Analytical study for calculating the axial velocity for blood flow through the catheter of a stenosed artery

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Abstract: The current study reviews the effect of blood flow characteristics through stenosis during angioplasty, considering that blood is a steady, Newtonian fluid with a steady and incompressible, these vessels are purified by the use of catheters. The engineering of the arterial part is a tube with overlapping stenosis in the lumen of the arteries of the patient, causing a defect in the cardiovascular system, was built Mathematically for the movement of physiological fluid representing blood in the gap between two eccentric tubes (eccentric - ring flows), where the inlet tube is a uniform solid representing the moving catheter while the other is a tapered cylindrical tube representing the artery with intervening stenosis. The problem was analyzed to find the axial velocity approximate solution.

Keywords— Blood flow, Stenosis, Newtonian, tube, eccentric, axial velocity

1. Introduction

In medicine, one of the major health hazards is atherosclerosis, which is the leading cause of death in many countries. Atherosclerosis or stenosis is a cardiovascular disease (CVDs), which refers to the narrowing of arterial lumen .The inner open space or cavity of an artery outstanding to deposition of fatty substances. Stenosis leads to an increase in the resistance to the flow and associated reduction in blood supply in the downstream which causes hypertension, myocardial infarction and cerebral strokes [1].

-**In (1998)** RK Dash et al [2] studied the fluid dynamics of blood flow in a catheter-curved artery with stenosis through mathematical analysis. Considering that the blood is a non-compressible Newtonian fluid and the flow is constant and laminar. It was solved by the method of approximate analysis through the double sequential perturbation method, reaching the insertion of a catheter in the artery to form an increasing number of secondary vortices.

-**In (2010)** V.P.Srivastava et al [3] studied the characteristics of the flow through the narrowed angioplasty with intervening stenosis probe. In order to calculate the presence of red blood cells, the blood was represented by a two-phase model (i.e., the suspension of red blood cells in the plasma). The characteristics of blood flow were flow rate resistance (resistance to flow), shear wall stress in the narrow area, it was found that any slight increase in catheter pilgrimage leads to a significant increase in the volume of resistance and change of blood properties.

-**In (2011)** Narendra et al [4] discussed the problem of blood flow through a symmetrical stenosis during angioplasty on the grounds that blood is a Newtonian fluid. They presented an analysis of expressions for the characteristics of blood flow, resistance and shear stress in the wall where the resistance increases with the size of the catheter while the impedance increases with the size of the stenosis. It was noted that there are opposite properties compared to the differences in the magnitude of the impedance (flow resistance).

-**In (2012**) Mekheimer and Kot [5] discussed blood flow between two eccentric tubes where the inner tube represents catheter while the outer tube was a tapered artery with stenosis. Blood flow was analyzed mathematically by approximate solution to calculate the characteristics of blood flow by considering it as a Newtonian fluid for a mild stenosed artery.

-**In (2017**) Rupesh K.Srivastava et al [6] studied the blood flow through a catheter stenosis artery with the influence of the external body. The acceleration and pulsatile behavior of the blood flow in the subject artery were considered. They studied the gradient of the pulse pressure and the velocity of the slide considering that the Newtonian fluid.

2. Mathematical Model

Consider axisymmetric flow of blood through a catheterized artery with mild intervening stenosis the(r,θ,z)coordinate of a physical point in the cylindrical polar coordinate system where the z-axis is taken along the axis of the artery while r,θ are along the radial and circumferential directions. The geometry with the overlapping arterial wall with time-varying intervening stenosis is determined by the function $R(z,t)$ and $r=\sigma+\epsilon\cos\theta$ as shown in Fig. 1 and Fig. 2 which can be written mathematically as: [7]

$$R(z,t)\begin{cases}\left[(mz+R_0)-\frac{\delta\cos\phi}{l_0}(z-d)\begin{Bmatrix}11-\frac{94}{3l_0}(z-d)\\+\frac{32}{l_0^2}(z-d)^2\\-\frac{32}{3l_0^3}(z-d)^3\end{Bmatrix}\right]\Omega(t), & d\le z\le d+\frac{3l_0}{2}\\ =(mz+R_0)\Omega(t) & otherwise\end{cases}\quad (1)$$

The time-variant parameter $\Omega(t)$ is taken to be

$$\Omega(t)=1-b(\cos\omega t-1)\exp[-b\omega t], \quad (2)$$

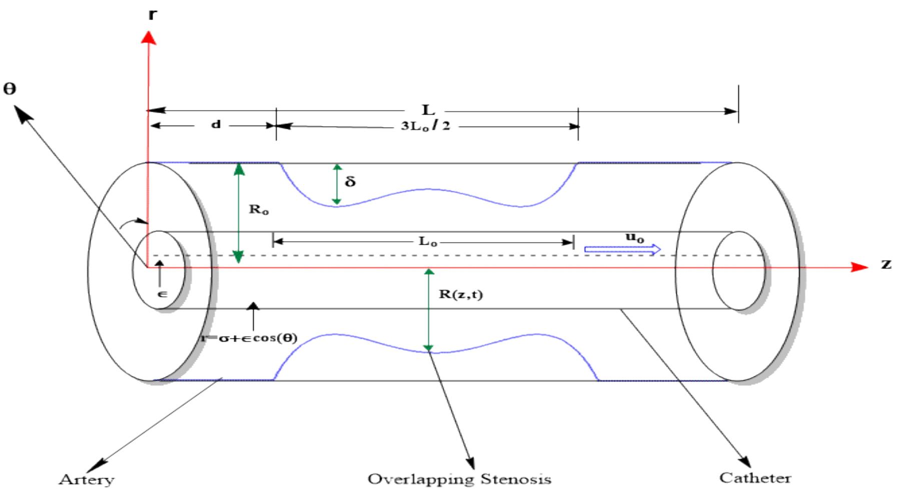

Figure 1: Geometry of Stenosis with Catheter

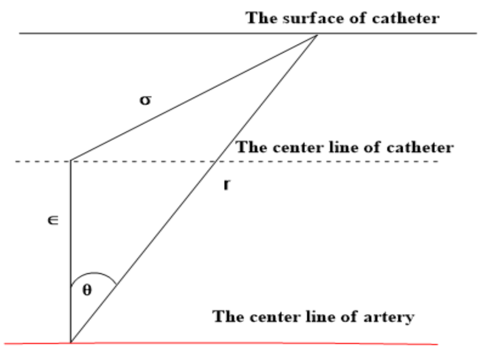

Figure 2: shows that $r = \sigma + \epsilon cos\theta$ in fig.1

where $R(z,t)$ is the radius of the tapered arterial section in the constricted region. $R_0$is the constant radius of the normal artery without stenosis, $\phi$is the angle of tapering, $3l_0/2$ is the lengh of overlapping stenosis, d is the location of the stenosis, δcos$\phi$is taken to be the critical value of the overlapping stenosis, $m = \tan\phi$ represents the slope of the tapered vessel, b is a constant t is the time and ω represents the angular frequency of forced oscillation The blood is Newtonian fluid .considering a steady with viscosity μ and density ρ is constant ,the laminar and incompressible The catheter is mobile as the catheter moves in the axial direction with velocityu_0

2.1 Governing equation

$$\frac{1}{r}\frac{\partial}{\partial r}(ru_r) + \frac{1}{r}\frac{\partial u_\theta}{\partial r} + \frac{\partial u_z}{\partial z} = 0 \tag{3}$$

$$\rho\left(\frac{\partial u_r}{\partial t} + u_r\frac{\partial u_r}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta} - \frac{u_\theta^2}{r} + u_z\frac{\partial u_r}{\partial z}\right) = -\frac{\partial p}{\partial r} + \mu\left(\nabla^2 u_r - \frac{u_r}{r^2} - \frac{2}{r^2}\frac{\partial u_\theta}{\partial u_\theta}\right) \tag{4}$$

$$\frac{\partial u_z}{\partial t} + u_r\frac{\partial u_z}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta} + u_z\frac{\partial u_z}{\partial z} = -\frac{\partial p}{\partial z} + \mu\nabla^2 u_z, \tag{5}$$

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}$$

2.2 The boundary codition [5]

$u_r = u_\theta = u_z = 0$ at $r = R(z,t)$ (6)

$u_r = u_\theta = 0, u_z = u_0$ at $r = \sigma + \epsilon cos\theta$ (7)

Where u_r, u_θ, u_z are the velocity components in the r, θ and z directions respectively, p is the fluid pressure,

2.3 Solution to the problem

We can solve the analytical solution to the system of linear equations (3),(4),(5) by using reduce order Considering the eccentricity parameter to be very small ($\epsilon \ll 1$)[8] so the axial velocity u_z can be expressed eq.(5) as:

$$\frac{\partial u_z^2}{\partial r^2} + \frac{1}{r}\frac{\partial u_z}{\partial r} = \frac{dp}{dz} \quad (8)$$

We multiply the equation (r) by

$$r\frac{\partial u_z^2}{\partial r^2} + \frac{\partial u_z}{\partial r} = r\frac{dp}{dz} \quad (9)$$

On integrating with respect to r, we get

$$\int r\frac{\partial u_z^2}{\partial r^2} + \int \frac{\partial u_z}{\partial r} = \int r\frac{dp}{dz} \quad (10)$$

$$= \int r\frac{\partial u_z^2}{\partial r^2} \; ,by\, I.P$$

$$u = r, \partial u = \partial r, \partial v = \frac{\partial u_z^2}{\partial r^2} \; , v = \frac{\partial u_z}{\partial r}$$

$$r\frac{\partial u_z}{\partial r} - \int \frac{\partial u_z}{\partial r}\, \partial r \rightarrow r\frac{\partial u_z}{\partial r} - u_z \quad (11)$$

In Eq. (10)

$$r\frac{\partial u_z}{\partial r} - u_z + \int \frac{\partial u_z}{\partial r} = \int r\frac{dp}{dz} \quad (12)$$

$$ru_z' - u_z + u_z = \frac{r^2}{2}\frac{dp}{dz} + A \quad (13)$$

We divide the equation (13) by r and we get

$$u_z` = \frac{r}{2}\frac{dp}{dz} + \frac{A}{r} \quad (14)$$

Integrating with respect to r, yields

$$\int u_z` = \int \frac{r}{2}\frac{dp}{dz} + \int \frac{A}{r} \quad (15)$$

$$u_z = \frac{r^2}{4}\frac{dp}{dz} + A\log r \ + B \tag{16}$$

Now, $u_z = 0, on\ \ r = R(z,t)$ gives

$$0 = \frac{R^2}{4}\frac{dp}{dz} + A\log R \ + B \tag{17}$$

Also, $u_z = u_0,\ on\ \ r = \sigma + \epsilon\cos(\theta)$

$$u_0 = \frac{(\sigma + \epsilon\cos(\theta))^2}{4}\frac{dp}{dz} + A\log(\sigma + \epsilon\cos(\theta)) \ + B \tag{18}$$

Subtracting equation (17) from equation (18), we obtain

$$0 = \frac{R^2}{4}\frac{dp}{dz} + A\log(R) - \frac{(\sigma + \epsilon\cos(\theta))^2}{4}\frac{dp}{dz} - A\log(\sigma + \epsilon\cos(\theta)) \ - u_0 \tag{19}$$

$$0 = \frac{1}{4}\frac{dp}{dz}\{R^2 - (\sigma + \epsilon\cos(\theta))^2\} + A\,[\log(R) - \log(\sigma + \epsilon\cos(\theta))] \ - u_0 \tag{20}$$

$$A = \left(-\frac{1}{4}\frac{dp}{dz}\{R^2 - (\sigma + \epsilon\cos(\theta))^2\} + u_0\right) \div [\log(R) - \log(\sigma + \epsilon\cos(\theta))] \tag{21}$$

$$\rightarrow A = -\frac{1}{4}\frac{dp}{dz}\left\{\frac{R^2 - (\sigma + \epsilon\cos(\theta))^2}{[\log(R) - \log(\sigma + \epsilon\cos(\theta))]}\right\} + \frac{u_0}{[\log(R) - \log(\sigma + \epsilon\cos(\theta))]} \tag{22}$$

Again, from equation (16):

$$0 = \frac{R^2}{4}\frac{dp}{dz} + \left[\frac{-\frac{1}{4}\frac{dp}{dz}\left\{\frac{R^2 - (\sigma + \epsilon\cos(\theta))^2}{[\log(R) - \log(\sigma + \epsilon\cos(\theta))]}\right\} + u_0}{[\log(R) - \log(\sigma + \epsilon\cos(\theta))]}\right]\log R \ + B \tag{23}$$

$$\rightarrow B = -\frac{R^2}{4}\frac{dp}{dz} + \left[\frac{\frac{1}{4}\frac{dp}{dz}\left\{\frac{R^2 - (\sigma + \epsilon\cos(\theta))^2}{[\log(R) - \log(\sigma + \epsilon\cos(\theta))]}\right\} - u_0}{[\log(R) - \log(\sigma + \epsilon\cos(\theta))]}\right]\log R \tag{24}$$

$$u_z = \frac{r^2}{4}\frac{dp}{dz} - \frac{1}{4}\frac{dp}{dz}\left\{\frac{R^2 - (\sigma + \epsilon\cos(\theta))^2}{[\log(R) - \log(\sigma + \epsilon\cos(\theta))]}\right\} + \frac{u_0}{[\log(R) - \log(\sigma + \epsilon\cos(\theta))]}\log r + \left\{-\frac{R^2}{4}\frac{dp}{dz} + \left[\frac{\frac{1}{4}\frac{dp}{dz}\left\{\frac{R^2 - (\sigma + \epsilon\cos(\theta))^2}{[\log(R) - \log(\sigma + \epsilon\cos(\theta))]}\right\} - u_0}{[\log(R) - \log(\sigma + \epsilon\cos(\theta))]}\right]\log R\right\} \tag{25}$$

$$u_z = \frac{1}{4}\frac{dp}{dz}\left\{(r^2 - R^2) + \left[\frac{(R^2 - (\sigma + \epsilon\cos(\theta))^2)}{\log(R) - \log(\sigma + \epsilon\cos(\theta))}\right](\log R - \log r\ )\right\} - u_0\left[\frac{\log R - \log r}{\log(R) - \log(\sigma + \epsilon\cos(\theta))}\right] \quad (26)$$

So we got the axial velocity,

$$u_z = \frac{1}{4}\frac{dp}{dz}\left\{(r^2 - R^2) + \left[\frac{(R^2 - (\sigma + \epsilon\cos(\theta))^2)}{\log R \backslash \sigma + \epsilon\cos(\theta)}\right](\log R \backslash r)\right\} - u_0\left[\frac{\log R \backslash r}{\log(R) \backslash \sigma + \epsilon\cos(\theta)}\right] \quad (27)$$

3. Results and Discussion

Analytical graphs of the results obtained for the axial velocity u_z. For various values of the eccentricity parameter ϵ , the radius of catheter σ, the velocity of catheter u_0 , the angle of circumferential direction θ, the taper angle ϕ and the maximum height of stenosis δ^* are presented and discussed.

The discuss the quantitative results of the present study, MATLAB is used analytical study results. The different parameters used the value velocity of catheter $u_0 = 0.1$, non - dimensional catheter radius $\sigma = 0.1, 0.15$, the angle of circumferential tion θ, $\theta = 0^\circ$, $\theta = 45^\circ$ $\delta^* = 0.2$,and time$t = 0.5$ has been taken from the literature(Mekheimer et al.,2012 [5]), with the normal body temperature$(37C^\circ)$.

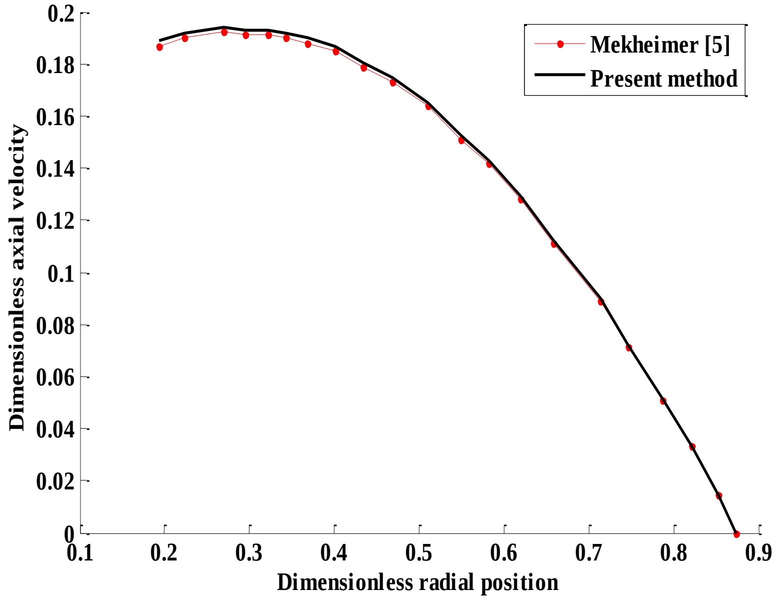

Figure 3: Comparison of axial velocity u_z with (Mekheimer et al .2012)[5] in artery.

In this figure, we made a comparison of the results of the axial velocity. It turned out that the results to (Mekheimer et al.,2012 [5]), very close, By equation of axial velocity u_z in Eq.(27) for different values of σ by fixing the other parameter. We can see that the axial velocity decreases by increasing the radius of catheter σ near the wall of stenosis, the axial velocity is independent approximately to σ.The axial velocity is higher for eccentric catheter compared to concentric catheter variation u_z versus r is shown for value at $u_0 = 0.1$ and $\theta = 0^o, t = 0.5, z = 1.2,\ \delta^* = 0.2, \phi = 0, \sigma = 0.1,$ and $\epsilon = 0$.

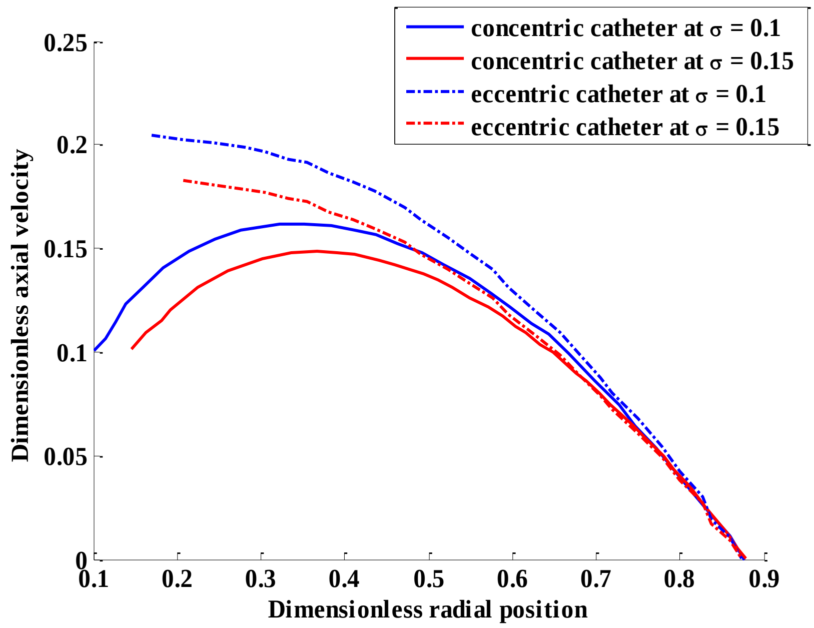

Figure 4: Variation of axial velocity u_0 with radial distance r for different values of catheter radius σ.

We can see that the axial velocity decreases by increasing the radius of catheter σ near the wall of catheter but near the wall of stenosis, the axial velocity is independent approximately to σ. The axial velocity is higher for eccentric catheter compared to concentric catheter is shown for value $u_0 = 0.1$, $\theta = 45$ $t = 0.5$, $z = 1.2, \delta^* = 0.2, \phi = 0, \sigma = 0.1, \sigma = 0.15$, $\epsilon = 0.05$.

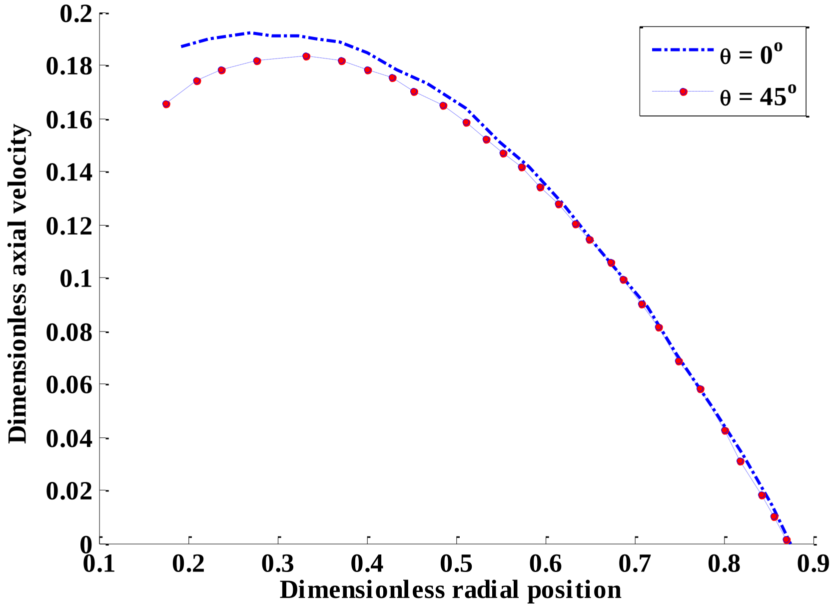

Figure 5: Variation of axial velocity of catheter u_z and angle of circumferential direction θ .

In this figure show the effect of the angle of circumferential direction on the flow. We can record that axial velocity decreases as θ increses. Clearly,the relative importance of eccentricty is dependent on circumferential angle the velocity value of the basic flow started at the radius of the catheter, where $u_0 = 0.1$, $\sigma = 0.1$ and $\theta = 0^{\circ}, \theta = 45^{\circ}, t = 0.5, z = 1.2, \delta^* = 0.2, \phi = 0, \epsilon = 0.05$

4. Conclusion

A mathematical model was simulated to see how the catheter affects the axial velocity Based on the achieved results the following conclusions are made.

The analytical solution are obtained using Analytical study and the results for axial velocity in horizontal position of the artery cylindrical polar coordinate are compared with the findings of (Mekheimer,et al[5]. The axial velocity of eccentric catheter is higher than that of concentric catheter. The transmission of axial velocity through a moving catheter is ($u_0 \neq 0$) substantially higher than that through a steady.

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