β^*-Regular supra topological spaces

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Abstract—Form the series of generalization of the topic of supra topology is the generalization of separation axioms . In this paper we have been introduced ($S\beta^*$ - $SS\beta^*$) regular spaces . Most of the properties of both spaces have been investigated and reinforced with examples . In the last part we presented the notations of supra β^*-R_i-space (i=0,1) and we studied their relationship with ($S\beta^*$ - $SS\beta^*$) regular spaces.

Keywords – supra closed, supra g-open , supra semi .open , supra closure and supra interior .

1. Introduction

Mashhour in [1] present the supra topological space and studied some concepts with it like S*-continuous maps, supra neighborhood and supra T_1- space. Recall that [2] a subset A of a supra space (X, τ^*) is called supra g-closed if $cl^{\tau^*}(A) \subset W$ whenever $A \subset W$ and W is supra open and A is supra g-open if $F \subset int^{\tau^*}(A)$ for any supra closed F contained in A , where $cl^{\tau^*}(A)$, $int^{\tau^*}(A)$ are the supra closure and supra interior operators of A respectively[3] .Also A called supra semi-open [4] if $A \subset cl^{\tau^*}(int^{\tau^*}(A))$. Supra semi-closed is the complement of supra semi-open .

T. M. Al-shami etal . [5] evolved the last concept and presented the notations of supra semi. neighborhood and supra semi T_1- space. In a supra topological space or supra space for short we introduced the notation of supra β^*-closed sets as follows : A subset A of a supra space (X, τ^*) is termed supra β^*-closed , if $cl^{\tau^*}(int^{\tau^*}(A)) \subset V$, wherever $A \subset V$ and V is supra -g-open .The complement of supra β^*-closed is called supra β^*-open [6] .By we supra β^*-open the researchers in [6] [10] defined supra β^*-T_1 and supra β^*-T_2 similar to the definitions of Mashhour , mentioned above .

Finally, a map f: $(X,\tau_1^*) \longrightarrow (Y,\tau_2^*)$ is said to be S^*-closed [1] (resp. supra semi*. closed [5] ,supra semi*.open[5]) if the image of a supra closed is supra closed (resp. the image of a supra semi-closed is supra semi-closed , the image of a supra semi-open is supra semi-open) . Further f is called [6] supra β^*-open (resp. $S\beta^*$-irresolute) if the image (resp. inverse image) of every supra β^*-open is supra β^*-open.

2. Supra β^*-regular

In this section, we shall present the concepts of Sβ^*-regular among the generalization of the separation axioms that pretain to supra topological spaces .

Definition2.1

A supra .space (X,τ^*) is called supra β^*-regular(Sβ^*-regular) if for each supra closed set F in X and $\nu \notin F$, there exist W_1 , W_2 $\in$S β^*O such that $\nu \in W_1$, $F \subset W_2$ and $W_1 \cap W_2 = \emptyset$.

Example 2.2

Let X= $\{b_1 , b_2 , b_3\}$ with supra topology $\tau^*= \{ X , \emptyset , \{ b_1\} , \{b_1, b_2\} , \{ b_2, b_3\} \}$, $\tau^{*c} = \{ \emptyset , X , \{b_2, b_3\} , \{b_3\} , \{b_1\} \}$, so Sβ^*C(X) $= \{ \emptyset , X , \{b_2, b_3\} , \{b_3\} , \{b_1\} , \{b_1, b_3\} ,\{ b_2\}\}$,hence S$\beta^*$O(X) $= \{ X , \emptyset , \{ b_1\} , \{b_1, b_2\} , \{ b_2, b_3\} ,\{ b_2\} \{b_1, b_3\}\}$. It is not difficult to show that (X ,τ^*) is not supra regular space white it is supra β^*-regular space.

Definition 2.3

A supra space(X,τ^***)** and $B \subset X$. A point w$\in X$ is called the supra β^*-cluster point of B if $B \cap W \neq \emptyset$ for each supra β^*-open set W of X containing w . The set of all supra β^*-cluster points of B is said to be supra β^*-closure ,we denoted by s-$cl_\beta{}^*$(B) .

The following useful proposition follows from definition (1.3) .

Proposition 2.4

Let B be a subset of a supra .space (X,τ^*) , then s-$cl_\beta{}^*(B) = \cap \{F \in S\beta^*C(X) | B \subset F \}$.

Proof. Suppose first $x \in$ s-$cl_\beta{}^*$(B) . Now we have to show $x \in \cap\{F \in S\beta^*C(X) | B \subset F\}$,so let $x \notin \cap\{F \in S\beta^*C(X) | B \subset F \}$, then there exists $F_i \in S\beta^*C(X)$ which containing B and $x \notin F_i$ implies $x \in X\text{-}F_i$ which is supra β^*-open but $(X\text{-}F_i) \cap B = \emptyset$ a contradiction ,since $x \in$ s-$cl_\beta{}^*$(B) .Conversely let $x \in \cap\{F \in S\beta^*C(X) | B \subset F\}$ and $x \notin$ s-$cl_\beta{}^*$(B), hence there exists $\mathcal{U}$ such that $x \in \mathcal{U}$ and $\mathcal{U} \cap B = \phi$ implies $B \subset (\mathcal{X} - \mathcal{U})$ which is supra β^*-closed and by the assumption we have $x \in X\text{-} \mathcal{U}$ that is impossible since $x \in \mathcal{U}$ and we have done . □

Some properties are realized in the supra β^*- regular spaces as we will see in the following three results . In fact the conditions of (1.5 ,1.6) are sufficient .

Theorem2.5

A supra space (X,τ^*) is Sβ^*- regular if and only if for every $\nu \in X$ and every supra open set W such that $\nu \in W$, there exists a supra β^*-open set V such that $\nu \in V \subset$ s-$cl_\beta{}^*(V) \subset W$.

Proof . For necessity , since $\nu \notin U^c$ and U^c is supra closed so by the Sβ^*-regularity of X , there exist disjoint supra β^*-open V and W containing x and U^c respectively. But s-$cl_\beta *(V) \subset$s-$cl_\beta * (W^c)$ and since s-$cl_\beta *(W^c)$ is supra β^*-closed [6], hence s-$cl_\beta *(V) \subset W^c \subset U$,therefore $x \in V \subset$ s-$cl_\beta * (V) \subset U$ .For sufficiency let E be a supra closed in (X,τ^*) and $x \notin E$, E^c is supra β^*-open containing x implies that there exist supra β^*-open set V such that $x \in V \subset$s-$cl_\beta{}^*(V) \subset E^c$. Now $E \subset (\text{s-}cl_\beta *(V))^c$, thus V and (s-$cl_\beta{}^*$(V)) are two supra β^*-open sets that containing x and E respectively ,implies (X,τ^*) is Sβ^*-regular . □

Theorem2.6

A supra space (X,τ^*) is Sβ^*-regular if and only if for each supra closed set E and each $x \notin E$, there exists a supra β^*-open V containing E such that $x \notin$ s-$cl_\beta *(V)$.

Proof . Since (X,τ^*) is $S\beta^*$-regular and $x \notin E$, so there exist disjoint supra β^* -open sets U,V such that $x \in U$, $E \subset V$ which is exactly mean x does not belong to $s\text{-}cl_{\beta^*}(V)$. For sufficiency Suppose E is a supra closed in (X,τ^*) and $\nu \notin E$ .By the a assumption we have $V \in S\beta^*O(X)$ such that $E \subset V$ and $\nu \notin s\text{-}cl_{\beta^*}(V)$, this means that there is a supra β^*-open set W containing x and $W \cap V = \emptyset$ implies (X,τ^*) is $S\beta^*$-regular . □

Proposition2.7

If X is $S\beta^*$-regular and $c \in X$, then each supra neighborhood of c contain a supra β^* -closed set consist of c.

Proof .Let $c \in X$ and a supra neighborhood M of c , then there is a supra open set $N \subset X$ such that $c \in N \subset M$ implies $c \notin X\text{-}N$. Since X is $S\beta^*$-regular ,so we have $U_1, U_2 \in S\beta^*O(X)$ such that $c \in U_1$ and $X\text{-}N \subset U_2$ and $U_1 \cap U_2 = \emptyset$. Hence $c \in U_1 \subset X\text{-}U_2 \subset N \subset M$,therefore $X\text{-}U_2$ will be the required supra β^*-closed . □

What are the characteristics of functions that transfer a supra β^*- regular space to a supra β^*- regular space or vice versa ? we will see this in the following two theorems .

Theorem2.8

Let $f : (X,\tau_1^*) \longrightarrow (Y,\tau_2^*)$ be an injective ,S^*.closed and $S\beta^*$- irresolute ,then X is supra β^*-regular if Y is supra β^*-regular .

Proof . Suppose $x \in X$ and B is any supra closed subset of X such that $x \notin B$. Since f is S^*.closed , so $f(B)$ is supra closed subset of Y such that $f(x) \notin f(B)$ thus by the supra β^*-regularity of Y we have disjoint V ,W containing $f(B)$ and $f(x)$ respectively . Clear that (for instance see [8]) $B \subset f^{-1}(f(B)) \subset f^{-1}(V)$ and $x \in f^{-1}(W)$, also $f^{-1}(V) \cap f^{-1}(W) = \emptyset$, hence $f^{-1}(V)$ and $f^{-1}(W)$ are the required supra β^*-open sets which complete proof . □

Theorem2.9

Let $f: (X,\tau_1^*) \longrightarrow (Y,\tau_2^*)$ be a bijective , supra β^*-open and S^*.continuous map, then Y is supra β^*-regular if X is supra β^*-regular .

Proof . Suppose $y \in Y$ and B is any supra closed subset of Y such that $y \notin B$, hence $f^{-1}(y) \notin f^{-1}(B)$. Since f is S^*.continuous , so $f^{-1}(B)$ is supra closed in X and by the supra β^*-regularity of X there are disjoint W_1 ,W_2 $\in S\beta^*O(X)$ which containing $f^{-1}(y)$ and $f^{-1}(B)$ respectively .Now it is easy to $f(W_1)$, $f(W_2)$ are the required supra β^* -open sets , implies Y is supra β^*-regular. □

The following example shows that not every supra β^*-regular space is supra β^*-T_2-space .

Example 2.10

Let $X= \{w_1 , w_2 , w_3, w_4\}$ with supra topology $\tau^*= \{ X , \emptyset ,\{w_1\},\{w_3\},\{w_1, w_3\}, \{ w_1, w_2\},\{ w_1 ,w_4\} ,\{ w_3 , w_4\} ,\{w_2 , w_4\},\{w_1 , w_2 , w_3\},\{ w_1 , w_2, w_4\}, \{ w_2 , w_3, w_4\}, \{ w_1 , w_3, w_4\}\}$,so $S\beta^*C(X)=\{ \emptyset, X ,\{ w_1 \} ,\{ w_2\} , \{ w_3\},\{w_4\} , \{w_1,w_3\},\{ w_1, w_2\},\{w_2,w_3\},\{w_3, w_4\},\{w_2 ,w_4\} ,\{w_1,w_2 ,w_4\},\{w_1,w_2, w_3\}\}$, hence $S\beta^*O(X)=\{X , \emptyset , \{ w_1\},\{w_3\},\{ w_1, w_3\},\{w_1,w_2\},\{ w_1 ,w_4\} , \{w_3 , w_4\} ,\{w_2 , w_4\},\{w_1 , w_2 , w_3\},\{ w_1 , w_2, w_4\},\{ w_2 , w_3, w_4\}, \{ w_1 , w_3, w_4\}\}$. It is not difficult to show that (X ,τ^*) is not $S\beta^*$-regular , but it is supra β^*-T_2.

Theorem 2.11

Any supra β^*- regular and supra T_1-space is supra β^*-T_2-space .

Proof . Assume that (X ,τ^*) be a supra β^*-regular , supra T_1-space and $\nu \neq y \in X$. From [1] we have $\{\nu\}$ is supra closed and since $y \notin \{\nu\}$, then we have disjoint W, $V \in S\beta^*O(X)$ such that $y \in W$ and $\{\nu\} \subset V$, implies X is supra β^*-T_2-space. □

The following example shows how the quotient space of supra β^* -regular might not supra β^* -regular .

Example2.12

Let $(\mathbb{R},\tau_u)$ be *the* real numbers with τ_u and let $\rho : \mathbb{R} \longrightarrow B$, when $B = \{b_1 , b_2 , b_3, b_4\}$ defined as follows .

$$\rho(x)=\begin{cases} b_2 & x<1 \\ b_3 & 1\le x<5 \\ b_4 & 5\le x<20 \\ b_1 & x\ge 20 \end{cases}$$

Thus the quotient supra topology on B is qsT (B)= {∅ , B ,{ b_2} , { b_2, b_3} , { b_2, b_3, b_4}} .Therefore Sβ^*O(B) = {∅ , B ,{ b_2} , { b_2, b_3} ,{ b_2, b_3, b_4} , { b_1, b_2, b_3} , { b_1, b_2, b_4}, { b_3, b_4} , { b_2, b_4},{ b_1,b_2},{ b_4},{ b_3} } . It is clear that (B ,qsT(B)) is not supra β^* -regular .

3 . strongly supra$\boldsymbol{\beta^*}$- regular

This section deals with a strong from of Sβ^*-regularity . In the beginning we will present the following definition .

Definition3.1

A supra space (X,τ^*) is called strongly supra β^*- regular (SSβ^*-regular) , if for each supra semi-closed set F and a ∉ F , there exist disjoint supra β^*-open sets U,V such that a ∈ U , F ⊂ V .

The next two theorems give characterizations for SSβ^*- regular spaces .

Theorem3.2

A supra space (X,τ^*) is SSβ^*- regular if and only if for each ν ∈ X and any supra semi -open set Ⱳ such that ν ∈Ⱳ, there exist a supra β^*-open set Ѵ such that ν ∈ Ѵ⊂ s-$cl_\beta{}^*$(Ѵ) ⊂ Ⱳ.

Proof . For necessity , since ν ∉ $Ⱳ^c$ and $Ⱳ^c$ is supra semi-closed so by the SSβ^*-regularity of X , ,so we have disjoint H ,K∈S β^*O such that ν ∈H and $Ⱳ^c$⊂K .But s-cl_β ∗(Ѵ) ⊂s-cl_β ∗ ($Ⱳ^c$) and since s-cl_β ∗($Ⱳ^c$) is supra β^*-closed [6], hence s-cl_β ∗(Ѵ) ⊂ $Ⱳ^c$⊂ Ⱳ,therefore ν∈ Ѵ⊂ s-cl_β ∗ (Ѵ) ⊂ Ⱳ.For sufficiency let E be a supra semi-closed in (X,τ^*) and x ∉ E , E^c is supra semi-open containing x implies that there exists a supra β^*-open set V such that x ∈ V ⊂s-$cl_\beta{}^*$(V)⊂ E^c. Now E ⊂ (s-cl_β ∗(V)$)^c$, thus V , (s-$cl_\beta{}^*$(V)) ∈S β^*O such that x ∈ V and E ⊂ (s-$cl_\beta{}^*$(V)) ,implies (X,τ^*) is SSβ^*- regular. □

Theorem3.3

A supra space (X,τ^*) is SSβ^*-regular if and only if for every supra semi closed set E and each ν ∉E , there exists a supra β^*-open Ѵ containing E such that ν ∉ s-$cl_\beta{}^*$(Ѵ)

Proof . Since (X,τ^*) is SSβ^*-regular and ν ∉ E , so there exist disjoint supra β^* -open sets Ⱳ, Ѵ such that ν ∈ Ⱳ , E ⊂ Ѵ , hence ν ∉ s-$cl_\beta{}^*$(Ѵ) . For sufficiency let E be a supra semi closed in (X,τ^*) and ν ∉ E . By the a assumption we have Ѵ∈ Sβ^*O(X) such that E ⊂ Ѵ and ν ∉s-cl_β ∗(Ѵ) . As a result, there is a supra β^*-open set Ⱳ containing x and Ⱳ ∩ Ѵ = ∅ implies (X,τ^*) is SSβ^*-regular. □

Theorem3.4

A supra space X is SS β^*-regular if for every ν ∈X the supra β^*-closed neighborhood of ν form a basis of a supra semi-open neighborhood of ν .

Proof . Suppose ν ∈X and let F be any supra semi-closed does not contain ν . Thus X-F is supra semi-open containing ν ,so there is a supra β^*-closed neighborhood W of ν such that ν ∈ $W \subset X$-F . Now let G= X-W ,then G is supra β^*-open and F⊂ G. On the other hand W is supra neighborhood of ν , hence there is a supra open set H such that ν∈ $H \subset W$ [6]. Therefore $G \cap H \subset X$-$W \cap W = \emptyset$ and since H is supra β^*-open(3.1.3)the proof complete. □

For a mapping between two SSβ^*- regular spaces we have the following theorems .

Theorem 3.5
let f : $(X,\tau_1^*) \longrightarrow (Y,\tau_2^*)$ be an injective, supra semi*.closed and supra β^*.continuous map , then X is supra SSβ^*-regular if Y is SSβ^*-regular .
Proof . Suppose x ∈X and B is any supra semi- closed subset of X such that x ∉B . Since f is supra semi*.closed, so f(B) is supra semi-closed subset of Y such that $f(x) \notin f(B)$ and by the SSβ^*-regularity of Y we have disjoint V ,W ∈ Sβ^*O(X) containing f(B) and f(x) respectively . Clear that (for instance see [9]) $B \subset f^{-1}(f(B)) \subset f^{-1}(V)$ and $x \in f^{-1}(W)$,also $f^{-1}(V) \cap f^{-1}(W) = \emptyset$ and we have done . □

Theorem 3.6
Let f: $(X,\tau_1^*) \longrightarrow (Y,\tau_2^*)$ be a bijective , supra semi*.open and S*.continuous map, if X is SSβ^*-regular then Y is SSβ^*-regular .
Proof . Suppose y ∈Y and B is any supra semi-closed subset of Y such that y ∉B, hence $f^{-1}(y) \notin f^{-1}(B)$. Since f is S*.continuous , so $f^{-1}(B)$ is supra semi-closed in X and by the SSβ^*-regularity of X there are disjoint W_1 ,W_2 ∈S β^*O(X) which containing $f^{-1}(y)$ and $f^{-1}(B)$ respectively . Now it is easy to $f(W_1)$, $f(W_2)$ are the required supra β^* -open sets , implies Y is SSβ^*-regular. □

The following example shows that not every SSβ^*-regular space is supra β^*-T_2-space .

Example3.7
Let X= $\{w_1 , w_2 , w_3, w_4\}$ with supra topology τ^*= {X , Ø , $\{w_1, w_3\}$, $\{w_1, w_2\}$, $\{ w_2, w_3\}$,$\{ w_3 , w_4\}$,$\{ w_2 , w_4\}$,$\{w_1, w_2 , w_3\}$,$\{ w_1 , w_2, w_4 \}$, $\{ w_2, w_3, w_4\}$, $\{ w_1 , w_3, w_4\}$}, so ssC(X)={X, Ø,$\{ w_2 , w_4\}$,$\{ w_3, w_4\}$, $\{ w_1 , w_4\}$, $\{ w_1, w_2 \}$, $\{w_1 ,w_3\}$, $\{w_1\}$, $\{w_2\}$,$\{w_3\}$,$\{w_4\}$},hence Sβ^*O(X) { X , Ø , $\{ w_1, w_3\}$, $\{ w_1, w_2\}$, $\{ w_2, w_3\}$, $\{ w_3 , w_4\}$,$\{ w_2 , w_4\}$,$\{w_1, w_2 , w_3\}$,$\{ w_1 , w_2, w_4 \}$, $\{ w_2, w_3, w_4\}$, $\{ w_1 , w_3, w_4\}$}.It is not difficult to show that (X ,τ^*) is not SSβ^*-regular space ,but it is supra β^*-T_2-space.

Theorem3.8
Any SSβ^*- regular and supra semi T_1-space is supra β^*-T_2-space .
Proof . Let (X ,τ^*) be a SSβ^*-regular , supra semi T_1-space and x ≠ y ∈X . From [5] we have {x} is supra semi-closed and since y ∉ {x} ,so we have disjoint U , V ∈ S β^*O (X) such that y ∈ U and {x} ⊂ V, implies (X ,τ^*)is supra β^*-T_2-space. □

4. Supra β^*- R_0-space and supra β^*- R_1-space .

Recall that [7] a space X is said to be R_0-space if for each open set V and c∈V we have cl({c})⊂V and X is called R_1-space if for each $d_1 \neq d_2$ in X with $cl(\{d_1\}) \neq cl(\{d_2\})$, then we have disjoint open sets V_1,V_2 such that $cl(\{d_1\}) \subset V_1$ and $cl(\{d_2\}) \subset V_2$.

Definition4.1
A supra space (X,τ^*) is called supra -R_0 if each supra open set contains the supra closure of each of its singleton .

Definition4.2
A supra .space (X,τ^*) is called supra -R_1 if for each c_1 ,c_2 in X with $cl^{\tau^*}\{c_1\} \neq cl^{\tau^*}(\{c_2\})$, there exist disjoint supra open sets W_1, W_2 such that $cl^{\tau^*}(\{c_1\}) \subset W_1$ and $cl^{\tau^*}(\{c_2\}) \subset W_1$.

Example4.3
Let X= $\{w_1, w_2, w_3\}$with supra topology τ^*= { X , Ø ,$\{ w_1\}$, $\{ w_2\}$, $\{ w_1, w_2\}$, $\{ w_2, w_3\}$,$\{ w_1, w_3\}$ } , τ^{*c} ={ Ø , X , $\{ w_1\}$, $\{ w_3\}$,$\{ w_2\}$,$\{ w_2, w_3\}$,$\{ w_1, w_3\}$}. It is not difficult to show that (X,τ^*) is supra R_0-space and supra R_1-space

Remark 4.4
Every supra R_1 is supra R_0 .
Proof. Assume U be a supra open subset of a supra space (X,τ^*) . We want to show that $cl^{\tau^*}(\{c\}) \subset U$ for each $c \in U$ .Now suppose $y \notin U$, hence $c \notin cl^{\tau^*}(\{y\})$ and hence $cl^{\tau^*}(\{c\}) \neq cl^{\tau^*}(\{y\})$. Since (X,τ^*) is supra R_1 ,so there exists supra open W_y such that $cl^{\tau^*}(\{y\}) \subset W_y$ implies $y \notin cl^{\tau^*}(\{c\})$ which complete the proof . □

The converse of (3.4) is not true as seen in the following example .

Example 4.5
Let $X=\{w_1,w_2,w_3\}$with supra topology $\tau^*=\{X ,\emptyset,\{w_1, w_2\},\{ w_2, w_3\},\{ w_1, w_3\}\}$, hence $\tau^{*c}=\{ \emptyset , X , \{ w_1\}, \{ w_2\} , \{ w_3\} \}$, it is clear that (X,τ^*) is not supra R_1-space, while it is supra R_0-space .

Definition4.6
A supra space (X,τ^*) is called supra β^*-R_0 if every supra β^*-open set contains the supra β^*-closure of each of its singleton .

Definition4.7
A supra space (X,τ^*) is said be to supra β^*-R_1 if for each c_1 ,c_2 in X with s-$cl_\beta*\{c_1\} \neq$ s-$cl_\beta*\{c_2\}$, there exist disjoint supra β^*-open sets V_1,V_2 such that s-$cl_\beta*\{c_1\} \subset V_1$ and s-$cl_\beta*\{c_2\} \subset V_2$.

Example 4.8
Let $X= \{w_1,w_2,w_3\}$with supra topology $\tau^*= \{ X , \emptyset , \{w_1\} ,\{ w_1, w_2\} ,\{w_2, w_3\}\}$, $\tau^{*c}=\{ \emptyset, X ,\{w_2,w_3\},\{w_3\} , \{w_1\}\}$ .Thus $S\beta^*C=\{ \emptyset , X , \{w_2, w_3\} , \{ w_3\} , \{ w_1\}, \{w_1, w_3\} ,\{w_2\}\}$,and $S\beta^*O =\{ X , \emptyset ,\{w_1\},\{w_1,w_2\} , \{w_2,w_3\} ,\{ w_2\},\{w_1 ,w_3\}\}$. It is clear that (X,τ^*) is supra β^*- R_0-space and supra β^*- R_1-space .

Remark 4.9
Every supra β^*- R_1-space is supra β^*- R_0-space.
Proof . The proof is similar to (3.4).

The following example demonstrates that the opposite of the aforementioned statement (3.9) is not true.

Example 4.10
Let $X= \{w_1,w_2,w_3\}$with supra topology $\tau^*=\{ X , \emptyset , \{w_1, w_2\} ,\{ w_2, w_3\}\}$,hence $\tau^{*c}=\{ \emptyset , X , \{ w_1\} , \{ w_3\}\}$ .Thus $S\beta^*C =\{ \emptyset , X ,\{w_1\} ,\{w_3\} ,\{w_1, w_3\}, \{ w_2\}\}$, implies $S\beta^*O=\{ X , \emptyset ,\{w_2, w_3\} ,\{w_1, w_2\} ,\{w_2\} ,\{w_1, w_3\}\}$. It is obvious that (X,τ^*) is not supra β^*- R_1-space, while it is supra β^*- R_0-space .

Theorem4.11
A supra space X is supra β^*-R_0 if and only if for any $c_1$and c_2 in X such that s-$cl_\beta^*(\{c_1\}) \neq$ s-$cl_\beta^*(\{c_2\})$,then s-$cl_\beta^*(\{c_1\}) \cap$ s-$cl_\beta^*(\{c_2\}) = \emptyset$.
Proof. Assume X is supra β^*-R_0- space and $c_1, c_2 \in X$ such that s-$cl_\beta^*(\{c_1\}) \neq$ s-$cl_\beta^*(\{c_2\})$,so let $w \in$s-$cl_\beta^*(\{c_1\})$ and $w \notin$ s-$cl_\beta^*(\{c_2\})$ .Thus there is $U \in S\beta^*O(X)$ such that $c_2 \notin U$ and $w \in U$ .But $w \in$ s-$cl_\beta^*(\{c_1\})$, hence $c_1 \in U$ and hence $c_1 \notin$ s-$cl_\beta^*(\{c_2\})$ implies $c_1 \in$ (s-$cl_\beta^*(\{c_2\}))^c$ which is supra β^* -open and by the supra β^*-R_0 of X we have s-$cl_\beta^*(\{c_1\}) \subset$ (s-$cl_\beta^*(\{c_2\}))^c$ implies s-$cl_\beta^*(\{c_1\}) \cap$ s-$cl_\beta^*(\{c_2\}) = \emptyset$. Conversely let U be a supra β^*-open and $c \in U$,we want to show s-$cl_\beta^*(\{c\}) \subset U$. Now suppose $w \notin U$, hence $c \notin$ s-$cl_\beta^*(\{w\})$ and hence s-$cl_\beta^*(\{w\}) \neq$ s-$cl_\beta^*(\{c\})$ and by the assumption we have s-$cl_\beta^*(\{w\}) \cap$ s-$cl_\beta^*(\{c\}) =\emptyset$ implies $w \notin$ s-$cl_\beta^*(\{c\})$ and we have done . □

Remark 4.12

Every Sβ^*-regular and every SSβ^*-regular are supra β^*-R_0 .

Proof .1. Assume X be a SSβ^*-regular , so by (2.2) we have for every $w \in X$ and any supra semi-open W_1 such that $w \in W_1$, there exists supra β^*-open W_2 such that $w \in W_2 \subset$ s-cl$_\beta^*$(W_2)$\subset W_1$ .In particular for each $v \in W_1$ we have s-cl$_\beta^*$({v }) $\subset W_1$, hence X is supra β^*-R_0 and since each supra open is also supra semi-open [4] ,thus the proof of the case X is Sβ^*-regular follows immediately. □

Theorem 4.13

Every supra β^*-R_1- space is supra β^* -regular restricted on s-cl$_\beta^*$({c})

Proof . Assume X be a supra β^*-R_1- space ,so by (3.4) it is supra β^*-R_0- space and let z $\notin$ s-cl$_\beta^*$({c}) , hence s-cl$_\beta^*$({c}) $\neq$ s-cl$_\beta^*$({z}) implies s-cl$_\beta^*$({c}) $\cap$s-cl$_\beta^*$({z}) = Ø (3.11) . Now since X is supra β^*- R_1- space ,then we have U, V$\in$Sβ^*O(X) such that s-cl$_\beta^*$({z}) $\subset$ U and s-cl$_\beta^*$({c})$\subset$V. But s-cl$_\beta^*$({c}) is supra β^*-closed (1.4) and since each supra closed is supra β^*-closed [6] . As a result every supra closed F of the form s-cl$_\beta^*$({c}) for each c in X and c $\notin$ F separated by supra β^*-open sets . □

5 . Conclusions

Our main target in this work was to generalize one of the properties of separation axioms in the supra topological spaces which is the supra regularity . The concept of (Sβ^*- SSβ^*) spaces was introduced and the properties of such kind of spaces were studied , also these types were linked to the concept of (R_0-R_1)spaces after their generalization to supra β^*-R_0 and supra β^*-R_1 .

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