Bayesian Estimation of Reliability for Multicomponent Stress-Strength Model Based on Topp-Leone Distribution

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Abstract— In this paper, the reliability function of a multicomponent stress-strength model was calculated when each stress and strength variable had a Topp-Leone distribution and an identically independent distribution (i.i.d). By assuming that the primary normal information is related to the Gamma Distribution and the Weibull Distribution on several occasions, the estimation was based on the Bayes using Lindley approximation methods, under the squared error loss function, in order to find the optimal method, the mathematical formulas for these estimators were developed and compared. Using a Monte Carlo simulation method, the estimators were compared based on the mean squares error (MSE)..

Keywords— Multicomponent model; Stress-Strength; Topp-Leone distribution; Lindley Approximation. 1 Introduction

1 Introduction

The stress and strength are widely used in measuring the reliability of systems, which can be defined according to the mathematical formula R=P (X <Y). In the formula, the variable X is a random variable that represents the strength and is subjected to a random stress Y. When the strength is greater than the applied stress, the system works; when it is smaller, the system fails. Due to the importance of reliability in production systems and components, many studies have investigated aspects related to the ability and accuracy of these systems. Different distributions were studied following the surrounding parameters of productive and practical data, where they were used to estimate the distribution parameters or to show the confidence interval of these systems' reliability. In the following, we mention some researchers who studied the reliability of stress and strength for single-component systems. In 2009, Kund

and Raqab were able to estimate the reliability function of the stress and strength model are independent and follow the three-parameter Weibull distribution [1]. In 2010, Rezaei et al. Present different methods for estimated the stress -strength reliability function of the Generalized Pareto distribution [2]. Saracoglu et al. 2012 [3] used type II stepwise control data to estimate the reliability of that system when follow Exponential distribution. Moreover, in 2013 Genc estimated reliability of the same system when stress and strength are independent and follow the Topp Leone distribution and its application to real data [4]. In the same year, Rao et al. found the reliability function of the stress and strength system when the stress and strength variables follow in Inverse Rayleigh distribution [5].AL-Mutairi and Aboukhamseen in 2015, reported the reliability of the system using Lindley distribution [6]. However, in the year 2017, Asgharzadeh et al. [7] estimated the reliability function of the stress and strength system using Bayesian and non-Bayesian methods, for Generalized Exponential distribution. In the same context, Sharma 2018 [8] Generalized Inverse Lindley stress-strength reliability mod for Bayesian analysis. In 2019, Selman and Hamad derived a mathematical formula stress and strength system for three different distributions [9]. In 2020, Al-Omari et al. [10] used the Median and Ranked Set Sampling methods to estimate of the stress-strength reliability for Exponentiated pareto Distribution. And lately Almarashi et al.2021 [11]. studied the reliability of the mentioned system for the Topp-Leone distribution using Advanced Sampling methods. The stress and strength model of a single component is developed and expanded to include two or more components. A system that contains more than one component is called a multicomponent system, and a system that contains k components functions properly if at least s of the components works ($1\leq s\leq k$), where the strength is higher than the applied stress. A multicomponent system could be serial or parallel, and these are considered special cases when $s = k$ and $s = 1$ are mentioned respectively. There are many examples of multicomponent systems in communication, marine, and industry. For instance, an aircraft with more than one engine assumes that there are at least $1\leq s\leq k$ of the necessary engines to take off. Another example is applied in the marine communication systems, where 6 out of 10 transmitters are operated to cover the area [12]. Many papers were published to study the reliability of multicomponent stress-strength systems following different distributions. These studies include, but not limited to, in 2011, Mokhlisa and Khames [13] studied the reliability of multicomponent stress-strength models for Marashall-Olkin exponential distribution. In 2012, Rao has estimated the reliability function for k independent compounds. He assumed that the the stress and strength variables follow the Generalized Rayleigh distribution [14]. Nadar and Kizilaslan in 2015 [15,16], Karam and Jani in 2016 [17]. Hassan and Alohali in 2018 [18]. And in the same year, too.2018, Ali et al. [19] estimated the reliability function of the same system when the stress and strength are non-independent components and follow the Weibull and BurrIII distribution. Kohansal in 2019 [20]. Pandit and Joshi in 2019[21] estimated the reliability function of the multicomponent stress and strength system using the Bayesian and non-Bayesian methods, where the stress and strength variables followed the Generalized Rayleigh distribution. In 2020, Mahato et al. [22] studied the reliability of the mentioned system under progressive Type II monitoring data when the failure times follow Inverted exponentiated Distribution. Sauer et al, in 2020 [23]. As well, Rasekhi et al. in 2020 [24], Mezaal et al. in 2020 [25], Lately in 2021, Kohansal and Shoaee in [26], reported the reliability of the system using the Weibull Distribution. And Wang et al. in same year 2021 [27] published a paper to find the reliability of stress and strength of multicomponent system for Type II monitoring data where the stress and strength variables followed Rayleigh Distribution.

Assuming that the random variables ($Y, X_1, X_2, \ldots, X_k$) are independent, $G(y)$ is a cumulative density function (CDF) for Y, and $F(x)$ is the CDF for ($X_1, X_2, \ldots, X_k$), the reliability function of a multicomponent stress and strength system is expressed by the following formula that was firstly presented by Bhattacharyya and Johnson in 1974 [28]:

$$R_{s,k} = P[\text{at least s of the } (X_1, X_2, \ldots, X_k) \text{exceed } Y]$$
$$= \sum_{i=s}^{k} \binom{k}{i} \int_{-\infty}^{\infty} [1 - F(y)]^{i} [F(y)]^{k-i} dG(y) \qquad (1)$$

This paper was organized to give a brief introduction in the first section. In the second section, the Topp-Leone distribution and a mathematical derivation for the reliability function of a multicomponent stress-strength system were discussed. In the third section, the Lindley approximation was used to estimate the reliability function following the Bayes method under the squared error loss function. The normal conjugate (initial distribution) of Gamma and Weibull Distributions were applied in two cases, s, k = 2,3 and s, k = 1,4, where a comparison between them was set. The fourth section contains the simulation results of the Monte Carlo method; while the last section summarizes the research results and concludes the outcome.

2 Topp-Leone Distribution

Topp-Leone is one of the failure models that examine the performance of equipment in addition to its importance in calculating reliability. It is one of the continuous distributions that was first introduced by Topp and Leone in 1955 [29] and investigated by Nadarajah and Kotz in 2003, where they studied the probability density function of the distribution. In the following, we show the probability and cumulative density functions [30].

$$f(x;\alpha) = 2\alpha\,(1-x)[x(2-x)]^{\alpha-1} \;\; ; \; 0 < x < 1, 0 < \alpha < \infty \qquad (2)$$

$$F(x) = [x(2-x)]^{\alpha} \quad ; 0 < x < 1, 0 < \alpha < \infty \qquad (3)$$

It was supposed that we have two variables $X \sim TL(\alpha 1)$ and $Y \sim TL(\alpha 2)$, where $\alpha 1$ represents the strength parameter and $\alpha 2$ represents the stress parameter. By assuming that these variables are identical, independent, and random that each follows the Topp-Leone distribution, we get the following equation after referencing equation (1) and substituting in it

$$R_{S,K=}\sum_{i=s}^{k}\binom{k}{i}\int_0^1 [\,1-\left(y(2-y)\right)^{\alpha 1}]^i[\left(y(2-y)\right)^{\alpha 1}]^{k-i}\,2\alpha_2(-y)\,\left(y(2-y)\right)^{\alpha 2-1}\ dy \quad (4)$$

Assuming that

$$t=\left(y(2-y)\right)^{\alpha 1}$$

$$dt=[-\alpha 1\ \ y^{\alpha 1}\ (2-y)^{\alpha 1-1}]+[\alpha 1\ \ y^{\alpha 1-1}\ (2-y)^{\alpha 1}]\ dy$$

$$=\ \sum_{i=s}^{k}\binom{k}{i}\int_0^1(1-t)^i\ t^{k-i}\ \ \frac{\alpha_2}{\alpha_1}\ \ \left(t^{\frac{1}{\alpha 1}}\right)^{\alpha 2}\ (t)^{-1}\ dt$$

And by considering that $\rho=\frac{\alpha_2}{\alpha_1}$

$$=\rho\sum_{i=s}^{k}\binom{k}{i}\ \int_0^1 t^{k-i+\rho-1}\ (1-t)^i dt$$

$$R_{s,k}\ =\rho\sum_{i=s}^{k}\binom{k}{i}\ \ \mathcal{B}\ (k-i+\rho\,,i+1) \quad (5)$$

$$R_{s,k}\ =\rho\sum_{i=s}^{k}\binom{k}{i}\ \frac{\Gamma(k-i+\rho)\Gamma(i+1)}{\Gamma(k+\rho+1)}$$

$$R_{s,k}=\rho\sum_{i=s}^{k}\frac{k!}{(k-i)!}\left[\prod_{j=0}^{i}(k+\rho-j)\right]^{-1} \quad (6)$$

3 Reliability Estimation in Multicomponent Stress-Strength System

3.1. Maximum Likelihood Method (MLM)

Suppose that a random sample of size n of strength variable (x_1, x_2, …, x_n) have Topp-Leone distribution with the parameter (α_1). Similarly, the random samples of size m of stress variable (y_1, y_2, …, y_m) have Topp-Leone distribution with the parameter (α_2). Here, the maximum likelihood function of random samples for the combined distribution of stress and strength variables becomes as follows :

$$L\ =(x,y\,|\ \alpha_1,\alpha_2)=\prod_{i=1}^{n}f(x_i,\alpha_1)\ \prod_{j=1}^{m}\ f\left(y_j,\alpha_2\right) \quad (7)$$

Thus, the maximum likelihood estimator for the two parameters α_1 and α_2 take the following order:

$$\widehat{\alpha}_{1mle}\ =-\frac{n}{\sum_{i=1}^{n}\ln\left(x_i(2-x_i)\right)} \quad (8)$$

$$\widehat{\alpha}_{2mle} = -\frac{m}{\sum_{j=1}^{m} \ln\left(y_j(2-y_j)\right)} \tag{9}$$

By substituting $\hat{\alpha}_1$mle, $\hat{\alpha}_2$mle into equation (6), we get the reliability function estimation Rs,k by the maximum likelihood estimator method as follows:

$$\widehat{R}_{(s,k)MLE} = \frac{\widehat{\alpha}_{2\ MLE}}{\widehat{\alpha}_{1\ MLE}} \sum_{i=s}^{k} \frac{k!}{(k-i)!} \left[\prod_{j=0}^{i}\left(k+\frac{\widehat{\alpha}_{2\ MLE}}{\widehat{\alpha}_{1\ MLE}}-j\right)\right]^{-1} \tag{10}$$

3.2 Bayesian Method

The Bayesian method generally focuses on the use of prior information to estimate unknown parameters (α= α1, α2, …, αn). This includes assuming that the parameters are random variables. where prior information is added to the current sample information [31]. This information is represented in the form of a priority probability function, which represents all the information and experience about parameters needed to be estimated. These parameters were previously identified through analysis or monitoring [32].

The Bayes estimator of any parameter depends on two functions, the first is the posterior probability distribution function, and the second is the loss function, and in this paper, we used the squared error loss function.

3.2.1 Bayesian Estimation Using a Prior (Gamma Distribution)

In this part, we deal with estimating the reliability function $R_{s,k}$ for the multicomponent stress-strength model when the variables are independent and follow the Topp-Leone distribution. The research methodology relies on the use of the Bayesian estimation method by employing the Lindley approximation, under the squared error loss function[33], and the natural conjugate (Gamma Distribution) in two cases ($R_{1,4}$ and $R_{2,3}$). Hence, the parameters α_1 and α_2 were therefore assumed to be statistically independent variables, and the gamma distribution Gamma (a_1, b_1) and Gamma (a_2, b_2), which are expressed as follows, is the primary distribution for each parameter.

$$\pi(\alpha_1) = \frac{b_1^{a_1}}{\Gamma a_1}\, \alpha_1^{a_1-1} e^{-\alpha_1 b_1} \tag{11}$$

$$\pi(\alpha_2) = \frac{b_2^{a_2}}{\Gamma a_2}\, \alpha_2^{a_2-1}\, e^{-\alpha_2 b_2} \tag{12}$$

Here, the joint prior distribution for the two parameters becomes:

$$\pi(\alpha_1,\alpha_2) = \pi(\alpha_1)\pi(\alpha_2) = \frac{b_1^{a_1}}{\Gamma a_1}\alpha_1^{a_1-1}e^{-\alpha_1 b_1}\ \frac{b_2^{a_2}}{\Gamma a_2}\alpha_2^{a_2-1}\ e^{-\alpha_2 b_2} \tag{13}$$

where (a_1, b_1) and (a_2, b_2) are known and positive values.
Using the inverse of the Bayes rule by integrating the possibility function in equation (7) with the primary conjunction distribution function in equation (13), the joint posterior distribution for the two estimated parameters α_1 and α_2 becomes as follows:

$$\pi^*(\alpha_1,\alpha_2|x,y) = \frac{L(x,y|\alpha_1,\alpha_2)\ \pi(\alpha_1,\alpha_2)}{\iint L(x,y|\alpha_1,\alpha_2)\ \pi(\alpha_1,\alpha_2)d\alpha_1 d\alpha_2}$$
$$\propto \alpha_1^{n+a_1-1}\alpha_2^{m+a_2-1}e^{-b_1\alpha_1}e^{-b_2\alpha_2}$$
$$\times\prod_{i=1}^{n}(1-x_i)\big(x_i(2-x_i)\big)^{\alpha_1-1}\prod_{j=1}^{m}(1-y_j)\left(y_j(2-y_j)\right)^{\alpha_2-1} \tag{14}$$

Thereby, the Bayesian estimator for the reliability function $R_{s,k}$ under the squared error loss function follows the below formula:

$$\hat{R}_{s,k,B}=\int_0^\infty\int_0^\infty R_{s,k}\ \pi^*(\alpha_1,\alpha_2|x,y)\ d\alpha_1 d\alpha_2\ . \tag{15}$$

From (15), we notice that there is difficulty in performing the integrals, and it became necessary to use a numerical approximation method to calculate these complex integrals. This utilized method is called the Lindley Approximation, which is used to find the Bayesian estimator. The details of the method are shown as follows:
This method was proposed by D.V. Lindley in 1980, which consisted of a method to find an approximate formula to calculate the integrals of the Bayesian estimator [34].

$$[u(\alpha)] = \frac{\int u(\alpha)\ e^{L(\alpha)+v(\alpha)}d\alpha}{\int e^{L(\alpha)+v(\alpha)}d\alpha} \tag{16}$$

$$E\big(u(\alpha)\big) = \left[u(\hat{\alpha}) + \tfrac{1}{2}\sum_i\sum_j\big(u_{ij}+2u_i v_j\big)\sigma_{ij} + \tfrac{1}{2}\sum_i\sum_j\sum_k\sum_l L_{ijk}\sigma_{ij}\sigma_{kl}u_l\right] \tag{17}$$

Where:
$u(\alpha)$: an optional function for the parameter α.
$\hat{\alpha}$: The maximum likelihood estimator for the parameter α.
$L(\alpha)$: The natural logarithm of the likelihood function.
$v(\alpha)$: The natural logarithm for the primary distribution function of parameter α.
σ_{ij}:(i, j) th element in the inverse of the matrix (-Lij) all are evaluated at the MLE of the parameters.
Here,

$\alpha = (\alpha_1, \alpha_2, \dots, \alpha_P)$, $i, j, k, l = 1,2$

$v = \ln \pi(\alpha)$, $u = u(\alpha_i)$

$$ui = \frac{\partial u}{\partial \alpha_i} \quad , uij = \frac{\partial^2 u}{\partial \alpha_{ij}} \quad , \quad L_{ijk} = \frac{\partial^3 \ln L}{\partial \alpha_i \, \partial \alpha_j \, \partial \alpha_k} \quad , \quad v_j = \frac{\partial v}{\partial \alpha_j}$$

$$v_{1G} = \frac{\partial v}{\partial \alpha_1} = \frac{a_1 - 1}{\alpha_1} - b_1 \quad , \quad v_{2G} = \frac{\partial v}{\partial \alpha_2} = \frac{a_2 - 1}{\alpha_2} - b_2$$

$$L_{111} = \frac{\partial^3 \ln L}{\partial \alpha_1^3} = \frac{2n}{\alpha_1^3} \quad , \quad L_{222} = \frac{\partial^3 \ln L}{\partial \alpha_2^3} = \frac{2m}{\alpha_2^3}$$

$$L_{112} = L_{121} = L_{122} = L_{221} = L_{212} = L_{211} = 0$$

$$\sigma_{11} = \frac{\alpha_1^2}{n} \quad , \sigma_{22} = \frac{\alpha_2^2}{m} \, , \sigma_{12} = \sigma_{21} = 0$$

In order to estimate the reliability function Rs,k of a multicomponent stress-strength system when the stress and strength variables follow the Topp-Leone distribution, further assumptions were added. Here, based on the standard Bayesian method in terms of the square loss function and the primary joint distribution (Gamma Distribution), and by using the Lindley Approximation method, we substitute within equation 17 to get:

$$\hat{R}_{s,k,L,G} = \hat{R}_{s,k} + \frac{1}{2}\left[\left(\hat{R}_{11} + 2\hat{R}_1 \hat{v}_{1G}\right)\sigma_{11} + \left(\hat{R}_{22} + 2\hat{R}_2 \hat{v}_{2G}\right)\sigma_{22}\right] + \frac{1}{2}\left[\left(\hat{L}_{111}\sigma_{11}^2 \hat{R}_1 + \hat{L}_{222}\sigma_{22}^2 \hat{R}_2\right)\right] \tag{18}$$

Where:

$$R_i = \frac{\partial R_{s,k}}{\partial \alpha_i} \quad , \quad R_{ii} = \frac{\partial^2 R_{s,k}}{\partial \alpha_i^2} \quad , i = 1,2$$

Furthermore, v_i , L_{111}, L_{222} , σ_{11}, and σ_{22} were defined and derived from the above formulas, and the first and second derivatives of $R_{s,\,k}$ when (s, k) = 2,3 and (s, k) =1,4 are shown below:

$$R_1 = \frac{\partial R_{2,3}}{\partial \alpha_1} = \frac{(12\alpha_1\alpha_2^2 + 30\alpha_1^2\alpha_2)}{[(3\alpha_1 + \alpha_2)(2\alpha_1 + \alpha_2)]^2}$$

$$R_2 = \frac{\partial R_{2,3}}{\partial \alpha_2} = \frac{(-30\alpha_1^3 - 12\alpha_1^2\alpha_2)}{[(3\alpha_1 + \alpha_2)(2\alpha_1 + \alpha_2)]^2}$$

$$R_{11} = \frac{\partial^2 R_{2,3}}{\partial \alpha_1^2} = \frac{12\alpha_2^2 + 60\alpha_1\alpha_2 - 588\alpha_1^2\alpha_2^2 - 120\alpha_1\alpha_2^3 - 720\alpha_1^3\alpha_2}{[(3\alpha_1 + \alpha_2)(2\alpha_1 + \alpha_2)]^3}$$

$$R_{22} = \frac{\partial^2 R_{2,3}}{\partial \alpha_2^2} = \frac{6\alpha_1^2(38\alpha_1^2 + 30\alpha_1\alpha_2 + 6\alpha_2^2)}{[(3\alpha_1 + \alpha_2)(2\alpha_1 + \alpha_2)]^3}$$

$$\acute{R}_1 = \frac{\partial R_{1,4}}{\partial \alpha_1} = \frac{(36\alpha_1^2\alpha_2 + 4\alpha_2^3 + 24\alpha_1\alpha_2^2)}{[(4\alpha_1 + \alpha_2)(3\alpha_1 + \alpha_2)]^2}$$

$$\acute{R}_2 = \frac{\partial R_{1,4}}{\partial \alpha_2} = \frac{(-36\alpha_1^3 - 24\alpha_1^2\alpha_2 - 4\alpha_1\alpha_2^2)}{[(4\alpha_1 + \alpha_2)(3\alpha_1 + \alpha_2)]^2}$$

$$\acute{R}_{11} = \frac{\partial^2 R_{1,4}}{\partial \alpha_1^2} = \frac{-864\alpha_1^3\alpha_2 - 288\alpha_1\alpha_2^3 - 864\alpha_1^2\alpha_2^2 - 32\alpha_2^4}{[(4\alpha_1 + \alpha_2)(3\alpha_1 + \alpha_2)]^3}$$

$$\acute{R}_{22} = \frac{\partial^2 R_{1,4}}{\partial \alpha_2^2} = \frac{8\alpha_1(27\alpha_1^3 + 27\alpha_1^2\alpha_2 + 9\alpha_1\alpha_2^2 + \alpha_2^3)}{[(4\alpha_1 + \alpha_2)(3\alpha_1 + \alpha_2)]^3}$$

3.2.2 Bayesian Estimation Using a Prior (Weibull Distribution)

In this part, the reliability function $R_{s,k}$ is estimated for the multicomponent stress and strength system when each variable follows the Topp-Leone distribution, uses the Bayesian method, follows the Lindley Approximation, and under the square loss function. The primary distribution for each parameter is Weibull (c_1, d_1) and Weibull (c_2, d_2), which are shown as follows [15]:

$$\pi(\alpha_1) = \frac{c_1}{d_1}\ \alpha_1^{c_1-1} e^{-\frac{\alpha_1^{c_1}}{d_1}} \tag{19}$$

$$\pi(\alpha_2) = \frac{c_2}{d_2}\ \alpha_2^{c_2-1} e^{-\frac{\alpha_2^{c_2}}{d_2}} \tag{20}$$

Therefore, the joint prior for the parameters α_1 and α_2 will be:

$$\pi(\alpha_1, \alpha_2) = \pi(\alpha_1)\pi(\alpha_2) = \frac{c_1}{d_1}\ \alpha_1^{c_1-1} e^{-\frac{\alpha_1^{c_1}}{d_1}}\ \frac{c_2}{d_2}\ \alpha_2^{c_2-1} e^{-\frac{\alpha_2^{c_2}}{d_2}} \tag{21}$$

Here, v_{1w} and v_{2w} are given in the following formula:

$$v_{1w} = \frac{(c_1 - 1)}{\alpha_1} - \frac{c_1\alpha_1}{d_1} \quad , v_{2w} = \frac{(c_2 - 1)}{\alpha_2} - \frac{c_2\alpha_2}{d_2}$$

Therefore, estimating the reliability function $R_{s,k}$ for the multicomponent stress-strength system of the Topp-Leone distribution could have a new form. In our work, after substituting within equation 17 and using the Lindley Approximation method, the formula became:

$$\hat{R}_{s,k,L,W} = \hat{R}_{s,k} + \tfrac{1}{2}\left[\left(\hat{R}_{11} + 2\hat{R}_1\hat{v}_{1W}\right)\sigma_{11} + \left(\hat{R}_{22} + 2\hat{R}_2\hat{v}_{1W}\right)\sigma_{22}\right] + \tfrac{1}{2}\left[\left(\hat{L}_{111}\sigma_{11}^2\hat{R}_1 + \hat{L}_{222}\sigma_{22}^2\hat{R}_2\right)\right] \tag{22}$$

Where the derivatives of $\sigma_{22}, \sigma_{11} L_{222}, L_{111}, R_{22}, R_2, R_{11},$ and R_1 were illustrated above.

4 Simulation Study

In this section, we show the simulation results to estimate the function Rs,k when the stress and strength independent random variables have Topp-Leone distribution. Default values for distribution parameters have been specified, as well as the selection of different default sample sizes for observations of the random variable X, Y strength and stress have been specified as (15, 30, 45, 60) for two cases of s, k = (2, 3) and (1,4). The Mean Square Error (MSE) is used to compare two different estimation methods, called the Bayes method and using the Lindley approximation under Gamma and Weibull prior distributions. The simulation results were based on the MSE comparison criterion with a repeating size of L = 1000 from the statistical program R. Two cases of the shape and scale parameters are used to perform the comparison of the Bayes method under Weibull and Gamma prior distributions. The first case, when the shape and scale parameters of Gamma distributions (a1, b1) = (1.5,1.5) and (a2, b2) = (2,2) and Weibull distributions (c1, d1) = (1.5,1.5) and (c2, d2) = (2,2) are equal, the Bayes method under Weibull distribution is better than it under Gamma distribution, because it achieved the lowest MSE. In the second case, when the shape and scale parameter of the prior Gamma distribution are (a1, b1) = (0.5,1), (a2, b2) = (0.5,1) and the Weibull distribution (c1, d1) = (0.5,1), (c2, d2) = (0.5,1) are not equal, The Bayes method dependent on Lindley's approximation and based on the Gamma prior distribution is better than the Bayes method based on the Weibull distribution because it achieves the lowest MSE for all experiments. The simulation results are shown in Tables 1, 2,3 and 4.

Table 1. $R_{s,k}$ and $\widehat{R}_{s,k}$ When the shape and scale parameter of the prior (Gamma distribution) and (Weibull distribution) is $(a_1, b_1) = (1.5,1.5)$, $(a_2, b_2) = (2,2)$, $(c_1, d_1) = (1.5,1.5)$, $(c_2, d_2) = (2,2)$

(α_1, α_2)	(s, k)	(n, m)	R	$\widehat{R}_{L(G)}$	$\widehat{R}_{L(W)}$
(0.4 ,0.4)	2,3	(15,15)	0.5	0.6637641	0.6626301
		(30,30)		0.5667557	0.5662339
		(45,45)		0.5433762	0.5430480
		(60,60)		0.5281150	0.5278562
	1,4	(15,15)	0.8	0.9663786	0.9653527
		(30,30)		0.8698180	0.8693171
		(45,45)		0.8428278	0.8425052
		(60,60)		0.8297161	0.8294656
(0.80.7,)	2,3	(15,15)	0.5385694	0.6653428	0.6642402
		(30,30)		0.5662859	0.5657655
		(45,45)		0.5431110	0.5427770
		(60,60)		0.5315068	0.5312609
	1,4	(15,15)	0.8205128	0.9636032	0.9624240
		(30,30)		0.8666117	0.8661134
		(45,45)		0.8405139	0.8401742
		(60,60)		0.8291277	0.8288774

Table 2. $R_{s,k}$ and $\widehat{R}_{s,k}$When the shape and scale parameter of the prior (Gamma distribution)and (Weibull distribution) is $(a_1, b_1) = (0.5,1)$, $(a_2, b_2) = (0.5,1)$, (c_1, d_1)

(α_1, α_2)	(s, k)	(n, m)	R	$\widehat{R}_{L(G)}$	$\widehat{R}_{L(W)}$
(0.4 ,0.4)	2,3	(15,15)	0.5	0.6812030	0.6812605
		(30,30)		0.5737990	0.5738060
		(45,45)		0.5474118	0.5474188
		(60,60)		0.5343086	0.5343132
	1,4	(15,15)	0.8	0.9731791	0.9731584
		(30,30)		0.8710786	0.8710934
		(45,45)		0.8426766	0.8426730
		(60,60)		0.8320667	0.8320723
(0.80.7,)	2,3	(15,15)	0.5385694	0.6688161	0.6687925
		(30,30)		0.5720923	0.5720949
		(45,45)		0.5462277	0.5462337
		(60,60)		0.5346914	0.5346969
	1,4	(15,15)	0.8205128	0.9800381	0.9800816
		(30,30)		0.8706267	0.8706450
		(45,45)		0.8419314	0.8419265
		(60,60)		0.8325078	0.8325118

$=(0.5,1)$, $(c_2,d_2) =(0.5,1)$

Table 3 . MSE for $\widehat{R}s, k$ When the shape and scale parameter of the prior (Gamma distribution) and (Weibull distribution) is $(a_1, b_1) = (1.5,1.5)$, $(a_2,b_2) = (2,2)$, $(c_1,d_1) =(1.5,1.5)$, $(c_2,d_2)=(2,2)$

(α_1, α_2)	(s, k)	(n, m)	$MSE_{(G)}$	$MSE_{(W)}$
(0.4 ,0.4)	2,3	(15,15)	0.0268186916	0.0264485594
		(30,30)	0.0044563300	0.0043869276
		(45,45)	0.0018814979	0.0018531287
		(60,60)	0.0007904544	0.0007759676
	1,4	(15,15)	0.0276818324	0.0273415010
		(30,30)	0.0048745566	0.0048048548
		(45,45)	0.0018342218	0.0018066954
		(60,60)	0.0008830489	0.0008682221
(0.8 ,0.7)	2,3	(15,15)	1.607148e-02	1.579313e-02
		(30,30)	7.682003e-04	7.396276e-04
		(45,45)	5.062578e-05	4.770393e-05
		(60,60)	2.341399e-05	1.988125e-05
	1,4	(15,15)	2.047485e-02	2.013880e-02
		(30,30)	2.125104e-03	2.079415e-03

		(45,45)	4.000418e-04	3.865691e-04
		(60,60)	7.421637e-05	6.996627e-05

Table 4. MSE for $\widehat{R}s,k$ When the shape and scale parameter of the prior (Gamma distribution) and (Weibull distribution) is (a_1,b_1) (0.5,1), (a_2, b_2) = (0.5,1), (c_1,d_1) =(0.5,1), (c_2,d_2)=(0.5,1)

(α_1, α_2)	(s, k)	(n, m)	$MSE_{(G)}$	$MSE_{(W)}$
(0.4 ,0.4)	2,3	(15,15)	0.0328345252	0.0328553833
		(30,30)	0.0054462967	0.0054473206
		(45,45)	0.0022478740	0.0022485464
		(60,60)	0.0011770821	0.0011773923
	1,4	(15,15)	0.0299909871	0.0299838481
		(30,30)	0.0050521666	0.0050542677
		(45,45)	0.0018212952	0.0018209815
		(60,60)	0.0010282713	0.0010286319
(0.8 ,0.7)	2,3	(15,15)	1.696421e-02	1.695805e-02
		(30,30)	1.123781e-03	1.123961e-03
		(45,45)	5.864953e-05	5.874120e-05
		(60,60)	1.499621e-05	1.503903e-05
	1,4	(15,15)	2.544831e-02	2.546221e-02
		(30,30)	2.511404e-03	2.513233e-03
		(45,45)	4.585475e-04	4.587539e-04
		(60,60)	1.438807e-04	1.439757e-04

5 Conclusions

. In this paper, the reliability function of the multicomponent stress and strength system was estimated when each of the stress and strength variables are independent and follows the Topp- Leone distribution using
the Bayes method, depending on the Lindley's approximation and under of a squared loss function, Prior distribution Gamma and Weibull distribution and the comparison between them. By specifying default values for parameters and different sample sizes and in two cases when s,k =(2,3)and (1,4) the results showed:

1. With an increase in sample size, both the real R value and the estimated R value get closer to one another, and the MSE value goes down.
2. The Bayes method using the Gamma prior distribution was the best when the parameters of the shape and scale were not equal because it achieved the lowest MSE.

3. The Bayes method using the Weibull prior distribution was the best when the parameters of shape and scale were equal because it achieved the lowest MSE.

6 References

[1] Kundu, D., & Rabab, M. Z. (2009). Estimation of R= P (Y< X) for three-parameter Weibull distribution. Statistics & Probability Letters, 79(17), 1839-1846.

[2] Rezaei, S., Tahmasebi, R., & Mahmoodi, M. (2010). Estimation of P [Y< X] for generalized Pareto distribution. Journal of Statistical Planning and Inference, 140(2), 480-494.

[3] Saraçoğlu, B., Kincaid, I., & Kundu, D. (2012). On estimation of R= P (Y< X) for exponential distribution under progressive type-II censoring. Journal of Statistical Computation and Simulation, 82(5), 729-744.

[4] Genç, A. İ. (2013). Estimation of P (X> Y) with Topp–Leone distribution. Journal of Statistical Computation and Simulation, 83(2), 326-339.

[5] Srinivasa Rao, G., Kantam, R. R. L., Rosaiah, K., & Pratapa Reddy, J. (2013). Estimation of stress–strength reliability from inverse Rayleigh distribution. journal of Industrial and production Engineering, 30(4), 256-263.

[6] Ghitany, M. E., Al-Mutairi, D. K., & Aboukhamseen, S. M. (2015). Estimation of the reliability of a stress-strength system from power Lindley distributions. Communications in Statistics-Simulation and Computation, 44(1), 118-136

[7] Asgharzadeh, A., Valiollahi, R., & Raqab, M. Z. (2017). Estimation of Pr (Y< X) for the two-parameter generalized exponential records. Communications in Statistics-Simulation and Computation, 46(1), 379-394.

[8] Sharma, V. K. (2018). Bayesian analysis of head and neck cancer data using generalized inverse Lindley stress–strength reliability model. Communications in Statistics-Theory and Methods, 47(5), 1155-1180.

[9] Selman, B. B. (2019). Comparison Different Estimation Methods for System Reliability in Stress-Strength Models (Doctoral dissertation, University of Baghdad).

[10] Al-Omari, A. I., Almanjahie, I. M., Hassan, A. S., & Nagy, H. F. (2020). Estimation of the stress-strength reliability for exponentiated Pareto distribution using median and ranked set sampling methods. CMC-Computers, Materials & Continua, 64(2), 835-857.

[11] Almarashi, A. M., Algarni, A., Hassan, A. S., Elgarhy, M., Jamal, F., Chesneau, C., ... & Nagy, H. F. (2021). A new estimation study of the stress-strength reliability for the Topp–Leone distribution using advanced sampling methods. Scientific Programming, 2021.

[12] Kızılaslan, F. (2017). Classical and Bayesian estimation of reliability in a multicomponent stress–strength model based on the proportional reversed hazard rate mode. Mathematics and Computers in Simulation, 136, 36-62.

[13] Mokhlis, N. A., & Khames, S. K. (2011). Reliability of multi-component stress–strength models. Journal of the Egyptian Mathematical Society, 19(3), 106-111.

[14] Rao, G. S. (2012). Estimation of reliability in multicomponent stress-strength based on generalized exponential distribution. Revista Colombiana de Estadística, 35(1), 67-76.

[15] Nadar, M., & Kızılaslan, F. (2015). Estimation of reliability in a multicomponent stress-strength model based on a Marshall-Olkin bivariate Weibull distribution. IEEE Transactions on Reliability, 65(1), 370-380.

[16] Kizilaslan, F., & Nadar, M. (2015). Classical and Bayesian estimation of reliability in multicomponent stress-strength model based on Weibull distribution. Revista Colombiana de Estadística, 38(2), 467-484.

[17] Karam, N. S., & Jani, H. H. (2016). Estimation of Reliability in Multicomponent Stress-Strength Model Following Burr-III Distribution. Al-Mustansiriya Journal of Science and Education, 17(1), 329-342.

[18] Hassan, M. K., & Alohali, M. I. (2018). Estimation of reliability in a multicomponent stress-strength model based on generalized linear failure rate distribution. Hacettepe Journal of Mathematics and Statistics, 47(6), 1634-1651.

[19] Ali, A., Khaliq, S., Ali, Z., & Dey, S. (2018). Reliability estimation of s-out-of-k system for non-identical stress–strength components. Life Cycle Reliability and Safety Engineering, 7(1), 33-41.

[20] Kohansal, A. (2019). On estimation of reliability in a multicomponent stress-strength model for a Kumaraswamy distribution based on progressively censored sample. Statistical Papers, 60(6), 2185-2224.

[21] Shubhashree, J. (2019). Estimation of multicomponent system reliability for a bivariate generalized Rayleigh distribution. International Journal of Computational and Theoretical Statistics, 6(1).

[22] Mahto, A. K., Tripathi, Y. M., & Kızılaslan, F. (2020). Estimation of reliability in a multicomponent stress–strength model for a general class of inverted exponentiated distributions under progressive censoring. Journal of Statistical Theory and Practice, 14(4), 1-35.

[23] Sauer, L., Lio, Y., & Tsai, T. R. (2020). Reliability inference for the multicomponent system based on progressively type II censored samples from generalized pareto distributions. Mathematics, 8(7), 1176.

[24] Rasekhi, M., Saber, M. M., & Yousof, H. M. (2020). Bayesian and classical inference of reliability in multicomponent stress-strength under the generalized logistic model. Communications in Statistics-Theory and Methods, 50(21), 5114-5125.

[25] Mezaal, A. R., Mansour, A. I., & Karm, N. S. (2020, July). Multicomponent stress-strength system reliability estimation for generalized exponential-poison distribution. In Journal of Physics: Conference Series (Vol. 1591, No. 1, p. 012042). IOP Publishing.

[26] Kohansal, A., & Shoaee, S. (2021). Bayesian and classical estimation of reliability in a multicomponent stress-strength model under adaptive hybrid progressive censored data. Statistical Papers, 62(1), 309-359.

[27] Wang, L., Lin, H., Ahmadi, K., & Lio, Y. (2021). Estimation of Stress-Strength Reliability for Multicomponent System with Rayleigh Data. Energies, 14(23), 7917.

[28] Bhattacharyya, G. K., & Johnson, R. A. (1974). Estimation of reliability in a multicomponent stress-strength model. Journal of the American Statistical Association, 69(348), 966-970.

[29] Al-Shomrani, A., Arif, O., Shawky, A., Hanif, S., & Shahbaz, M. Q. (2016). Topp–Leone family of distributions: Some properties and application. Pakistan Journal of Statistics and Operation Research, 443-451.

[30] Vicari, D., Van Dorp, J. R., & Kotz, S. (2008). Two-sided generalized Topp and Leone (TS-GTL) distributions. Journal of Applied Statistics, 35(10), 1115-1129.

[31] Carlin, B. P., & Louis, T. A. (2008). Bayesian methods for data analysis. CRC Press.

[32] Yousif, A. H. (2020). PROPOSED BAYESIAN ESTIMATORS OF THE SCALE PARAMETER IN NAKAGAMI DISTRIBUTION. Pak. J. Statist, 36(4), 355-362.

[33] Akgül, F. G. (2019). Reliability estimation in multicomponent stress–strength model for Topp-Leone distribution. Journal of Statistical Computation and Simulation, 89(15), 2914-2929.

[34] Lindley, D. V. (1980). Approximate bayesian methods. Trabajos de estadística y de investigation operativa, 31(1), 223-245.

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