Domination (set and Number) in Neutrosophic Soft over Graphs

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Abstract: By combining the ideas of neutrosophic soft sets and oversets in graphs and replacing each crisp or neutrosophic set by overset at each vertex and edge, we present a new framework for treating neutrosophic soft information (NSI) in this research.

Keywords: Single valued neutrosophic overgraph, dominating set, domination number.

1. Introduction

There is a lot of study being done in the subject of dominance in graph theory.a book on dominance that lists 1222 works in this field was published in 1998.A collection of vertices D that each vertex is either in or adjacent to a vertex in D is referred to as a dominant set in a graph[1]. The fuzzy set theory is a concept that Zadeh (1965) developed as a remedy for the problem of handling uncertainties. As an extension of the fuzzy set concept, Atanassov [3] offered intuitionistic fuzzy sets. When the information at hand is insufficient to allow the typical fuzzy set to find the imprecision, the idea of intuitionistic fuzzy(IF) sets might be taken into consideration as an alternate approach.

In fuzzy sets, the only factor taken into account is the degree of approval. If not, an (IF) set's (truth(T) and falsity(F)) memberships functions are described, where the sum of both values must be smaller than one [4]. Fuzzy graph theory was first developed in 1975 by Azriel Rosenfied [8].

Despite being so young, it is expanding quickly and has significant applications across many different fields. Actually, a significant portion of issues include ambiguous, insufficient, and inconsistent data. The most effective mathematical tool for handling this kind of problem is thought to be neutrosophic sets [2].Smarandache started the neutrosophic set concept in

1995.(T, I, F) memberships are three components of a neutrosophic set that are declared independently to address problems with inconsistent, imprecise, and indeterminate data.
In order to specify set theoretic operators on a certain kind of neutrosophic set known as single valued neutrosophic set(SVNS), Wang came up with the concept of (SVNS)[6]. The soft set notion had introduced by Molodtsov [5] as a ground-breaking method for handling uncertainty in mathematics. From a parameter standpoint, Molodtsov's soft sets offer us a fresh method to achieve this. Soft sets have been found to have potential uses in many different domains. in 1995 when sumrand [9]studded the state of over loud the membership > 1, contract to the traditional set with membership<= 1 he introduced overset concept. In this article we will introduce the conception of soft overset and soft overgraph

2. Preliminaries

Definition: 2.1. A Single-Valued Neutrosophic Overset (SVNOS) A is defined as:

$$A = \left\{ \begin{matrix} (v, < T(v), I(v), F(v) >): at\ least\ one\ of\ 1 \leq\ T(v), I(v), F(v) \leq 2 \\ and\ non\ of\ them < 0\ , for\ all\ v \in V \end{matrix} \right\}, [7,10]$$

Definition: 2.2. *Let X be a universe and Z be a set of parameters. Consider $A \subset Z$ and P (X) be the set of all Single valued neutrosophic oversets of X.*

The pair (F, A) is known to be the neutrosophic soft overset (NSOS) over X, where F is a function given by $F: A \to P(X)$

Definition: 2.3.*let $G^* = (V, E)$ is an underline graph and, A is nonempty set of parameters.*

A neutrosophic soft overgraph (NSOG) $G = (G^*, F, K, A)$ is a quadruple set *if it meets the conditions below:*

(1) (F, A) *is a (NSOS) over V. i.e* $F: A \to P(V)$

(2) (K, A)*is a (NSOS) over E. i.e* $K: A \to P(E)$

(3) $(F(e), K(e))$ *is aneutrosophic over graph of G^* then*

$T_{K(e)}(v_i v_j) \leq min\{T_{F(e)}(v_i), T_{F(e)}(v_j)\}$,

$I_{K(e)}(v_i v_j) \leq Min\{I_{F(e)}(v_i), I_{F(e)}(v_j)\}$,

$F_{K(e)}(v_i v_j) \geq Max\{I_{F(e)}(v_i), I_{F(e)}(v_j)\}$,

$\forall\ e \in A\ and\ v_i, y \in V\ st\ v_i v_j \in E \subset V \times V$

Such that not all of $T_{K(e)}v_j, I_{K(e)}v_j, F_{K(e)}v_j\}$ <1 and no one of them <0.

Also there exist at lest $T_{K(e)}v_i v_j, I_{K(e)}v_i v_j, F_{K(e)}v_i v_j\}$ >1 and all of them<0.

Note: *the (NSOG) is called pure if all of it vertices and edges are over sets*

Example: 2.1 *Consider a simple graph $G^* = (V, E)$ and such that*

$v = \{v_{1,} v_{2,} v_{3,} v_4\}$ *and* $E = \{v_1 v_{2,} v_1 v_{3,} v_1 v_4, v_2 v_4, v_3 v_4\}$ *and* $A = \{e_1, e_2\}$ *set of parameters.*

let (F, A) be a (NSOS) over V with approximation function (AF) by $F: A \to P(X)$, *and let (K, A) be a (NSOS) over E with (AF)* $K: A \to P(E)$ *are defined by*

$H(e_1)$	$F\ (e_1)$	$(v_1, 0.5,1.4,0.6)$	$(v_2, 1.2,0.6,0.7)$	$(v_3, 1.5,0.4,0.5)$	$(v_4, 0.1,0.4,1.3)$

	$K\,(e_1)$	$(v_1v_2, 0.5,0.6,0.7)$	$(v_1v_3, 0.5,0.4,0.6)$	$(v_1v_4, 0.1,0.4,1.3)$	
$H(e_2)$	$F\,(e_2)$	$(v_1, 0.2,0.3,1.5)$	$(v_2, 0.4,1.7,0.3)$	$(v_3, 1.6,0.7,0.4)$	$(v_4, 1.3,0.4,0.5)$
	$K(e_2)$	$(v_1v_3, 0.2,0.3,1.5)$	$(v_2v_4, 0.4,0.4,0.5)$	$(v_3v_4, 1.3,0.4,0.5)$	

then $H\ (e_1) = \{\ F\ (e_1), K(e_1)\}$ *and* $H\ (e_2) = \{\ F\ (e_2), K(e_2)\}$ *are neutrosophic Overgraphs according to the parameters* e_1 *and* e_2 *respectively as seems as follows: -*

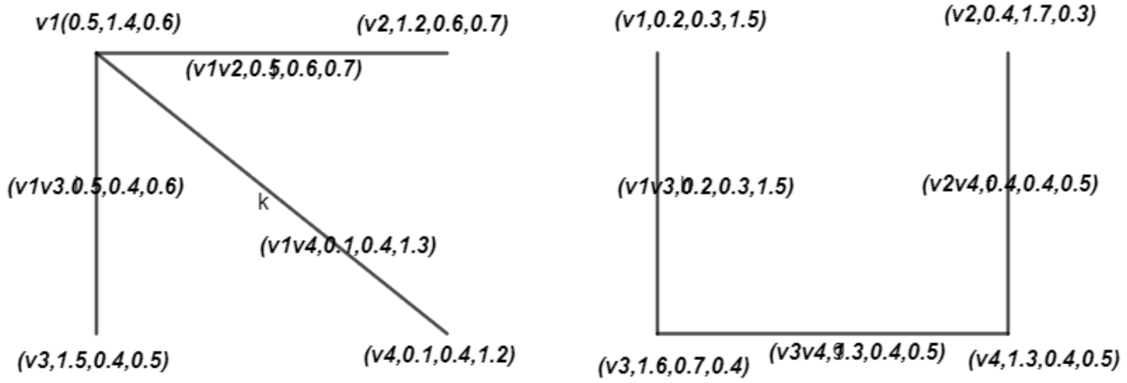


Figure 2.1 G=(H(e1),H(e2)) Neuyrosophic soft overgraph

Example: *2.2. Let* $a(1.4,0.7,0.5), b(1.2,0.5,0.2), c(0.2,0.3,1.3)$ *and* $ab(1.2,0.5,0.5), ac(0.2,0.3,1.3), bc(0.2,0.3,1.3)$

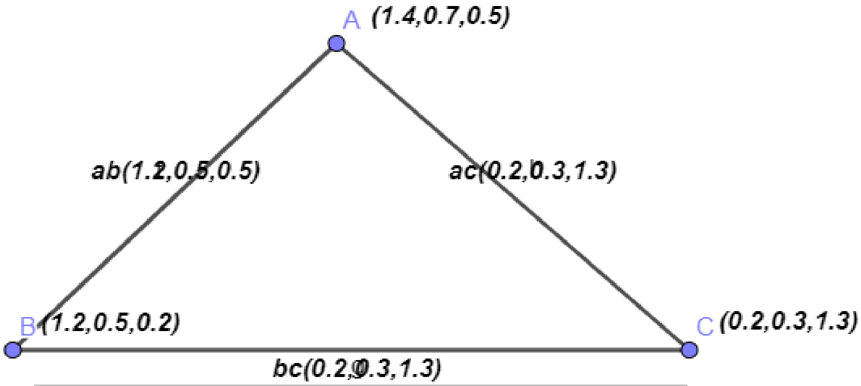


Figure 2.2 H is Neutrosophic overgraph but not soft

Example: 2.3. *Consider the graph* $G^* = (V, E), where\ V = \{a, b, c, d\} and\ E = \{ab, bd, cd\}$.

Let

$A = \{e_1, e_2\} and\ (F, A)\ be\ a\ neutrosophic\ soft\ set\ (NSS) over\ V\ with\ approximate$

function by $F: A \to P(X)$ *and (K, A) be a (NSS) over E with* $K: A \to P(E)$ *gevin by*

H (e_1)	F (e_1)	a(,0.5,0.6,0.7)	(b,0.4,0.5,0.3)	(c,0.7,0.5,0.8)	(d,0.4,0.9,0.5)
	K (e_1)	(ab,0.3,0.4,0.5)	(bd,0.3,0.4,0.4)	(ac,0.4,0.3,0.6)	
H (e_2)	F (e_2)	a(,0.4,0.5,0.2)	(b,0.3,0.6,0.8)	(c,0.3,0.4,0.5)	(d,0.7,0.8,0.5)
	K (e_2)	(ab,0.2,0.3,0.5)	(bc,0.1,0.3,0.4)	(cd,0.2,0.2,0.4)	

It is clearly that H (e_1) *and H* (e_2) *are neutrosophic graphs according to the parameters* e_1 *and* e_1 *respectiviely as follows*

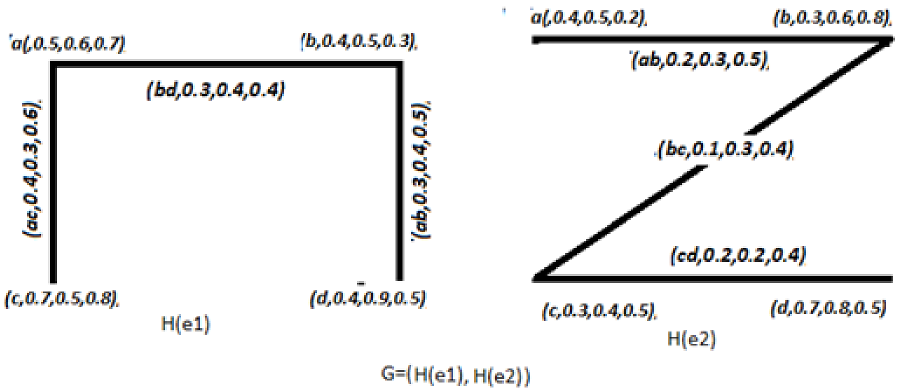


Figure 2.3 Neutrosophic soft graph but not overgraph

Definition: 2.4. *In a (NSOG)* $G = (G^*, F, K, A)$, *the vertex* v_i *is adjacent to* v_j *if*

$$T_{K(e)}(v_i v_j) \le \text{Min}\{T_{F(e)}(x), T_{F(e)}(y)\},$$
$$I_{K(e)}(v_i v_j) \le Min\{I_{F(e)}(x), I_{F(e)}(y)\}$$
$$F_{K(e)}(v_i v_j) \ge Max\{I_{F(e)}(x), I_{F(e)}(y)\},$$
$$\forall \; e \in A \text{ and } x, y \in V \text{ st } xy \in E \subset V \times V$$

Definition: 2.5. *The degree of any vertex* u *in a (NSOG)* $G = (G^*, L, K, A)$, *is denoted by* $d(u) = (d_{TL(e)}(u), (d_{IL(e)}(u), (d_{FL(e)}(u))$ *where*
$d_{TL(e)}(u) = \sum_{e\in A}(\sum_{u\in V, u\ne v} T_{K(e)}(v,u)$, $d_{IL(e)}(u) = \sum_{e\in A}(\sum_{u\in V, u\ne v} I_{K(e)}(v,u)$
$d_{FL(e)}(u) = \sum_{e\in A}(\sum_{u\in V, u\ne v} F_{K(e)}(v,u)$ *are known as the degree of (T,I,F)-membership vertex such that* v *and* u *are adjacent for all* $e \in A$.

Notes: *-1)* $\delta(G) = min\{d_g(v)/v \in V, e \in A.\}$ *is the minimum degree of* v

2) is $\Delta(G) = max\{d_g(v)/v \in V, e \; A.\}$ *is the maximum degree of* v

Definition 2.6. *If* $G = (G^*, J, K, A)$ *be (NSOG). The total degree* $td(x)$ *of* $x \in G$ *is obtained by* $td(x) = (td_{TJ(e)}(x), td_{IJ(e)}(x), td_{FJ(e)}(x))$ *where*

$$td_{TJ(e)}(x) = \sum_{e \in A} (\sum_{x \neq y \in V} T_{K(e)}(x,y) + T_{J(e)}(x,y)$$

$$td_{IJ(e)}(x) = \sum_{e \in A} \left(\sum_{x \neq y \in V} I_{K(e)}(x,y) + I_{J(e)}(x,y) \right)$$

$$td_{FJ(e)}(x) = \sum_{e \in A} (\sum_{x \neq y \in V} F_{K(e)}(x,y) + F_{J(e)}(x,y))$$ *Called the t- degree of (T, I and F) membership vertex respectively for all* $e \in A, x, y \in V$.

Example 2.4 let $G^* = (V, E)$ be a simple graph with V = {a, b, c, d} .

Suppose A = (J, A) be a(NS) over a set V with (NAF) J: A → ρ (V) and K: A -→ ρ (E) represented as

H (e_1)	*j* (e_1)	*a(,1.5,0.6,1.2)*	*(b,0.7,1.6,0.5)*	*(c,1.6,0.5,1.7)*	*(d,1.6,1.5,0.7)*
	K (e_1)	ab(0.7,0.6,1.2)	bd(0.7,1.4,0.7)	cd(1.3,0.5,1.7),	ac(1.5,0.5,1.4)
H (e_2)	*j* (e_2)	b(0.7,1.6,0.5)	*(b,0.3,0.6,0.8)*	c(1.7,0.6,0.5)	d(0.8,1.9,1.4)
	K (e_2)	ac(1.6,0.6,1.8)	dc(0.7,0.6,1.4)	bd(0.8,0.6,1.4),	ad(0.8,0.7,1.6)

obviously, H (e_1) = (J (e_1)), K (e_1))) and H (e_2) = (J (e_2), K (e_2)) are neutrosophic graphs with respect to the e1 and e_2 as shown in Figure 2.4

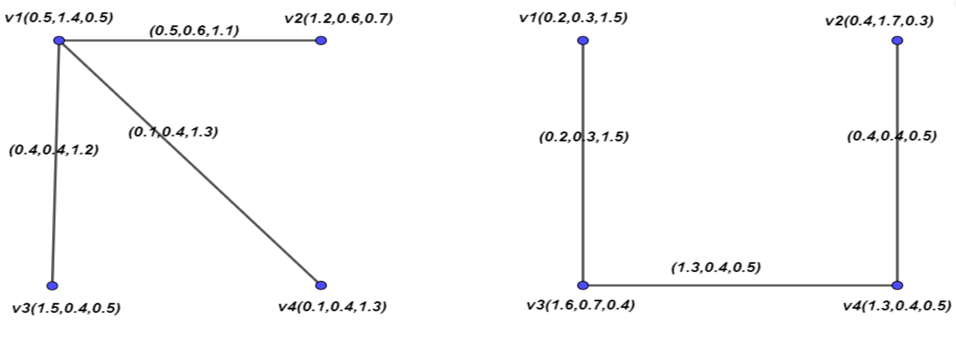


figure 2.1 Neutrosophic soft overgraph G=(H(e1),H(e2))

The degree of the graph H (e_1) vertices are as follows, deg(a) = (3.5,1.7,3.6), deg(b) = (1.4,2,1.7), deg(c) = (2.8,1.0,2.8),deg(d) = (3.3,2.5,3.1)

and total degree td(a)= (5,2.3,4.8), td(b)2.1,3.6,2.2), td(c)= (4.4,1.5,4.5), td(d)= (4.9,4,3.8).

Also, for H (e_2) degree of a, b, c, d will be as bellow,

deg(a) = (2.4,1.3,3.4), deg(b) = (0.8,0.6,1.4), deg(c) = (2.3,1.2,3.2), deg(d) = (2.3,1.9,4.4)

td(a)= (4,2,5.2), td(b)= (2.3,1.2,2.1), td(c)= (4,1.8,3.7), td(d)= (3.1,3.8,5.8)

Definition: 2.7. *a simple graph G= (V, E) with same degree for all* $v \in V$ *is said to be regular.*

Definition: 2.8. *suppose that* $G = (G^*, J, K, A)$ *be (NSOG) of a simple graph* $G^* = (V, E)$*, then G is defined as regular graph where H (e) is regular* $\forall\ e \in A$*,*

Definition: 2.9. $G = (G^*, J, K, A)$ *is called k- regular (NSOG) if H (e) be a regular with* $d_G(v) = k\ ,\forall v \in V and\ \forall\ e \in A$

Definition 2.10. *a graph* $H(e)$ *is called a totally regular neutrosophic ovregraph (NOG) if* $td_G(v) = td_G(u)\ ,\forall\ u, v \in H(e)$ *where* $e \in A$*. and it is k−totally regular (NOG) if* $td_G(v) = td_G(u) = k\ ,\forall\ u, v \in H(e_i)\ , e_i \in A$

Definition: 2.11. *a graph* $G = (G^*, J, K, A) = (H(e_1), \dots H(e_n))$ *is a totally regular (NSOG) where* $H\ (e_i)$ *be totally regular graph* $\forall\ e_i \in A, i = 1 \dots n.$

Definition: 2.12. *the order O (G) of the (NSOG)* $G = (G^*, F, K, A)$ *calculated by*
O (G) $=\sum_{e\in A}\{ \sum_{I=1}^{n} T_{F(e)}(v_i)\ , \sum_{I=1}^{n} I_{F(e)}(v_i)\ , \sum_{I=1}^{n} F_{F(e)}(v_i)\}$*, and the size S (G) of G calculated by*
S (G) = $\sum_{e\in A}\{ \sum_{I=1}^{n} T_{K(e)}(v_i v_j)\ , \sum_{I=1}^{n} I_{K(e)}(v_i v_j)\ , \sum_{I=1}^{n} F_{K(e)}(v_i v_j)\}$*,*

Definition: 2.13. Let $G = (G^*, J, K, A)$ be a (NSOG). then

1) Vertex- cardinality is $|V| = \sum_{e\in A}(\left|\sum_{v_i} \frac{T_{j(e)}(x) + I_{j(e)}(x) - F_{j(e)}(x)}{2}\right| + \frac{n}{2}$

   $where\ n\ is\ number\ of\ vertices$ in G
2) Edge -cardinality is $|E| = \left|\sum_{v_i v_j} \frac{T_{j(e)}(xy) + I_{j(e)}(xy) - F_{j(e)}(xy)}{2}\right| + \frac{m}{2}$

   Where m is number of edges in G

3) cardinality of G will be $|G| = \sum_{e\in A}(\ |V| +\ |E|)$

Example 2.5 Consider the above example 2.4, here H (e_1) and H (e_2) are (NSOG)s of G with respect to the parameter e_1, the vertex cardinality=4.75, the edge cardinality=3.35 and cardinality of G =8.1

according to the parameter e_2, $|V|$= 4.5, $|E|$ = 2.6 and $|G|$= 7.1

Definition: 2.14. Let $G_1 = (G^*, F_1, K_1, A)$ *and* $G_2 = (G^*, F_2, K_2, B)$ *be two (NSOG).*

Then G_1 *be a neutrosophic soft subovergraph of* G_2 if

$i)\ A \subseteq B$ $ii)\ H_1(e)$ is partial subovergraph of $H_2(e)\ \forall\ e \in A$

Definition: 2.15. The graph $G_1 = (G^*, F_1, K_1, A)$ is known as spanning neutrosophic soft sub-overgraph of $G = (G^*, F, K, B)$ if $i)\ B \subseteq A$ $ii)\ V_1 = V$

$ii)$

$\{T_{F1(e)}(v) = T_{F(e)}(v)$, $I_{F1(e)}(v) = I_{F(e)}(v)$,

$F_{F1(e)}(v) = F_{F(e)}(v)\}$ $for\ all\ e \in B, v \in V$

Definition: 2.16.*let* $G_1 = (F_1, K_1, A)$ *and* $G_2 = (F_2, K_2, B)$ *be two neutrosophic soft Overgraphs. The intersection of* G_1*and* G_2 *is denoted by* $G = G_1 \cap G_2 = (F, K, A \cup B), where$

1) $(F, A \cup B) is\ a\ neutrosophic\ overset\ over V = V_1 \cap V_2$,

2) $(K, A \cup B) is\ a\ neutrosophic\ overset\ over\ E = E_1 \cap E_2$, *where memberships function of* G $defined\ by$

$a)$ $T_{F(e)}(v) = \{T_{F_1(e)}(v)\ if\ e \in A - B\ T_{F_2(e)}(v)\ if\ e \in B - A\ T_{F_1(e)}(v) \vee T_{F_2(e)}(v)\ \ Where\ One\ of\ T_{F_1(e)}(v), T_{F_2(e)}(v) \geq 1\ T_{F_1(e)}(v) \wedge T_{F_2(e)}(v)\ \ Otherwise\ \ if\ e \in A \cap B\}$

$I_{F(e)} = \{I_{F_1(e)}(v)\ if\ e \in A - B\ I_{F_2(e)}(v)\ if\ e \in B - A\ I_{F_1(e)}(v) \vee I_{F_2(e)}(v)\ \ Where\ One\ of\ I_{F_1(e)}(v), I_{F_2(e)}(v) \geq 1\ I_{F_1(e)}(v) \wedge I_{F_2(e)}(v)\ \ Otherwise\ \ if\ e \in A \cap B\}$,

$$F_{F(e)} = \begin{Bmatrix} F_{F_1(e)}(v) if\ e \in A - B \\ F_{F_2(e)}(v) if\ e \in B - A, \\ F_{F_1(e)}(v) \vee F_{F_2(e)}(v)\ \ if\ e \in A \cap B \end{Bmatrix}$$

$$b)\ T_{k(e)}(uv) = \begin{Bmatrix} T_{k_1(e)}(uv) if\ e \in A - B \\ T_{k_2(e)}(uv) if\ e \in B - A \\ T_{K_1(e)}(uv) \vee T_{K_2(e)}(uv)\ Where\ One\ of\ T_{F_1(e)}(v), T_{F_2(e)}(v) \geq 1 \\ T_{k_1(e)}(uv) \wedge T_{k_2(e)}(uv)\ \ Otherwise\ \ if\ e \in A \cap B \end{Bmatrix}$$

$$I_{k(e)}(uv) = \begin{Bmatrix} I_{k_1(e)}(uv) if\ e \in A - B \\ I_{k_2(e)}(uv) if\ e \in B - A \\ I_{K_1(e)}(uv) \vee I_{K_2(e)}(uv)\ Where\ One\ of\ I_{K_1(e)}(v), I_{K_2(e)}(v) \geq 1 \\ I_{k_1(e)}(uv) \wedge I_{k_2(e)}(uv)\ \ if\ e \in A \cap B \end{Bmatrix}$$

$$F_{k(e)}(uv) = \left\{ \begin{array}{c} F_{k_1(e)}(uv) if \ e \in A - B \\ F_{k_2(e)}(uv) if \ e \in B - A \\ F_{K_1(e)}(uv) \vee \ I_{K_2(e)}(uv) \ \ if \ e \in A \cap B \end{array} \right\}, for\ all\ u, v \in V$$

Definition: 2.17. *Let* $G_1 = (F_1, K_1, A)$ *and* $G_2 = (F_2, K_2, B)$ *be two neutrosophic soft Overgraphs. The union of* G_1 *and* G_2 *is denoted by* $G = G_1 \cup G_2$
Then $G = (F, K, A \cup B), where$
1) $(F, A \cup B) is\ a\ neutrosophic\ overset\ over\ V = V_1 \cup V_2$,
2) $(K, A \cup B) is\ a\ neutrosophic\ overset\ over\ E = E_1 \cup E_2$,
3) *The (T, I, F) membership functions of G for all* $u, v \in V$
$defined\ by\ same\ way\ in\ the\ intersection\ definition\ above.$

Example: 2.5. *Let* $A = \{e_1, e_2\}$ *and* $B = \{e_1, e_3\}$ *be two parameters sets.*
Let G_1 *and* G_2 *be two neutrosophic soft overgraphs defined by* $G_1 = \{H_1(e_1), H_1(e_2)\}$, *where*

$H_1(e_1)$	$F_{(e1)}$	$(u_1, 0.4,1.5,0.6)$	$(u_2, 0.3,0.5,1.2)$	$(u_3, 0.5,0.3,1.4)$	$(u_4, 1.6,0.3,0.8)$
	$k_{(e1)}$	$(u_1u_2, 0.3,0.5,1.2)$	$(u_1u_4, 0.4,0.3,0.8)$	$(u_2u_3, 0.3,0.3,1.4)$	$(u_2u_4, 0.3,0.3,1.2)$
$H_1(e_2)$	$F_{(e2)}$	$(u_1, 1.3,0.4,0.6)$	$(u_2, 1.2,0.5,0.4)$	$(u_3, 1.5,0.3,0.4)$	$(u_4, 1.4,0.5,1.2)$
	$k_{(e2)}$	$(u_1u_2, 1.2,0.4,0.6)$	$(u_2u_3, 1.2,0.3,0.4)$	$(u_2u_4, 1.2,0.3,0.4)$	

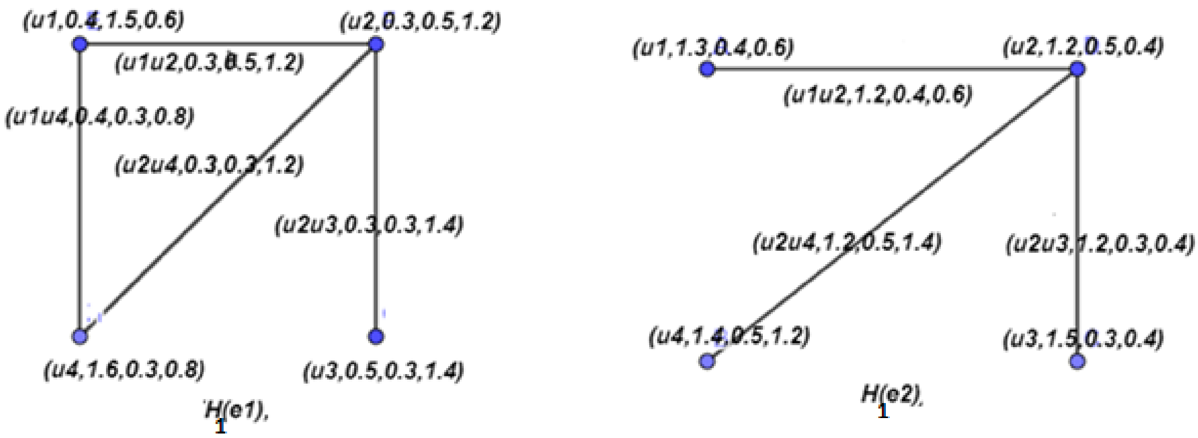


Figure 2.5 G=(H(e1),H(e2))

and $G_2 = \{H_2(e_1), H_2(e_3)\}$,*where*

$H_2(e_1)$	$F_{(e1)}$	$(u_1, 1.3,0.4,0.5)$	$(u_2, 0.6,0.7,1.4)$	$(v_1, 1.2,0.5,1.5)$	$(u_3, 0.7,0.4,1.2)$
	$k_{(e1)}$	$(u_1v_1, 1,0.3,1.6)$	$(u_1u_2, 0.6,0.4,1.4)$	$(u_2u_3, 0.4,0.3,1.7)$	
$H_2(e_3)$		$(u_1, 0.4,1.7,1.3)$	$(u_2, 1.2,1.5,1.6)$	$(v_1, 1.3,0.4,1.6)$	$(u_3, 0.6,1.5,0.7)$

	$F_{(e2)}$				
	$k_{(e2)}$	$(u_1v_1, 0.4,0.4,1.8)$	$(u_2v_1, 1,0.4,1.7)$	$(u_2u_3, 0.5,1.3,1.6)$	

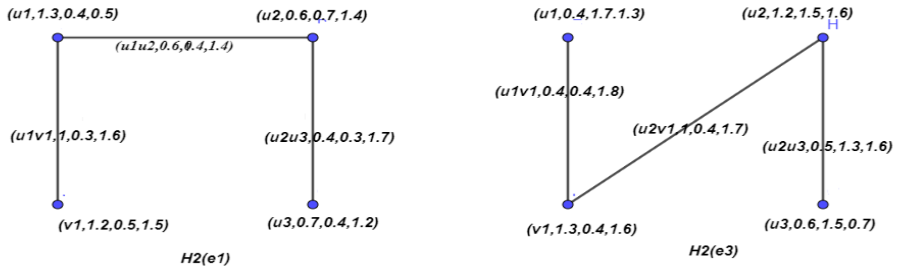
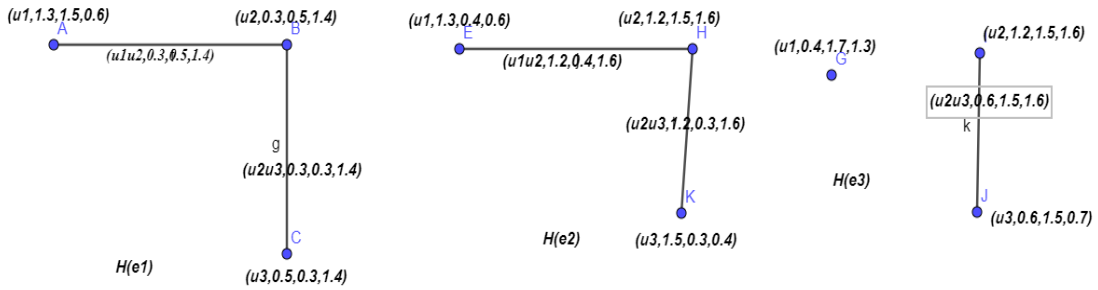


Figure 2.6 G2=(H2(e1),H2(e3))

Then $G_1 \cap G_2$ *will be*

$H(e_1)$	$F_{(e1)}$	$(u_1, 1.3,1.5,0.6)$	$(u_2, 0.3,0.5,1.4)$	$(u_3, 0.5,0.3,1.4)$
	$k_{(e1)}$	$(u_2u_3,0.3,0.3,1.4)$	$(u_1u_2,0.3,0.5,1.4)$	
$H(e_2)$	$F_{(e2)}$	$(u_1, 1.3,0.4,0.6)$	$(u_2, 1.2,1.5,1.6)$	$(u_3, 1.5,0.3,0.4)$
	$k_{(e2)}$	$(u_1u_2, 1.2,0.4,1.6)$	$(u_2u_3, 1.2,0.3,1.6)$	
$H(e_2)$	$F_{(e3)}$	$(u_1, 0.4,1.7,1.3)$	$(u_2, 1.2,1.5,1.6)$	$(u_3, 0.6,1.5,0.7)$
	$k_{(e3)}$	$(u_1u_2,0.6,1.5,1.6)$		



G =(H(e1),H(e2),H(e3))

and $G_1 \cup G_2$ *will be*

$H(e_1)$	$F_{(e1)}$	$(u_1, 1.3,1.5,0.5)$	$(u_2, 0.3,0.5,1.4)$	$(u_3, 0.5,0.3,1.4)$	$(u_4, 1.6,0.3,0.8)$	$(v_1, 1.2,0.5.5,1.1.5)$
	$k_{(e1)}$	$u_1u_2, 1.2,0.3,0.5)$	$(u_2u_3, 1.3,0.4,0.6)$	$(u_2u_4, 1.2,0.5,1.1)$	$(u_1v_1, 0.2,0.3,1.2)$	$u_2v_1, 0.3,1.1,1.4)$
$H(e_2)$	$F_{(e2)}$	$(u_1, 1.3,0.4,0.6)$	$(u_2, 1.2,0.5,0.4)$	$(u_3, 1.5,0.3,0.4)$	$(u_4, 1.4,0.5,1.2)$	
	$k_{(e2)}$	$(u_1u_2, 0.2,0.5,1.1)$	$(u_2u_3, 1.2,0.5,0.6)$	$(u_2u_4, 0.2,0.1,1.1)$	$(u_2v_1, 0.3,0.5,1.2)$	$(u_3v_1, 0.4,0.5,1.2)$
$H(e_2)$		$(u_1, 0.4,1.7,1.3)$	$(u_2, 1.2,1.5,1.6)$	$(u_3, 0.6,1.5,0.7)$	$(V_1, 1.3,0.4,1.6)$	

	$F_{(e3)}$					
	$k_{(e3)}$	$(u_1u_2, 0.2,0.5,1.1)$	$(u_2u_3, 1.2,0.5,0.6)$	$(u_2u_4, 0.2,0.1,1.1)$	$(u_2v_1, 0.3,0.5,1.2)$	$(u_3v_1, 0.4,0.5,1.2)$

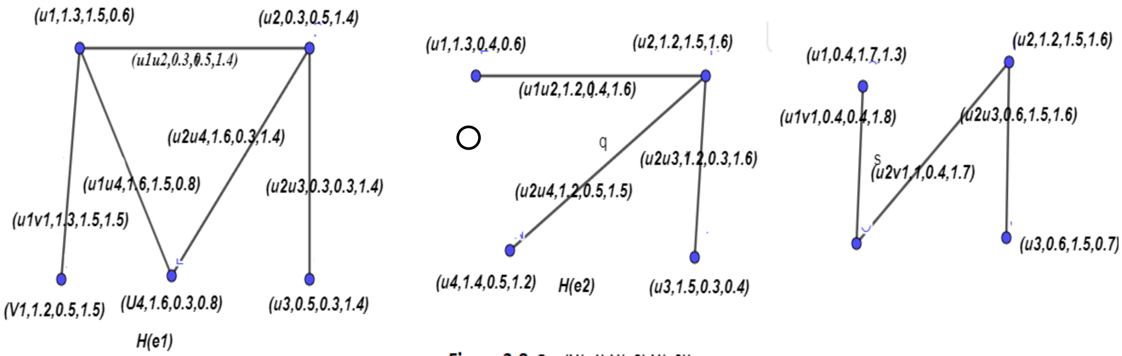


Figure 2.8 G =(H(e1),H(e2),H(e3))

Proposition: 2.1. *If G_1, G_2 are neutrosophic soft pure overgraphs then $G_1 \cup G_2$ is a neutrosophic soft pure overgraph if for each pair of vertices v_i, v_j have corresponding component which is greater than 1 or either $F_i \geq 1$ or $F_j \geq 1$*

Proof: *Case1: if all the components ≥ 1 the result is trivially*

Case2: if $\forall i \neq j, T_i \geq 1$, or $I_j \geq 1$ then either $T_K(v_i, v_j 1$ or $I_K(v_i, v_j)1$ which implies that $(T_K(v_i, v_j), I_K(v_i, v_j), F_K(v_i, v_j))$ is overset

Case 3: if either $F_i \geq 1$ or $F_j \geq 1$ then (T_K, I_K, F_K) is also overset

***Definition:* 2.18.** the graph $G^c = (F, K^c, A)$ *is a complement* of a neutrosophic soft overgraph $G = (F, K, A)$ where *the memberships at* K^c are defined as:

$i)\ T_{k^c(e)}(u,v) = T_{F(e)}(u) \wedge T_{F(e)}(v) - T_{K(e)}(u,v),$

$ii)\ I_{k^c(e)}(u,v) = I_{F(e)}(u) \wedge I_{F(e)}(v) - I_{K(e)}(u,v),$

$iii)\ F_{k^c(e)}(u,v) = F_{F(e)}(u) \vee F_{F(e)}(v) - F_{K(e)}(u,v),$

$for\ all\ u, v \in V, for\ all\ e \in A$

Example; 2.5. Consider simple graph $G* = (V, E)$, where $V = \{v_1, u_2, u_3, u_4\}$ and $E = \{u_1u_2, u_2u_4, u_3u_4\}$

Let A = $\{e_1, e_2\}$ and let (F, A) be a (NSOS) over V with F: A → P (V)

and (K, A) be a (NSOS) over E with its (AF) K: A → P(E) as follows:

$H_2(e_1)$	$F_{(e1)}$	{(a,1.5,0.6, 0.7)	(b, 0.4, 0.5, 1.3)	(c, 0.7, 1.5, 0.8)	(d,1.4,0.9,0.5)
	$k_{(e1)}$	$(ab, 0.3,0.4,1.4)$	$(bd, 0.3,0.4,1.3)$	$(ac, 0.6,0.6,0.8)$	
$H_2(e_2)$	$F_{(e2)}$	{(a,0.4,0.5, 1.2)	(b,1.3, 0.6, 0.8)	(c,0.3, 1.4, 0.5),	(d,0.7, 1.8, 0.5)
	$k_{(e2)}$	$(ab, 0.2,0.3,1.3)$	$(bc, 0.1,0.5,0.8)$	$(cd, 0.2,1.5,0.5)$	

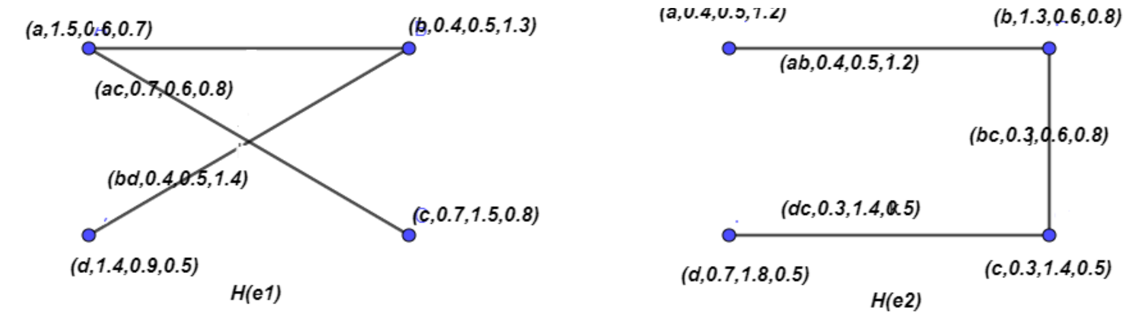


Figure 2.9 G=(H(e1),H(e2))

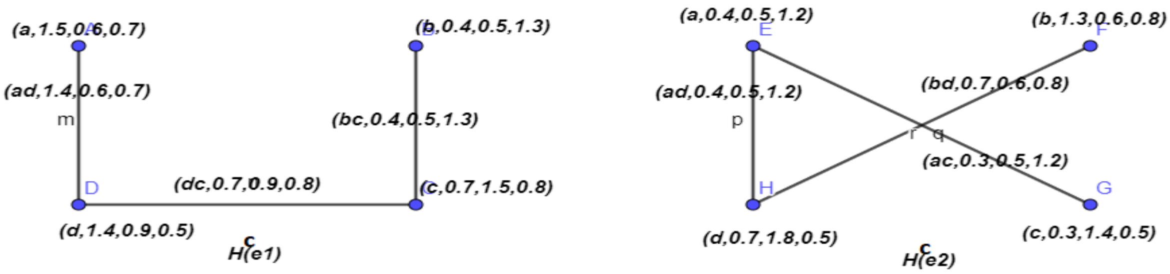


Figure 2.10 $G^C=(H^C(e1),H^C(e2))$

$I_{F(e)}(v)$

iii) $F_{K(e)}(uv) = F_{F(e)}(u) \vee F_{F(e)}(v)$

Definition: 2.19. a neutrosophic overgraph on a simple graph H^*= (V, E)

i) Is said to be strong neutrosophic overgraph (SNOG) if uv *be an effective edge for all* $uv \in E$

ii) is complete (CVOG) if $\forall u, v \in V \;\exists$ *an effective edge* $uv \in E$

Definition: 2.20. a (NSOG) is complete neutrosophic soft overgraph(CNSOG) if H (e) is complete (NOG) for all $e \in A$

Definition: 2.21. a (NSOG) is Strong neutrosophic soft overgraph (SNSOG) if H (e) is strong neutrosophic overgraph for each $e \in A$

Note: complement of pure (NSOG) is not necessary to be pure (NSOG)

Proposition: 2.2. complement of strong neutrosophic soft overgraph is strong neutrosophic soft overgraph

Proposition: 2.3. if G **be** *(SNSOG) then* $G \cup G^C$ *is (SNSOG)*

Definition: 2.22. *In a (NSOG) G of a simple graph* $G^* = (V, E)$ *for any* $x, y \in V$ *are called pair of neighbors in G if at least two of the components (*$T_{K(e)}$ *(xy),* $I_{K(e)}$ *(xy),* $F_{K(e)}$ *(xy)) are not equal to zero* $\forall\, e \in A$.

Definition: 2.23. *A path in (NSOG) is a sequence of various vertices* $v_1, v_2, \ldots, v_n$*, and each pair of* v_i, v_{i+1} $, i = 1,2, \ldots, n$ *are neighbors*.

Definition: 2.24. *The length of the path* $P = v_1, v_2, \ldots, v_n (n > 0)$ *in (NSOG) is* $l(p) = n - 1$

Definition: 2.25.*If* $v_i, v_j \in A$ *in G and if there exist a path p which connect them , the strength of p is defined as*

$$\left(min_i T_{K(e)}(v_i\ \ v_{i+1}), min_i I_{K(e)}(v_i\ \ v_{i+1}), max_i F_{K(e)}(v_i\ v_{I+1})\right)\ for\ i = 1 \ldots j - 1$$

Definition: 2.26. *If* $v_i, v_j \in V \subseteq G$*, then each of*

$$T^{\infty}{}_{K(e)}\,(vi\,, vj) = sup\left\{T^{l}{}_{K(e)}\left(v_i\,, v_j\right): l = 1,2, \ldots n, e \in A\right\},$$

$$I^{\infty}{}_{K(e)}\,(vi\,, vj) = sup\left\{I^{l}{}_{K(e)}\left(v_i\,, v_j\right): l = 1,2, \ldots n, e \in A\right\}\ and$$

$$F^{\infty}{}_{K(e)}\,(vi\,, vj) = inf\left\{F^{l}{}_{K(e)}\left(v_i\,, v_j\right): l = 1,2, \ldots n, e \in A\right\}$$

where $T_{K(e)}, I_{K(e)}, F_{K(e)}$ *are strength of connectedness of (truth, indeterminace, facility) membership between* vi *and* vj *respectively*.

Definition: **2.27.**: *If* $vi\,, vj$ *are connected by path of length m then*

$$T^{m}{}_{K(e)}\left(v_i\,, v_j\right) = sup\ sup \left\{\begin{matrix} T_{K(e)}(u, v_1) \wedge T_{K(e)}(v_1\,, v_2) \wedge T_{K(e)}(v_2\,, v_3) \ldots \wedge T_{K(e)}(v_{m-1}\,, v_m) \\ : u, v, v_1 \ldots v_{m-1}\,, v \in V \end{matrix}\right\},$$

$$I^{k}{}_{K(e)}\left(v_i\,, v_j\right) = sup\ sup \left\{\begin{matrix} I_{K(e)}(u, v_1) \wedge I_{K(e)}(v_1\,, v_2) \wedge I_{K(e)}(v_2\,, v_3) \ldots \wedge I_{K(e)}(v_{m-1}\,, v_m): \\ u, v, v_1 \ldots v_{m-1}\,, v \in V \end{matrix}\right\}, and$$

$$F^{k}{}_{K(e)}\left(v_i\,, v_j\right) = inf \left\{\begin{matrix} F_{K(e)}(u, v_1) \vee F_{K(e)}(v_1\,, v_2) \vee F_{K(e)}(v_2\,, v_3) \ldots \vee F_{K(e)}(v_{m-1}\,, v_m) \\ : u, v, v_1 \ldots v_{m-1}\,, v \in V \end{matrix}\right\}, e \in A.$$

Definition: 2.28. *neutrosophic soft overgraph G is said to be connected if for each* $u, v \in V$ *are connected by at least one uv-path* $\forall e \in A$

Definition: 2.29. *An* $edge(v_1 v_2)$ *is an effective edge, If the following holds*

$T_{K(e)}(v_1 v_2) \geq T^{\infty}{}_{K(e)}(v_1 v_2), I_{K(e)}(v_1 v_2) \geq I^{\infty}{}_{K(e)}(v_1 v_2)$ *and*

$F_{K(e)}(v_1 v_2) \geq F^{\infty}{}_{K(e)}(v_1 v_2)$.

Definition: 2.30. *let G be a (NSOG) of* $G^* =$ *(V, E),* $x \in V$ *,then* $N(x) = \{y: y \in V\}$ *is called neighborhood set of* x, *where* xy *is an effective edge*

Definition:2.31. *If* $x \in V$ *of a (NSOG) G of* $G^* =$ *(V, E) is an "isolated vertex" if* $T_{K(e)}(x, y) = I_{K(e)}(x, y) = F_{K(e)}(x, y) = 0$, *which means that a vertex x does not dominate any vertex y in G.* $\forall e \in A$.

Definition: 2.32. Let *G be a (NSOG) of* $G^* =$ *(V, E).*

If u, v ∈ V, so u is dominate v in G if uv be an effective edge.

Note: 1) Domination (D) is symmetric relation i.e., if x D y then y D $x, \forall x, y \in V$

2) $N(x) = \{y: y \in V, x\ D\ y\}$ is exactly collection of all y in V that dominated by x.

3) $If\ T_{K(e)}(x, y) < T^{\infty}{}_{K(e)}\ (x, y) and\ I_{K(e)}(x, y) < I^{\infty}{}_{K(e)}\ (x, y) and$

$F_{K(e)}(x, y) < F^{\infty}{}_{K(e)}\ (x, y), for$ all $x, y \in$ V, e ∈ A, then V be the single dominating set of G.

Definition: 2.33. The set S ⊂ V is dominating set (DS) in G if $\forall v \in V - S\ \exists u \in S$ such that u D v. for all $e \in A, u, v \in V$.

Definition: 2.34. A dominating set X of an (NSOG) is called minimal dominating set if

$\nexists Z \subset X$ *such that Z is dominating set. for all* $e \in A, u, v \in V$.

Definition: 2.35. *Let* $D_1, D_2 \ldots D_n$ be minimal dominating sets of (NSOG) G on $G^* = (V, E)$,then

1) Minimum cardinality of $D_i, i = 1, \ldots n$ is known as lower domination number (LDN) of G, i.e., $LDN = \sum_{e \in A}\left(d_{NS}(G)\right)\ \forall e \in A, u, v \in V$.
2) Maximum cardinality of $D_i, i = 1, \ldots n$ is called upper domination number (UDN) of G, i.e. UDN=$\sum_{e \in A}(D_{NS}(G))\ \ \forall e \in A, u, v \in V$.

Example: 2.6. Suppose G be a (NSOG) of G^* = (V, E), such that

$V = \{a, b, c, d\}$ *and* $E = \{(ab), (bc), (cd), (da), (ac)\}'$ A = {e1, e2} and let an approximation function J: A → ρ (v) over V and,

(K, A) be a function K: A → ρ (E)

K(e2) = ab(0.5,1.6,0.7), bc(0.5,1.4,0.7), cd(0.5,1.4,0.7), ad(0.5,1.6,0.7), ac(0.5,1.4,06)

are defined by

$H_2(e_1)$	$F_{(e1)}$	a(1.5,0.5,0.6)	b(0.5,1.6,0.7)	c(0.6,1.8,0.7)	d(0.4,1.2,1.3)	
	$k_{(e1)}$	ab(0.4,0.4,0.8)	bc(0.5,1.6,0.7)	cd(0.4,1.2,1.3)	ad(0.4,0.4,1.3)	bd(0.4,1.2,1.3)
$H_2(e_2)$	$F_{(e2)}$	a(0.6,0.7,1.7)	b(0.6,0.8,1.6)	c(0.4,0.6,1.2),	d(0.5,0.4,1.8)	
	$k_{(e2)}$	ab(0.4,0.7,1.8)	bd(0.5,0.4,1.9)	cd(0.5,0.4,1.8)	ad(0.5,0.3,1.9)	ac(0.4,06,1.7)

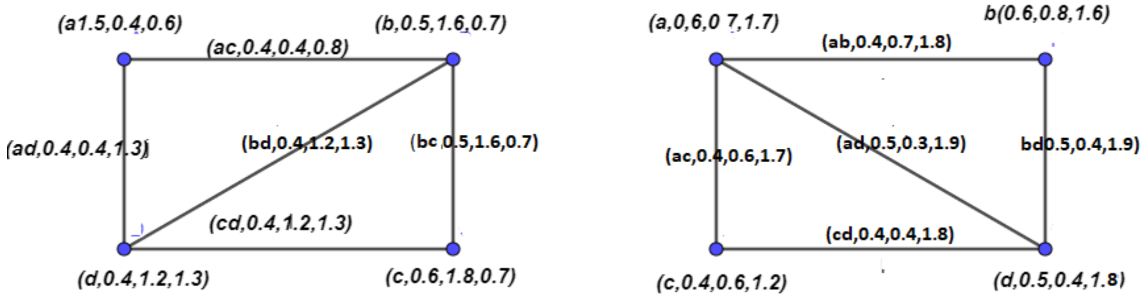


Figure 2.11 G=(H(e1),H(e2))

Here, in H (e1) with respect to e1, the dominating set is {(a, b), (c, d), (b, c), (d, a), (a, b, c), (b, d, a), (d, c, a), (d)}, and

1) a minimum dominating set (MinDS) is {d}, and a maximum dominating set (MaxDS) are {a, b}, {a, d}.
2) a minimum dominating number (MinDN)=0.65 and a maximum dominating number (MaxDN)= 2.35.

Similarly, in H (e2) corresponding to e2, the (DS)s are {(a, b), (c, d), (a, b, c), (b, c), (d, c, a)}, and, 1) the (MinDS) is {c, d}, and the (MaxDS) is {a, b}.

2) the (MinDN)=0.45, the (MaxDN)=0.7. and then clearly the domination number is

$\sum_{e\in A}(d_{NS}(G))$ = 0.65 +0.45 = 1.1 and $\sum_{e\in A}(D_{NS}(G))$ = 2.35 + 0.7 = 3.05

Definition: 2.36. $\forall\, u, v \in V$ in (NSOG) G on $G^* = (V, E)$ are called independent vertices if there exist no effective uv edge between u, v .

Definition: 2.37. The S ⊂ V is called independent set(IDS) of G if $T_{K(e)}(u, v) < T^{\infty}_{K(e}\ (u, v)\ and\ I_{K(e)}(u, v) < I^{\infty}_{K(e}\ (u, v) and\ F_{K(e)}(u, v) < F^{\infty}_{K(e}\ (u, v)\ \ \forall e \in A, u, v \in S.$

Definition: 2.38. The maximal (IDS) S of (NSOG) G is an (IDS) such that $\forall\, v \in V - S,$ the set $S \cup \{v\}$ is not independent.

Definition: 2.39. let $S_1, S_2 \ldots S_n$ be all maximal (IDS) in (NSOG) G then:

1) minimum cardinality among $S_i, i = 1, \ldots n$ is said to be lower (ID) number of G, and it is indicated by $\sum_{e \in A}(i_{NS}(G))$
2) maximum cardinality among $S_i, i = 1, \ldots n$ is called upper independence number of G, and it is determined by $\sum_{e \in A}(I_{NS}(G))$

Example 2.7. Consider the example 2.6.

According to parameter e_1,

1) Min (ID) set (IDS) is {a, c}, and the Maximum (IDS) is {a, c}.
2) Min (ID) number is 2.5, and the maximum “ID” number is 2.5.

With respect to e_2,

1) Minimum (IDS) is {c, a}, and the maximum (IDS) is {d, b}.
2) Mini (independent dominating) number = 0.45; the max (independent dominating) number = 0.45.

Then the independent domination number is
$\sum_{e \in A}(i_{NS}(G)) = 2.95\ and\ \sum_{e \in A}(I_{NS}(G)) = 2.95$

3. EFFECTIVE NEIGHBORHOOD DOMINATION

Definition:3.1. Consider G be (NSOG) on G^*= (V, E) and$x \in V$. Then

i) $\forall\, y \in V$ is called an effective neighbor of $x\ if\ xy$ is an effective edge.

ii) $Ns(x)$ the collection of all effective neighbors of x is called effective neighborhood set of x

iii) a closed effective neighborhood of x is determined by $Ns[x] = Ns(x) \cup x. \forall\, x \in V, e \in A.$

Definition: 3.2. Let (NSOG) G on a simple graph $G^* = (V, E)$ and $v \in V.$

the effective degree of v is defined as

1) $d_s(x) = \sum_{e \in A} \left(\sum_{\in N_{(s)}(x)} T_{K(e)}(x) \;, \sum_{\in N_{(s)}(x)} I_{K(e)}(x) \;, \sum_{\in N_{(s)}(x)} F_{K(e)}(x) \right)$

and the effective neighbor degree of v is defined as

2) $d_s N(x) = \sum_{e \in A} \left(\sum_{\in N_{(s)}(x)} T_{K(e)}(xy) \,, \sum_{\in N_{(s)}(x)} I_{K(e)}(xy) \;, \sum_{\in N_{(s)}(x)} F_{K(e)}(xy) \right)$

Definition: 3.3. Let G be (NSOG) on $G^* = (V, E)$ and $v \in V.$

The effective degree cardinality is defined by

1) $|\, d_s(v)| = \sum_{e \in A} (\left(\sum_{u \in N_{(s)}(v)} \frac{T_{K(e)}(uv) + I_{K(e)}(uv) - F_{K(e)}(uv)}{2} \right) + \frac{n}{2})$

The effective neighborhood degree cardinality is defined by

2) $|\, d_s N(v)| = \sum_{e \in A} (\left(\sum_{u \in N_{(s)}(v)} \frac{T_{K(e)}(u) + I_{K(e)}(u) - F_{K(e)}(u)}{2} \right) + \frac{M}{2})$

3) The minimum effective degree of G is defined as

$$\delta s(G) = \wedge \;|\, d_s(v)| \forall v \in V, e \in A.$$

4) Also, the maximum effective degree of G as

$$\Delta s(G) = \vee \;|\, d_s(v)|\; \forall u, v \in V, e \in A.$$

Example 3.1. Consider a (NSOG) in example 2.6

Corresponding to the parameter e_1 *the edges* $(ad), (dc)$ are effective edges in H(e_1)

And $d_s(a) = (0.4, 0.4, 1.3), d_s(b) = (1.3, 3.2, 2.8), d_s(c) = (0.9, 2.8, 2),$

$d_s(d) = (1.2, 2.8, 3.9)$, $|d_s(a)|$=0.25, $|d_s(b)|$=1.85, $|, d_s(c) = 1.35$, $|d_s(d)|$=0.55 and $\delta_s(G) = 0.25$ *and* $\Delta_s(G) = 1.85$

$d_sN(a) = (0.4,1.2,1.3), d_sN(b) = (1,3,2). d_sN(c) = (0.9,2.8,3), d_sN(d) = (2.6,3.8,2)$

$|d_sN(a)|$=0.65, $|d_sN(b)|$=1.5, $|d_sN(c)|$=0.85, $|d_sN(d)|$=2.7

$\delta_sN(G) = 0.65$ *and* $\Delta_sN(G) = 2.7$

also, for corresponding to the parameter e_2 *the edges*(ad), (dc) are effectives.

$d_s(a) = (0.4,0.6,1.7), d_s(b) = (0,0,0), d_s(c) = (0.8,1,3.5), d_s(d) = (0.4,0.4,1.8)$,

$|d_s(a)|$ =0.15, $|d_s(b)$=0|,$|d_s(c) = 0.35|$,$|d_s(d)|$=0 and $\delta_s(G) = 0$ *and* $\Delta_s(G) = 0.35$

$$d_sN(a) = (0.4,0.6,1.7), d_sN(b) = (0,0,0), d_sN(c) = (1.1,1.1,3.5),$$

$d_sN(d) = (0.4,0.6,1.2)$

And $|d_sN(a)|$=0.15, $|d_sN(b)|$=0, $|d_sN(c)|$=0.1, $|d_sN(d)|$=0.4

$$\delta_sN(G) = 0 \textit{ and } \Delta_sN(G) = 0.4$$

Definition: 3.4. Let $G^* = (V, E)$ be a simple graph and G be (NSOG) onG^* and $v \in V$. The effective size $S_{NS}(G)$ is defined by

$S_{NS}(G) =$

$\{\sum_{e\in A}(\sum_{uv\in E}\frac{T_{K(e)}(uv)+I_{K(e)}(uv)-F_{K(e)}(uv)}{2} + \frac{m}{2} \setminus uv \textit{ is an effective edge}\}$

and the effective order $O_{NS}(G)$ *is defined by*

$$O_{NS}(G) = \{\sum_{e\in A}(\sum_{u\in E}\frac{T_{J(e)}(u) + I_{J(e)}(u) - F_{J(e)}(u)}{2} + \frac{n}{2} \setminus uv \textit{ is an effective edge}\}$$

<u>Conclusion:</u> The concepts of neutrosophic soft overgraphs, effective neutrosophic soft overgraphs, complete (NSOG)s, and domination (set and number) of neutrosophic soft overgraphs were introduced in this work, and they were then appropriately illustrated with examples. Furthermore, some noticeable properties of strong neighborhood domination, independent domination number, and, the concept for the neutrosophic soft overgraph are the proposed concepts that were investigated and described with appropriate examples.

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