a guard. Therefore, a guard may be dispatched from a location only if two guards are stationed there.

In terms of military strategy, the Italian domination model stipulates that any unguarded vertex must either be adjacent to a vertex with two guards or to at least two surrounding vertices, each with one guard. The presence of two adjacent locations without guards increases their vulnerability, making them more susceptible to attack. A more secure arrangement occurs when an unguarded location is surrounded by guarded locations. This leads to the concept of an Italian dominating set, in which each vertex labeled 0 (unguarded) must have at least one neighboring vertex labeled 1 or 2 (guarded).

A further refinement arises when an unguarded location is adjacent to another unguarded location. In this scenario, minimizing costs becomes a priority, motivating the definition of a total Italian dominating function. Under this strategy, every vertex with value 1 must be adjacent either to another vertex with value 1 or to a vertex labeled 2, thereby maintaining control while supporting the empire's legions. This reflects the logic underlying total Italian domination in the context of the empire's strategic planning.

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Assuming that any location with guards can be adjacent to another guarded location—equivalently, any vertex with a positive assignment has a neighboring vertex with a positive label—this defense strategy strengthens and supports the legions while ensuring enhanced security. Moreover, this approach offers improved robustness and flexibility, formalized as the Total Italian Dominating Function.

2. PRELIMINARIES AND DEFINITIONS

In the present paper, we focus on G, a simple graph of finite order having vertex set $\mathcal{V} = \mathcal{V}(G)$, edge set $E = E(G)$, for terminology and notation not explicitly introduced here, refer to [13] for more information. Given a subset $D \subseteq V$, the open neighborhood of S is defined as $N(D) = \cup_{v\in D} N(v)$ while the closed neighborhood is $N[D] = N(D) \cup D$. For a vertex $v \in D \subseteq V$, if $m \in N[v] - N[D - \{v\}]$ we called to this vertex *a private neighbor for* v in relation to D.

If we have two subsets $Q, R \subseteq \mathcal{V}(G)$, the notation $[Q, R]$ denotes the collection of all edges with once endpoint in Q and the other in R. Let C be a subgraph of G; the number of edges incident to a vertex v in C is called degree v and given by $\deg_C(v) =| N(v) \cap V(C) |$. Under the same assumption, the graph $G - C$ is generated by removing from G all vertices in $\mathcal{V}(C)$ along with any edge's incident to those vertices. For any subset $D \subseteq \mathcal{V}(G)$, $G[D]$ indicates the subgraph of G induced by D.
The $\deg_G(v)$ in the graph G is represented by $d_G(v)$, more simply put $d(v)$ when the context is understood. A vertex without any connections (degree 0) is termed an *isolated vertex,* while a vertex with once connection (degree 1) is termed a *leaf* or *pendant vertex*. The collection of all leaves in G is indicates by $L(G)$.
The minimum degree of a graph G, is defined as the smallest degree among all vertices in G, denoted by $\delta(G)$. By similar way, the maximum degree of G, is the largest degree among its vertices, denoted by $\Delta(G)$. For any subset $D \subseteq \mathcal{V}(G)$, the generates subgraph on D, indicates by $G[D]$, its a maximal subgraph of G whose vertex set is D and which includes all edges of G with both endpoints in D.
A **total dominating set** (abbreviated as *TD-set*) of a graph $G = (\mathcal{V}, E)$, assuming G has no isolated vertices, is a subset $D \subseteq \mathcal{V}$ such that each vertex in $\mathcal{V}$ is adjacent to at least 1 vertex in D. The **total domination number** of G, denoted by $\gamma_t(G)$, is the minimum cardinality among all TD-sets of G. A TD-set of size $\gamma_t(G)$ is referred to as a $\gamma_t(G) - \mathbf{set}$.
Let $f: \mathcal{V}(G) \to \{0,1,2\}$ be a function. For each $i \in \{0,1,2\}$, define $\mathcal{V}_i^f = \{v \in \mathcal{V}(G) \mid f(v) = i\}$. When the context is clear, we may simply write V_i instead of $\mathcal{V}_i^f$. The **weight** of f, denoted by $\omega(f)$, is defined as $\omega(f) = f(\mathcal{V}(G)) = \sum_{v\in\mathcal{V}(G)} f(v)$.
A **Roman dominating function** (RD function) on a graph G is a function $f: \mathcal{V}(G) \to \{0,1,2\}$ such that every vertex $v \in \mathcal{V}_0$ has at least one neighbor $z \in N(v)$ with $f(z) = 2$, i.e., $z \in \mathcal{V}_2$. The **Roman domination number** of G, indicated by $\gamma_R(G)$, is the $\omega(f)$ over all Roman dominating functions on G [5].
Chellali et al. [4] introduced the concept of an **Italian dominating function** (also referred to as a **Roman {2}-dominating function**) as follows. An **Italian dominating function** (ID function) for graph G is a function $f: \mathcal{V}(G) \to \{0,1,2\}$ provided that for each vertex $m \in \mathcal{V}(G)$ with $f(m) = 0 f(m) = 0$, the condition $f(N(m)) \geq 2$ holds. In other words, either there no fewer once exists neighbor $v \in N(m)$ with $f(v) = 2 f(v) = 2$, or there are at least twice neighbors $n, s \in N(m)$ with $f(n) = f(s) = 1 f(n) = 1 f(s) = 1$. The **Italian domination number**, indicated by $\gamma_I(G)$, is the $\omega(f)$ taken over all Italian dominating functions on G.
Samadi et al. [8] introduced the concept of a **restrained Italian dominating function** (RID function) as a refinement of the Italian dominating function. A **restrained Italian dominating function** is considered a function $f: \mathcal{V}(G) \to \{0,1,2\}$ satisfying the conditions of an Italian dominating function, with the additional requirement a subgraph generated by the vertex set assigned weight 0 under f contains **no isolated vertices**.
Ridha Abdulhasan and Mojdeh [9] introduced the concept of a **total restrained Italian dominating function** (TRID function) as a refinement of the Italian dominating function. A **total restrained Italian dominating function** is considered a function $f: \mathcal{V}(G) \to \{0,1,2\}$ satisfying the conditions of an Italian dominating function, with two additional requirements, first, a subgraph generated by the vertex set assigned weight 0 under f contains **no isolated vertices**, second, a subgraph generated by the vertex set assigned positive weight under f also contains **no isolated vertices**.

The **middle graph** of a graph G, denoted by $\mathcal{M}(G)$, is defined as the graph whose vertex set is $\mathcal{V}(G) \cup E(G)$. In $\mathcal{M}(G)$, two vertices are adjacent $if\ and\ only\ if$ either they associate to pairs of edges connected via a common vertex in G, or one represents a vertex and the other an edge of G adjacent to that vertex.

Consider a simple graph G (without loops or multiple edges). An **edge cover** of G is defined as a collection of edges in which every vertex of the graph is incident to at least one edge from the set [13]. The **complete graph** with n vertices is indicated by K_n, while the **complete bipartite graph** with partite sets of sizes n and m is written as $K_{n,m}$. The special case $K_{1,n}$ is referred to as a **star graph**. Denoted by $S_{p,q}$ to **double star**, is a tree structure composed of exactly

twice support vertices, where one is adjacent to p leaves and the other to q leaves. A graph consisting of a single vertex is known as a **trivial graph**. Additionally, C_n and P_n represent the **cycle** and **path** on n vertices, respectively.
This present paper is arranged the accurate value of TID the behavior of the Middle function over standard graphs is now well-established.

3. NEW RESULT OF THE TOTAL ITALIAN DOMINATION IN MIDDLE GRAPHS OF SOME STANDARD GRAPHS:

Here the total Italian dominating function for various families of graphs is investigated, with particular attention given to determining the minimum total Italian domination number (TID number) for each case. More specifically, we establish the following results. For the path graph P_n and cycle C_n, the value of $\gamma_{tI}(P_n)$ & $\gamma_{tI}(C_n)$ is straightforward for $1 \le n \le 2$, while for $n \ge 3$, we derive the general form as follows.

Theorem 1. ([1] Proposition 8) For graph G with order $3 \le n$ then $\gamma_{tI}(P_n) = \left\lceil \frac{2n+2}{3} \right\rceil$.

We now turn our attention to determining the **(TID)** of the **middle graph** of the path graph P_n.

Theorem 2. Let P_n be a path and $n \ge 3$. Then $\gamma_{tI}(\mathcal{M}(P_n)) = n + 1$.

Proof. Assume that the set of vertices $\mathcal{V}(P_n) = \{r_1, r_2, \cdots, r_n\}$. And the vertex set of $\mathcal{M}(P_n)$ is $\mathcal{V}(P_n) \cup \{m_{12}, m_{23}, \cdots, m_{n-1,n}\}$ wherein $m_{i,i+1}$ is the vertex added to represent to edge $e = r_i r_{i+1}; \{i; i \ge 1, i \le n-1\}$.
We designate label 1 for vertices $m_{i,i+1}\{i; i \ge 1, i \le n-1\}$, designate 1 to the vertices r_1, r_n and 0 to the others. So $\gamma_{tI}(\mathcal{M}(P_n)) \le n+1$. In the other direction, given that the 1st and end vertices are required to be designated with 1 value, and the subgraph generated by 1 does not admit any isolated vertices, each vertex $m_{i,i+1}$ has to be designated by 1 and 0 for others. According to the designation, we obtain a (TID) of $\mathcal{M}(P_n)$ such that $\gamma_{tI}(\mathcal{M}(P_n)) \le n+1$. Therefore $\gamma_{tI}(\mathcal{M}(P_n)) = n+1$. See figure 1

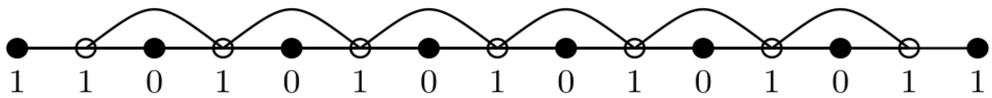

Figure 1 TID of $\mathcal{M}(P_8)$

As derived from Corollary 10 in reference [4], it follows that

Observation 3. ([4] Corollary 10) when C_n is a cycle graph then $\gamma_I(C_n) = \left\lceil \frac{n}{2} \right\rceil$.

In [1] the authors showed that any cycle C_n for $n \ge 3$.

Theorem 4. ([1] Proposition 9) If $n \ge 3$, $\gamma_{tI}(C_n) = \left\lceil \frac{3n}{2} \right\rceil$.

We now turn our attention to determining the **(TID)** of the **middle graph** of the cycle graph C_n.

Theorem 5. Let C_n be a cycle and $n \ge 3$. Then then $\gamma_{tI}(\mathcal{M}(C_n)) = n$.

Proof. Assume that the vertex set $\mathcal{V}(C_n) = \{r_1, r_2, \cdots, r_n\}$. And the vertex set of $\mathcal{M}(C_n)$ is $\mathcal{V}(C_n) \cup \{m_{12}, m_{23}, \cdots, m_{n-1,n}, m_{n,1}\}$ where $m_{i,i+1}$ is the vertex added to represent to edge $e = r_i r_{i+1}$ $\{i: i \ge 1, i \le n-1)$.
We designate value 1 to the new vertices $m_{i,i+1}$ $\{i; i \ge 1, i \le n-1\}$, and designate 0 to the others, then $\gamma_{tI}(\mathcal{M}(C_n)) \le n$. In the other direction, the vertices r_i and r_j in $\mathcal{M}(C_n)$ are not contiguous. Considering that r_i lies on both e_i and e_{i-1}, but in $\mathcal{M}(C_n)$ the vertex r_i has two contiguous $m_{i-1,i}$ and $m_{i,i+1}$, and $deg(m_{i,i+1}) = 4$. Thus, $f(N[r_i]) \ge 1$ and $f(N[m_{i,i+1}]) \ge 2$. This is achievable if we designate $f(r_i) = 1$ and $f(m_{i,i+1}) = 0$. Thus $\gamma_{tI}(\mathcal{M}(C_n)) \ge n$ & then $\gamma_{tI}(\mathcal{M}(C_n)) = n$. See figure 2

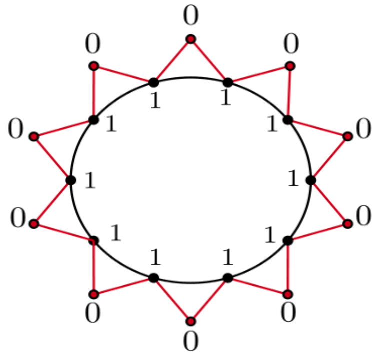

Figure 2 TID of $\mathcal{M}(C_{10})$

From the definition of Italian domination function that's easy to see that

Preposition 6. Let K_n represented to complete graph then $\gamma_I(K_n) = 2$ & $\gamma_{tI}(K_n) = 2$.

Theorem 7. Let K_n be a complete graph, then $\gamma_{tI}(\mathcal{M}(K_n)) = n$.

Proof. Assume that the vertex set $\mathcal{V}(K_n) = \{r_1, r_2, \cdots, r_n\}$. And the vertex set of $\mathcal{M}(K_n)$ is $\mathcal{V}(K_n) \cup \{m_{i,j}\}\{i; i \geq 1, i < j, i \leq n\}$ wherein $m_{i,j}$ is the vertex added to represent to edge $e = r_i r_j$ wherein $\{i; i \geq 1, i \leq n\}$. We designate value 1 to the new vertices $m_{i,i+1}$ $\{i; i \geq 1, i \leq n\}$ and designate 0 to the others, then $\gamma_{tI}(\mathcal{M}(K_n)) \leq n$. In the other direction, when $deg(r_i) = 3$. Therefore $f(N[r_i + r_{i+1}]) \geq 2$. This is achievable if we designate $f(r_i) = 0$ and $f(m_{i,i+1}) = 1$ and then $\gamma_{tI}(\mathcal{M}(K_n)) \geq n$. Hence $\gamma_{tI}(\mathcal{M}(K_n)) = n$.
Also, we can find the Italian domination function easily.

Preposition 8. Let $K_{1,n}$ represented to star graph, then $\gamma_I(K_{1,n}) = 2$ and $\gamma_{tI}(K_{1,n}) = 3$.

We here establish total Italian domination of middle graph for star, $\mathcal{M}(K_{1,n})$.

Theorem 9 Let $K_{1,n}$ be a star graph, then $\gamma_{tI}\big(\mathcal{M}(K_{1,n})\big) = 2n$.

Proof. Assume that the vertex set $\mathcal{V}(K_{1,n}) = \{r_0, r_1, r_2, \cdots, r_n\}$. And the vertex set of $\mathcal{M}(K_{1,n})$ is $\mathcal{V}(K_{1,n}) \cup \{m_{0,1}, m_{0,2}, m_{0,3}, \ldots, m_{0,n}\}$ wherein $m_{i,j}$ is the vertex added to represent to edge $e = r_0 r_i$ $\{i; i \geq 1, i \leq n\}$.
The middle graph for star is formed from K_{n+1} with vertices r_0 and $m_{0,i}$; $1 \leq i \leq n$, provided that the vertex r_i is a leaf neighbor of $m_{0,i}$; $1 \leq i \leq n$. Assume f be an $\gamma_{tI} - function$. Then $f(r_i) \geq 1$ and $f(m_{0,i}) \geq 1$. This case proves that $\gamma_{tI}\big(\mathcal{M}(K_{1,n})\big) \geq 2n$. In the other direction, the designation 2 to each $m_{0,i}$ for $0 \leq i \leq n$ yields a TID function of $\mathcal{M}(K_{1,n})$ of weight $2n$. Therefore $\gamma_{tI}\big(\mathcal{M}(K_{1,n})\big) = 2n$. See figure 3

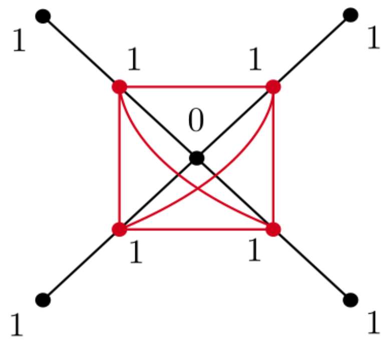

Figure 3 TID of $\mathcal{M}(K_{1,4})$

Here we go to the complete bipartite graph to get exact value for (ID), (TID) for this graph and (TID) for middle of complete bipartite graph $K_{m,n}$.

Preposition 10. If $n \leq 3$ then $\gamma_I(K_{n,m}) = 3$

Preposition 11. Let $K_{n,m}$ is a complete bipartite graph if $4 \leq n \leq m$ then $\gamma_I(K_{n,m}) = 4$ and $\gamma_{tI}(K_{n,m}) = 4$.

Theorem 12. Let $K_{n,m}$ is a complete bipartite graph if $m \leq n \leq 2$ then $\gamma_{tI}(\mathcal{M}(K_{n,m})) = 2m$.

Proof. Let Q, R be two partite sets of graphs $K_{n,m}$ and $n \geq m$ where $Q = \{q_1, q_2, \cdots, q_m\}$ and $R = \{r_1, r_2, \cdots, r_n\}$. Assume $\mathcal{M}(K_{n,m})$ represents a middle of $K_{n,m}$ with vertex set $\mathcal{V}(\mathcal{M}(K_{n,m})) = \mathcal{V}(K_{n,m}) \cup \{p_{i,j}: 1 \leq i \leq m \; and \; 1 \leq j \leq n\}$, where $p_{i,j}$ is the vertex added to represent to edge $e = q_i r_i$. Assume that $m \geq n$. Designate value 1 to two vertices $p_{i,j}$ inside q_i , $1 \leq i \leq m$ &$1 \leq j \leq n$ and designate 0 to others, deducing that $\gamma_{tI}(M(K_{m,n})) \leq 2m$. In the other direction, for totality and Italian condition we must assign at least two vertices $p_{i,j}$ inside q_i. Hence $\gamma_{tI}(\mathcal{M}(K_{m,n})) \geq 2m$.Therefore $\gamma_{tI}(\mathcal{M}(K_{n,m})) = 2m$.

For graph $S_{p,q}$ with $p, q \geq 2$. It is trivial to show that $\gamma_I(S_{p,q}) = \gamma_{tI}(S_{p,q}) = 4$.

Theorem 13. For double star $S_{p,q}$ with $p, q \geq 2$, then $\gamma_{tI}(\mathcal{M}(S_{p,q})) = 2(p + q)$.

Proof. Assume that $\mathcal{V}(S_{p,q}) = \{u_0, u_1, \cdots, u_p, v_0, v_1, \cdots, v_q\}$. And the vertex set of $\mathcal{M}(S_{p,q})$ is $\mathcal{V}(S_{p,q}) \cup \{a_{0,1}, a_{0,2}, \dots, a_{0,p}, b_{0,1}, b_{0,2}, \dots, b_{0,q}\} \cup \{c_0\}$ where where $x_{0,i}, y_{0,j}$ is the vertex added to represent to edge e $= u_0u_i, v_0v_j$ and c_0 is corresponding to $e = u_0v_0$. As shown in the figure below, $\{u_0, a_{0,1}, a_{0,2}, \dots, a_{0,p}\}$ creates a clique of order $p + 1$, also $\{v_0, b_{0,1}, b_{0,2}, \dots, b_{0,q}\}$ creates a clique of order $q + 1$ in $\mathcal{M}(S_{p,q})$. The vertex c_0 is incident to all vertices of these twice cliques.
The vertex u_i is a leaf neighbor of $a_{0,i}$ for $1 \leq i \leq p$ and vertex v_j is a leaf neighbor of $b_{0,j}$ for $1 \leq j \leq q$ in $\mathcal{M}(S_{p,q})$. It is manifest, under any TID function the leaf is Designated with positive label 1 and all $a_{0,i}, b_{0,j}$ then $\gamma_{tI}(\mathcal{M}(S_{p,q})) \geq 2(p + q)$. In the other direction, if we designate all leaf vertices with 0 for Italian condition we must designate all $a_{0,i}, b_{0,j}$ with value 2. Hence $\gamma_{tI}(\mathcal{M}(S_{p,q})) \leq 2(p + q)$. Thats lead to a TID function with $\omega(f) = 2(p + q)$, as a result$\gamma_{tI}\left(\mathcal{M}(S_{p,q})\right) = 2(p + q)$. See figure 4

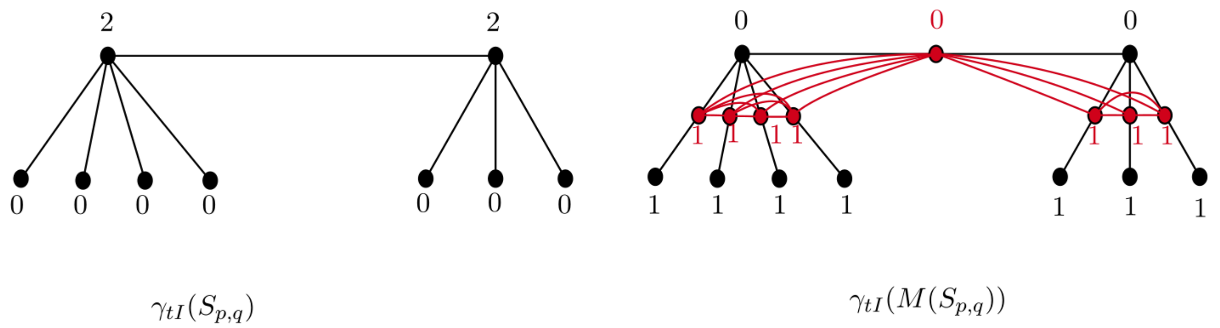

Figure 4 TID of $S_{4,3}$ and $\mathcal{M}(S_{4,3})$

4. DISCUSSIONS

The findings of this study underscore the significance of the Total Italian Domination technique in the strategic deployment of military units to ensure effective coverage of designated positions. This approach systematically assigns either one or two units per position as needed, while positions lacking direct coverage are supported by exactly two neighboring units, thereby reinforcing defensive integrity and minimizing vulnerabilities within the formation.
The results demonstrate that this method achieves an optimal balance between minimizing the number of deployed units and maximizing coverage efficiency. Such a balance is particularly valuable in reducing operational expenses without compromising essential security and defense requirements. The integration of precise mathematical modeling with strategic planning underpins the effectiveness of this deployment approach.
Furthermore, the Total Italian Domination technique provides a flexible and scalable framework adaptable to diverse tactical scenarios, enhancing operational responsiveness. Its applicability may extend beyond military contexts to fields such as civil security and disaster management, where the efficient allocation of limited resources is critical.
Nonetheless, further empirical validation is needed to assess the real-world applicability of this technique under varying environmental and tactical conditions. Future research may also incorporate additional criteria, including response time and adaptability to dynamic threats, to enhance the robustness and practicality of this deployment strategy.

5. CONCLUSION

In this manuscript, we discuss the results related to Total Italian Domination, which governs the deployment of military units according to the Italian technique. This approach ensures that positions are assigned either one or two units, while unguarded positions are supported by exactly two neighboring units. The proposed technique achieves minimal deployment with high efficiency, aiming to reduce overall operational costs.

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