**β*-Regular supra topological spaces**

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**Abstract**—Form the series of generalization of the topic of supra topology is the generalization of separation axioms. In this paper we have been introduced \( (\text{S}^* - \text{SS}^* ) \) regular spaces. Most of the properties of both spaces have been investigated and reinforced with examples. In the last part we presented the notations of \( \text{supra} \beta^*-R_i \)-space \((i=0,1)\) and we studied their relationship with \( (\text{S}^* - \text{SS}^* ) \) regular spaces.

**Keywords**— supra closed, supra g-open, supra semi-open, supra closure and supra interior.

1. Introduction

Mashhour in [1] present the supra topological space and studied some concepts with it like \( \text{S}^*\)-continuous maps, supra neighborhood and supra \( \text{T}_{1^*} \) space. Recall that [2] a subset \( A \) of a supra space \( (X, \tau^*) \) is called supra \( g^-\)closed if \( \text{cl}^{\tau^*}(A) \subseteq W \) whenever \( A \subseteq W \) and \( W \) is supra open and \( A \) is supra \( g^-\)open if \( F \subseteq \text{int}^{\tau^*}(A) \) for any supra closed \( F \) contained in \( A \), where \( \text{cl}^{\tau^*}(A) \), \( \text{int}^{\tau^*}(A) \) are the supra closure and supra interior operators of \( A \) respectively[3]. Also \( A \) called supra semi-open [4] if \( A \subseteq \text{cl}^{\tau^*}(\text{int}^{\tau^*}(A)) \). Supra semi-closed is the complement of supra semi-open.

T. M. Al-shami etal. [5] evolved the last concept and presented the notations of supra semi. neighborhood and supra semi \( \text{T}_{1^*} \) space. In a supra topological space or supra space for short we introduced the notation of supra \( \beta^-\)closed sets as follows: A subset \( A \) of a supra space \( (X, \tau^*) \) is termed supra \( \beta^-\)-closed, if \( \text{cl}^{\tau^*}(\text{int}^{\tau^*}(A)) \subseteq V \) whenever \( A \subseteq V \) and \( V \) is supra open and \( A \) is supra \( \beta^-\)-open if \( F \subseteq \text{int}^{\tau^*}(A) \) for any supra closed \( F \) contained in \( A \), where \( \text{cl}^{\tau^*}(A) \), \( \text{int}^{\tau^*}(A) \) are the supra closure and supra interior operators of \( A \) respectively[3]. Also \( A \) called supra semi-open [4] if \( A \subseteq \text{cl}^{\tau^*}(\text{int}^{\tau^*}(A)) \). By we supra \( \beta^-\)-open the researchers in [6] [10] defined supra \( \beta^-\)-\( T_1 \) and supra \( \beta^-\)-\( T_2 \) similar to the definitions of Mashhour, mentioned above.

Finally, a map \( f: (X,\tau_1^*) \rightarrow (Y,\tau_2^*) \) is said to be \( \text{S}^-\)-closed [1] (resp. supra semi-. closed [5] ,supra semi-.open[5]) if the image of a supra closed is supra closed (resp. the image of a supra semi-closed is supra semi-closed , the image of a supra semi-open is supra semi-open ). Further \( f \) is called [6] supra \( \beta^-\)-open (resp. \( \text{S}^\beta^-\)-irresolute) if the image (resp. inverse image) of every supra \( \beta^-\)-open is supra \( \beta^-\)-open.
2. Supra $\beta^*$-regular

In this section, we shall present the concepts of $S\beta^*$-regular among the generalization of the separation axioms that pertain to supra topological spaces.

**Definition 2.1**

A supra space $(X, \tau^*)$ is called supra $\beta^*$-regular($S\beta^*$-regular) if for each supra closed set $F$ in $X$ and $v \not\in F$, there exist $W_1, W_2 \in S\beta^O$ such that $v \in W_1$, $F \subset W_2$ and $W_1 \cap W_2 = \emptyset$.

**Example 2.2**

Let $X = \{b_1, b_2, b_3\}$ with supra topology $\tau^* = \{X, \emptyset, \{b_1\}, \{b_1, b_2\}, \{b_2, b_3\}\}$, $\tau^* = \{\emptyset, X, \{b_2, b_3\}, \{b_2\}\}$. Then $S\beta^O(X) = \{X, \emptyset, \{b_1\}, \{b_1, b_2\}, \{b_2, b_3\}\}$, hence $S\beta^O(X) = \{X, \emptyset, \{b_1\}, \{b_2, b_3\}, \{b_2\}\}$. It is not difficult to show that $(X, \tau^*)$ is not supra regular space as it is supra $\beta^*$-regular space.

**Definition 2.3**

A supra space $(X, \tau^*)$ and $B \subset X$. A point $w \in X$ is called the supra $\beta^*$-cluster point of $B$ if $B \cap W \neq \emptyset$ for each supra $\beta^*$-open set $W$ of $X$ containing $w$. The set of all supra $\beta^*$-cluster points of $B$ is said to be supra $\beta^*$-closure, we denote by $s-cl_{\beta^*}(B)$.

The following useful proposition follows from definition (1.3).

**Proposition 2.4**

Let $B$ be a subset of a supra space $(X, \tau^*)$, then $s-cl_{\beta^*}(B) = \bigcap\{F \in S\beta^O(X) \mid B \subset F\}$.

**Proof.** Suppose first $x \in s-cl_{\beta^*}(B)$. Now we have to show $x \in \bigcap\{F \in S\beta^O(X) \mid B \subset F\}$. So let $x \not\in \bigcap\{F \in S\beta^O(X) \mid B \subset F\}$, then there exists $F_i \in S\beta^O(X)$ which containing $B$ and $x \not\in F_i$ implies $x \in X - F_i$ which is supra $\beta^*$-open but $(X - F_i) \cap B = \emptyset$ a contradiction since $x \in s-cl_{\beta^*}(B)$. Conversely let $x \in \bigcap\{F \in S\beta^O(X) \mid B \subset F\}$ and $x \not\in s-cl_{\beta^*}(B)$, hence there exists $U$ such that $x \in U$ and $U \cap B = \emptyset$ implies $B \subset (X - U)$ which is supra $\beta^*$-closed and by the assumption we have $x \in X - U$ that is impossible since $x \in U$ and we have done.

Some properties are realized in the supra $\beta^*$-regular spaces as we will see in the following three results. In fact the conditions of (1.5, 1.6) are sufficient.

**Theorem 2.5**

A supra space $(X, \tau^*)$ is $S\beta^*$-regular if and only if for every $v \in X$ and every supra open set $W$ such that $v \in W$, there exists a supra $\beta^*$-open set $V$ such that $v \in V \subset s-cl_{\beta^*}(V) \subset W$.

**Proof.** For necessity, since $v \not\in U^c$ and $U^c$ is supra closed so by the $S\beta^*$-regularity of $X$, there exist disjoint supra $\beta^*$-open $V$ and $W$ containing $x$ and $U^c$ respectively. But $s-cl_{\beta^*}(V) \subset s-cl_{\beta^*}(W^c)$ and since $s-cl_{\beta^*}(W^c)$ is supra $\beta^*$-closed, hence $s-cl_{\beta^*}(V) \subset W^c \subset U$, therefore $x \in V \subset s-cl_{\beta^*}(V) \subset U$. For sufficiency let $E$ be a supra closed in $(X, \tau^*)$ and $x \notin E$, $E^c$ is supra $\beta^*$-open containing $x$ implies that there exist supra $\beta^*$-open set $V$ such that $x \in V \subset s-cl_{\beta^*}(V) \subset E^c$. Now $E \subset (s-cl_{\beta^*}(V))^c$, thus $V$ and $(s-cl_{\beta^*}(V))$ are two supra $\beta^*$-open sets that containing $x$ and $E$ respectively implies $(X, \tau^*)$ is $S\beta^*$-regular.

**Theorem 2.6**

A supra space $(X, \tau^*)$ is $S\beta^*$-regular if and only if for each supra closed set $E$ and each $x \notin E$, there exists a supra $\beta^*$-open $V$ containing $E$ such that $x \not\in s-cl_{\beta^*}(V)$. 

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The following example shows how the quotient space of \( A \) is \( S^* \)-regular and \( x \not\in E \), so there exist disjoint supra \( \beta^* \)-open sets \( U,V \) such that \( x \in U \), \( E \subset V \) which is exactly mean \( x \) does not belong to \( s\text{-cl}_{\beta^*}(V) \). For sufficiency Suppose \( E \) is a supra closed in \( (X,\tau^*) \) and \( v \not\in E \). By the a assumption we have \( V \in S^*\beta^*O(X) \) such that \( E \subset V \) and \( v \in s\text{-cl}_{\beta^*}(V) \), this means that there is a supra \( \beta^* \)-open set \( W \) containing \( x \) and \( W \cap V = \emptyset \) implies \( (X,\tau^*) \) is \( S^* \)-regular.

\[ \square \]

**Proposition 2.7**

If \( X \) is \( S^* \)-regular and \( c \in X \), then each supra neighborhood of \( c \) contain a supra \( \beta^* \)-closed set consist of \( c \).

**Proof.** Let \( c \in X \) and a supra neighborhood \( M \) of \( c \), then there is a supra open set \( N \subset X \) such that \( c \in N \subset M \) implies \( c \not\in X-N \). Since \( X \) is \( S^* \)-regular, so we have \( U_1, U_2 \) \( \in S^* \beta^*O(X) \) such that \( c \in U_1 \) and \( X-N \subset U_2 \) and \( U_1 \cap U_2 = \emptyset \). Hence \( c \in U_1 \subset X-U_2 \subset N \subset M \), therefore \( X-U_2 \) will be the required supra \( \beta^* \)-closed.

\[ \square \]

What are the characteristics of functions that transfer a supra \( \beta^* \)-regular space to a supra \( \beta^* \)-regular space or vice versa? We will see this in the following two theorems.

**Theorem 2.8**

Let \( f : (X,\tau_1^*) \rightarrow (Y,\tau_2^*) \) be an injective \( S^* \)-closed and \( S^* \)-irresolute, then \( X \) is supra \( \beta^* \)-regular if \( Y \) is supra \( \beta^* \)-regular.

**Proof.** Suppose \( x \in X \) and \( B \) is any supra closed subset of \( X \) such that \( x \not\in B \). Since \( f \) is \( S^* \)-closed, so \( f(B) \) is supra closed subset of \( Y \) such that \( f(x) \not\in f(B) \) thus by supra \( \beta^* \)-regularity of \( Y \) we have disjoint \( V,W \) containing \( f(B) \) and \( f(x) \) respectively. Clear that (for instance see [8]) \( B \subset f^{-1}(f(B)) \subset f^{-1}(V) \) and \( x \in f^{-1}(W) \), also \( f^{-1}(V) \cap f^{-1}(W) = \emptyset \), hence \( f^{-1}(V) \) and \( f^{-1}(W) \) are the required supra \( \beta^* \)-open sets which complete proof.

\[ \square \]

**Theorem 2.9**

Let \( f : (X,\tau_1^*) \rightarrow (Y,\tau_2^*) \) be a bijective \( \beta^* \)-open and \( S^* \)-continuous map, then \( Y \) is supra \( \beta^* \)-regular if \( X \) is supra \( \beta^* \)-regular.

**Proof.** Suppose \( y \in Y \) and \( B \) is any supra closed subset of \( Y \) such that \( y \not\in B \), hence \( f^{-1}(y) \not\subset f^{-1}(B) \). Since \( f \) is \( S^* \)-continuous, so \( f^{-1}(B) \) is supra closed in \( X \) and by the supra \( \beta^* \)-regularity of \( X \) there are disjoint \( W_1, W_2 \) \( \in S^* \beta^*O(X) \) which containing \( f^{-1}(y) \) and \( f^{-1}(B) \) respectively. Now it is easy to \( f(W_1), f(W_2) \) are the required supra \( \beta^* \)-open sets, implies \( Y \) is supra \( \beta^* \)-regular.

\[ \square \]

The following example shows that not every supra \( \beta^* \)-regular space is supra \( \beta^* \)-T_2-space.

**Example 2.10**

Let \( X=\{w_1,w_2,w_3,w_4\} \) with supra topology \( \tau^* = \{X,\emptyset,\{w_1\},\{w_2\},\{w_3\},\{w_4\},\{w_5\},\{w_6\},\{w_7\},\{w_8\},\{w_9\},\{w_{10}\},\{w_{11}\},\{w_{12}\},\{w_{13}\},\{w_{14}\}\} \), with supra topology \( \tau_1^* = \{X,\emptyset,\{w_1\},\{w_2\},\{w_3\},\{w_4\},\{w_5\},\{w_6\},\{w_7\},\{w_8\},\{w_9\},\{w_{10}\},\{w_{11}\},\{w_{12}\},\{w_{13}\},\{w_{14}\}\} \). It is not difficult to show that \( (X,\tau^*) \) is not \( S^* \)-regular, but it is supra \( \beta^* \)-T_2.

**Theorem 2.11**

Any supra \( \beta^* \)-regular and supra \( T_1^* \)-space is supra \( \beta^* \)-T_2-space.

**Proof.** Assume that \( (X,\tau^*) \) be a supra \( \beta^* \)-regular, supra \( T_1^* \)-space and \( v \not\in X \). From [1] we have \( \{v\} \) is supra closed and since \( y \not\in \{v\} \), then we have disjoint \( W \), \( V \in S^* \beta^*O(X) \) such that \( y \in W \) and \( \{v\} \subset V \) implies \( X \) is supra \( \beta^* \)-T_2-space.

\[ \square \]

The following example shows the quotient space of supra \( \beta^* \)-regular might not supra \( \beta^* \)-regular.
Example 2.12
Let \((\mathbb{R}, \tau_u)\) be the real numbers with \(\tau_u\) and let \(\rho : \mathbb{R} \rightarrow B\), when \(B = \{b_1, b_2, b_3, b_4\}\) defined as follows.
\[
\rho(x) = \begin{cases} 
  b_2 & x < 1 \\
  b_3 & 1 \leq x < 5 \\
  b_4 & 5 \leq x < 20 \\
  b_1 & x \geq 20
\end{cases}
\]
Thus the quotient supra topology on \(B\) is \(qsT(B) = \emptyset, \{b_2\}, \{b_2, b_3\}, \{b_2, b_3, b_4\}\) . Therefore \(S\beta^*O(B) = \emptyset, \{b_2\}, \{b_2, b_3\}, \{b_2, b_3, b_4\}\) . It is clear that \(B, qsT(B)\) is not supra \(\beta^*\)-regular.

3. strongly supra\(\beta^*\)-regular

This section deals with a strong from of \(S\beta^*\)-regularity. In the beginning we will present the following definition.

Definition 3.1
A supra space \((X, \tau^*)\) is called strongly supra \(\beta^*\)-regular \((SS\beta^*\text{-regular})\), if for each supra semi-closed set \(F\) and a \(\notin F\), there exist disjoint supra \(\beta^*\)-open sets \(U, V\) such that \(a \in U\), \(F \subset V\).

The next two theorems give characterizations for \(SS\beta^*\)-regular spaces.

Theorem 3.2
A supra space \((X, \tau^*)\) is \(SS\beta^*\)-regular if and only if for each \(v \in X\) and any supra semi-open set \(W\) such that \(v \in W\), there exist a supra \(\beta^*\)-open set \(V\) such that \(v \in V \subset s-cl_{\beta^*}(V) \subset W\).

Proof. For necessity, since \(v \notin W^c\) and \(W^c\) is supra semi-closed so by the \(SS\beta^*\)-regularity of \(X\), so we have disjoint \(H, K \in S\beta^*O\) such that \(v \notin H\) and \(W^c \subset K\). But \(s-cl_{\beta^*}(V) \subset s-cl_{\beta^*}(W^c)\) and since \(s-cl_{\beta^*}(W^c)\) is supra \(\beta^*\)-closed \([6]\), hence \(s-cl_{\beta^*}(V) \subset V \subset W\), therefore \(v \in V \subset s-cl_{\beta^*}(V) \subset W\). For sufficiency let \(E\) be a supra semi-closed in \((X, \tau^*)\) and \(x \notin E\), \(E^c\) is supra semi-open containing \(x\) implies that there exists a supra \(\beta^*\)-open set \(V\) such that \(x \in V \subset s-cl_{\beta^*}(V) \subset E^c\). Now \(E \subset (s-cl_{\beta^*}(V))^c\), thus \(V \subset (s-cl_{\beta^*}(V))^c\) \(\subseteq S\beta^*O\) such that \(x \in V\) and \(E \subset (s-cl_{\beta^*}(V))^c\), implies \((X, \tau^*)\) is \(SS\beta^*\)-regular.  

Theorem 3.3
A supra space \((X, \tau^*)\) is \(SS\beta^*\)-regular if and only if for every supra semi closed set \(E\) and each \(v \notin E\), there exists a supra \(\beta^*\)-open \(V\) containing \(E\) such that \(v \notin s-cl_{\beta^*}(V)\).

Proof. Since \((X, \tau^*)\) is \(SS\beta^*\)-regular and \(v \notin E\), so there exist disjoint supra \(\beta^*\)-open sets \(W, V\) such that \(v \in W\), \(E \subset V\), hence \(v \notin s-cl_{\beta^*}(V)\). For sufficiency let \(E\) be a supra semi-closed in \((X, \tau^*)\) and \(v \notin E\). By the assumption we have \(V \in S\beta^*O(X)\) such that \(E \subset V\) and \(v \notin s-cl_{\beta^*}(V)\). As a result, there is a supra \(\beta^*\)-open set \(W\) containing \(x\) and \(W \cap V = \emptyset\) implies \((X, \tau^*)\) is \(SS\beta^*\)-regular. 

Theorem 3.4
A supra space \(X\) is \(S\beta^*\)-regular if for every \(v \in X\) the supra \(\beta^*\)-closed neighborhood of \(v\) form a basis of a supra semi-open neighborhood of \(v\).

Proof. Suppose \(v \in X\) and let \(F\) be any supra semi-closed does not contain \(v\). Thus \(X-F\) is supra semi-open containing \(v\), so there is a supra \(\beta^*\)-closed neighborhood \(W\) of \(v\) such that \(v \in W \subset X-F\). Now let \(G = X-W\), then \(G\) is supra \(\beta^*\)-open and \(F \subset G\). On the other hand \(W\) is supra neighborhood of \(v\), hence there is a supra open set \(H\) such that \(v \in H \subset W\) \([6]\). Therefore \(G \cap H \subset X-W \cap W = \emptyset\) and since \(H\) is supra \(\beta^*\)-open \([3.1.3]\) the proof complete. 

For a mapping between two \(SS\beta^*\)-regular spaces we have the following theorems.
Theorem 3.5
let \( f : (X,\tau_1) \to (Y,\tau_2) \) be an injective, supra semi\(^*\)-closed and supra \( \beta^*\)-continuous map , then X is supra \( SS\beta^*\)-regular if Y is \( SS\beta^*\)-regular.

Proof. Suppose \( x \in X \) and \( B \) is any supra semi-closed subset of \( X \) such that \( x \notin B \). Since \( f \) is supra semi\(^*\)-closed, \( f(B) \) is supra semi-closed subset of \( Y \) such that \( f(x) \notin f(B) \) and by the \( SS\beta^*\)-regularity of \( Y \) we have disjoint \( V,W \in SS\beta^*O(X) \) containing \( f(B) \) and \( f(x) \) respectively . Clear that \( (X,\tau_1) \) is supra \( SS\beta^*\)-regular.

Theorem 3.6
Let \( f: (X,\tau_1) \to (Y,\tau_2) \) be a bijective , supra semi\(^*\)-open and \( S^*\)-continuous map, if X is \( SS\beta^*\)-regular then Y is \( SS\beta^*\)-regular.

Proof. Suppose \( y \in Y \) and \( B \) is any supra semi-closed subset of \( Y \) such that \( y \notin B \), hence \( f^{-1}(y) \notin f^{-1}(B) \). Since \( f \) is \( S^*\)-continuous , \( f^{-1}(B) \) is supra semi-closed in \( X \) and by the \( SS\beta^*\)-regularity of \( X \) there are disjoint \( W_1,W_2 \in SS\beta^*O(X) \) which containing \( f^{-1}(y) \) and \( f^{-1}(B) \) respectively . Now it is easy to show that \( f(W_1), f(W_2) \) are the required supra \( SS\beta^*\)-open sets , implies Y is \( SS\beta^*\)-regular.

The following example shows that not every \( SS\beta^*\)-regular space is supra \( \beta^*-T_2\)-space.

Example 3.7
Let \( X=\{w_1, w_2, w_3, w_4\} \) with supra topology \( \tau^* = \{ X,\emptyset, \{w_1, w_2\}, \{ w_3, w_4\}, \{ w_5, w_6\}, \{ w_7, w_8\}, \{ w_9, w_{10}\}, \{ w_{11}, w_{12}\}, \{ w_{13}, w_{14}\}, \{ w_{15}, w_{16}\}, \{ w_{17}, w_{18}\}, \{ w_{19}, w_{20}\}, \{ w_{21}, w_{22}\}, \{ w_{23}, w_{24}\}, \{ w_{25}, w_{26}\}, \{ w_{27}, w_{28}\}\} \). It is not difficult to show that \( (X,\tau^*) \) is not \( SS\beta^*\)-regular space , but it is supra \( \beta^*-T_2\)-space.

Theorem 3.8
Any \( SS\beta^*\)-regular and supra semi \( T_1\)-space is supra \( \beta^*-T_2\)-space.

Proof. Let \( (X,\tau^*) \) be a \( SS\beta^*-regular \) , supra semi \( T_1\)-space and \( x \neq y \in X \). From [5] we have \( \{x\} \) is supra semi-closed and since \( y \notin \{x\} \), so we have disjoint \( U,V \in S\beta^*O(X) \) such that \( y \in U \) and \( \{x\} \subset V \), implies \( (X,\tau^*) \) is supra \( \beta^*-T_2\)-space.

4. Supra \( \beta^*-R_0\)-space and supra \( \beta^*-R_1\)-space.
Recall that [7] a space \( X \) is said to be \( R_0\)-space if for each open set \( V \) and \( c\in V \) we have \( cl\{c\} \subset V \) and \( X \) is called \( R_1\)-space if for each \( d_1 \neq d_2 \) in \( X \) with \( cl\{d_1\} \neq cl\{d_2\} \), then we have disjoint open sets \( V_1,V_2 \) such that \( cl\{d_1\} \subset V_1 \) and \( cl\{d_2\} \subset V_2 \).

Definition 4.1
A supra space \( (X,\tau^*) \) is called supra \(-R_0\) if each supra open set contains the supra closure of each of its singleton.

Definition 4.2
A supra space \( (X,\tau^*) \) is called supra \(-R_1\) if for each \( c_1,c_2 \) in \( X \) with \( cl\{c_1\} \neq cl\{c_2\} \), there exist disjoint supra open sets \( W_1,W_2 \) such that \( cl\{c_1\} \subset W_1 \) and \( cl\{c_2\} \subset W_2 \).

Example 4.3
Let \( X=\{w_1, w_2, w_3\} \) with supra topology \( \tau^* = \{ X,\emptyset, \{w_1\}, \{ w_2\}, \{ w_3\}, \{ w_4\}, \{ w_5\}, \{ w_6\}, \{ w_7\}, \{ w_8\}\} \). It is not difficult to show that \( (X,\tau^*) \) is supra \( R_0\)-space and supra \( R_1\)-space.
Remark 4.4
Every supra $R_1$ is supra $R_0$.

**Proof.** Assume $U$ be a supra open subset of a supra space $(X,\tau)$. We want to show that $\text{cl}^\tau((c)) \subset U$ for each $c \in U$.

Now suppose $y \notin U$, hence $c \notin \text{cl}^\tau((y))$ and hence $\text{cl}^\tau((c)) \neq \text{cl}^\tau((y))$. Since $(X,\tau)$ is supra $R_1$, so there exists supra open $W_y$ such that $\text{cl}^\tau((y)) \subset W_y$ implies $y \notin \text{cl}^\tau((c))$ which complete the proof. \hfill $\Box$

The converse of (3.4) is not true as seen in the following example.

**Example 4.5**
Let $X=\{w_1, w_2, w_3\}$ with supra topology $\tau^*=\{X, \emptyset, \{w_1, w_2\}, \{w_2, w_3\}, \{w_1, w_3\}, \{w_1, w_2, w_3\}\}$, hence $\tau^*=\{\emptyset, X, \{w_1\}, \{w_2\}, \{w_3\}, \{w_1, w_2\}, \{w_2, w_3\}, \{w_1, w_3\}, \{w_1, w_2, w_3\}\}$. It is clear that $(X,\tau^*)$ is not supra $R_1$-space, while it is supra $R_0$-space.

**Definition 4.6**
A supra space $(X,\tau)$ is called supra $\beta^*-R_0$ if every supra $\beta^*$-open set contains the supra $\beta^*$-closure of each of its singleton.

**Definition 4.7**
A supra space $(X,\tau)$ is said to be supra $\beta^*-R_1$ if for each $c_1, c_2 \in X$ with $s-cl_{\beta^*}(\{c_1\}) \neq s-cl_{\beta^*}(\{c_2\})$, there exist disjoint supra $\beta^*$-open sets $V_1, V_2$ such that $s-cl_{\beta^*}(\{c_1\}) \subset V_1$ and $s-cl_{\beta^*}(\{c_2\}) \subset V_2$.

**Example 4.8**
Let $X=\{w_1, w_2, w_3\}$ with supra topology $\tau^*=\{X, \emptyset, \{w_1\}, \{w_1, w_2\}, \{w_2, w_3\}, \{w_1, w_3\}, \{w_1, w_2, w_3\}\}$. Hence $\tau^*=\{\emptyset, X, \{w_1\}, \{w_2\}, \{w_3\}, \{w_1, w_2\}, \{w_2, w_3\}, \{w_1, w_3\}, \{w_1, w_2, w_3\}\}$. It is clear that $(X,\tau^*)$ is supra $\beta^*-R_0$-space and supra $\beta^*-R_1$-space.

**Remark 4.9**
Every supra $\beta^*-R_1$-space is supra $\beta^*-R_0$-space.

**Proof.** The proof is similar to (3.4).

The following example demonstrates that the opposite of the aforementioned statement (3.9) is not true.

**Example 4.10**
Let $X=\{w_1, w_2, w_3\}$ with supra topology $\tau^*=\{X, \emptyset, \{w_1\}, \{w_1, w_2\}, \{w_2, w_3\}\}$. Hence $\tau^*=\{\emptyset, X, \{w_1\}, \{w_2\}, \{w_3\}\}$. Thus $S\beta^*C=\{\emptyset, X, \{w_1, w_3\}, \{w_1, w_2\}, \{w_2, w_3\}\}$, and implies $S\beta^*O=\{\emptyset, X, \{w_1\}, \{w_2\}, \{w_3\}\}$. It is obvious that $(X,\tau^*)$ is not supra $\beta^*-R_1$-space, while it is supra $\beta^*-R_0$-space.

**Theorem 4.11**
A supra space $X$ is supra $\beta^*-R_0$ if and only if for any $c_1$ and $c_2$ in $X$ such that $s-cl_{\beta^*}(\{c_1\}) \neq s-cl_{\beta^*}(\{c_2\})$, then $s-cl_{\beta^*}(\{c_1\}) \cap s-cl_{\beta^*}(\{c_2\}) = \emptyset$.

**Proof.** Assume $X$ is supra $\beta^*-R_0$ space and $c_1, c_2 \in X$ such that $s-cl_{\beta^*}(\{c_1\}) \neq s-cl_{\beta^*}(\{c_2\})$, so let $w \in s-cl_{\beta^*}(\{c_1\})$ and $w \notin s-cl_{\beta^*}(\{c_2\})$. Thus there is $U \in S\beta^*O(X)$ such that $c_2 \notin U$ and $w \notin U$. But $w \in s-cl_{\beta^*}(\{c_1\})$, hence $c_1 \notin U$ and hence $c_1 \notin s-cl_{\beta^*}(\{c_2\})$, which is supra $\beta^*$-open and by the supra $\beta^*-R_0$ of $X$ we have $s-cl_{\beta^*}(\{c_1\}) \subset (s-cl_{\beta^*}(\{c_2\}))^c$ implies $s-cl_{\beta^*}(\{c_1\}) \cap s-cl_{\beta^*}(\{c_2\}) = \emptyset$. Conversely let $U$ be a supra $\beta^*$-open and $c \in U$, we want to show $s-cl_{\beta^*}(\{c\}) \subset U$. Now suppose $w \notin U$, hence $c \notin s-cl_{\beta^*}(\{w\})$ and hence $s-cl_{\beta^*}(\{w\}) \neq s-cl_{\beta^*}(\{c\})$ and by the assumption we have $s-cl_{\beta^*}(\{w\}) \cap s-cl_{\beta^*}(\{c\}) = \emptyset$, implies $w \notin s-cl_{\beta^*}(\{c\})$ and we have done. \hfill $\Box$
Remark 4.12
Every Sβ*-regular and every SSβ*-regular are supra β'-R0.
Proof.1. Assume X be a SSβ*-regular, so by (2.2) we have for every w∈X and any supra semi-open W such that w∈W, there exists supra β'-open W2 such that w∈W2⊂s-clβ* (W2)⊂W1. In particular for each v∈W1 we have s-clβ*((v))⊂W1, hence X is supra β'-R0. and since each supra open is also supra semi-open [4], thus the proof of the case X is Sβ*-regular follows immediately. □

Theorem 4.13
Every supra β'-R1- space is supra β*-regular restricted on s-clβ*((c))
Proof. Assume X be a supra β'-R1- space, so by (3.4) it is supra β*-R0- space and let z∉s-clβ*((c)), hence s-clβ*((c))≠s-clβ*((z)) implies s-clβ*((c))∩s-clβ*((z))=Ø (3.11). Now since X is supra β'-R1- space, then we have U, V∈β*O(X) such that s-clβ*((z))⊂U and s-clβ*((c))⊂V. But s-clβ*((c)) is supra β*-closed (1.4) and since each supra closed is supra β*-closed [6]. As a result every supra closed F of the form s-clβ*((c)) for each c in X and c∉F separated by supra β*-open sets . □

5. Conclusions
Our main target in this work was to generalize one of the properties of separation axioms in the supra topological spaces which is the supra regularity . The concept of (Sβ*-SSβ*) spaces was introduced and the properties of such kind of spaces were studied, also these types were linked to the concept of (R0-R1) spaces after their generalization to supra β'-R0 and supra β'-R1.
6. References


