

Improved the Focal Length of a Concave Mirror Using Digital Image Analysis Technology

Hassan, H. D¹, Malik, M. I²

^{1,2}Department of Physics, College of Science, Wasit University, IRAQ

*Corresponding Author: Hassan, H. D

DOI: <https://doi.org/10.31185/wjps.498>

Received 20 August 2024; Accepted 19 September 2024; Available online 30 September 2024

ABSTRACT: This study explores the use of digital image processing technology to measure the focal lengths of simple concave mirrors. The proposed method is efficient, quick to implement, and offers high accuracy in calculating focal lengths. The process involves capturing multiple images of the reflection formed by the concave mirror on a screen, positioned at equal intervals. The point spread function (PSF) of these images is then calculated to identify the one with the least spread, indicating the clearest image. In this research, image number 7 was identified as having the lowest PSF value. This was determined using mathematical algorithms and equations implemented in MATLAB, with the corresponding graph shown in Figure 3. Subsequently, the focal length was calculated using the general equation for lenses and mirrors. A comparison was made with the traditional approach, which relies on students in the optics lab at the College of Science, University of Wasit, visually selecting the sharpest image. The digital method demonstrated superiority in determining focal lengths more accurately. A statistical analysis of the students' measurements revealed discrepancies with some margin of error, highlighting the advantages of this method in terms of precision and time-saving, as confirmed by the experimental findings.

Keywords: Focal length, concave mirror, digital image processing, statistical analysis, point spread function.



1. INTRODUCTION

The focal length of a lens refers to the distance between the lens's focus (or one of its two focal points). Lenses are scientifically classified based on their focal length, which is a critical factor in lens performance and plays a key role in the advancement of lens technology. By influencing the field of view and magnification, the focal length directly impacts the perspective of the image, the depth of field, visual effects, and the overall magnification of the camera lens. It is also a measure of the lens's optical power, indicating its capacity to converge or diverge light. Therefore, focal length serves as a defining characteristic for various lens properties [1]. Numerous previous studies have addressed methods for calculating focal length. For instance, in 2002, Brian J. De Boo and colleagues introduced a technique using holograms for precise focal length measurement. The holograms utilized first-order diffraction to mimic the reflective behavior of a convex spherical mirror in zero-point tests with a phase-shift interferometer [2]. In 2014, Chong ming Yang and others developed a method employing the Fiesta interferometer to measure the focal lengths of lenses and mirrors [3]. In 2015, Jiang-Hua Xu et al. used the Hartmann-Schack sensor to measure focal lengths within the 4F system [4]. In 2019, Zhangji Lu and his team proposed a technique based on the Gaussian optical formula and the geometric relationship between the camera and lens system to derive the focal length [5]. By 2021, Y. Mejía presented a simple method for determining the effective focal length of optical systems by measuring tangential magnification and calculating object and image positions relative to two reference points, deriving a linear equation for the distances of the principal planes [6]. In 2024, J. Dou et al. introduced a method based on vortex beam interference to measure focal lengths [7]. Currently, we propose an enhanced method for calculating the focal length of concave mirrors using digital image processing techniques, with algorithms and equations being developed using MATLAB.

2. RESEARCH PROBLEM

The main problem lies in the difficulty and accuracy of calculating the focal length of concave mirrors using traditional methods, which rely heavily on manual estimation and laboratory experiments. These methods may be inaccurate and suffer from variability in results. As a result, there is a need to develop MATLAB-based algorithms to increase the accuracy of these calculations and reduce potential errors, which contributes to providing more effective solutions in various applied and scientific fields.

3. SIGNIFICANCE OF THE RESEARCH:

This research gains its importance by presenting an innovative methodology for calculating the focal length of a concave mirror using algorithms programmed in MATLAB. This research will help simplify calculations and increase accuracy compared to traditional methods, making it easier for researchers and students to analyze the properties of concave mirrors more effectively. Furthermore, MATLAB is a powerful program for data processing and analysis, making it an excellent choice for such calculations.

4. RESEARCH OBJECTIVES

1. **Developing efficient algorithms for calculating focal length:** The research aims to design and develop accurate algorithms using MATLAB to calculate the focal length of a concave mirror in an automated manner.
2. **Improving measurement accuracy:** Comparing the results calculated using the developed algorithms with traditional methods to verify the improvement in accuracy and reduce errors.
3. **Providing an integrated software tool:** Researchers and students can quickly and accurately calculate the focal length by using an easy-to-use software tool within the MATLAB environment.
4. **Applying algorithms in practical experiments:** Testing algorithms in practical applications such as designing optical systems and analyzing devices based on concave mirrors.
5. **Disseminating knowledge and stimulating development:** Contributing to publishing the research results as a reliable reference for developing similar tools and software in other fields.

4. DIGITAL IMAGE & IMAGE PROCESSING

There are many types of digital images. Grayscale images are one of the most popular, with a range of (0-255) grey levels for each pixel (the smallest element in a digital image). [8] If the grayscale image is closer to zero, it is close to darkness, but if the value is close to (255), it is close to brightness. [9]. Digital images consist of columns and rows of pixels. The more pixels there are, the higher the image's resolution. Image processing is one of the important fields in computer science that performs certain operations on digital images to improve, analyze, and extract information from them to represent two-dimensional images. This technique requires software and computers. [10].

5. EXPERIMENTAL DESIGN

5.1 MATERIALS & METHOD

In this research, we conducted a simple system for this work in the Optics Laboratory at the College of Science, Wasit University, to measure the focal length of a concave mirror, as shown in Figure (2). The tools for conducting the experiments included the following: Flashlight +concave mirror, Two Holder, Ruler, plate, MATLAB A program, Camera (d5200 nikon, 24 mega pixel, full HD)



Figure (1) The label of the used concave mirror



Figure (2) Improved System

6. PRQCTICAL ACTIVE

In this research

1. We are working on building a simple system, as shown in Figure No. (2), for the purpose of the practical part.
2. We used the camera to take pictures of the shape, and then fed the pictures into the computer to do the calculations.
3. Pictures are taken at equal distances to get the best picture. Next, we calculate the focal length of the concave mirror using trigonometric relationships. $1/f = 1/u + 1/v$
4. The computer receives the data and uses MATLAB to perform calculations on it.
5. We analyze the images with developed code and determine the optimal image by comparing the point spread function (PSF) of each image. The lower the PSF value, the sharper and less distorted the image becomes.
6. We compare the results obtained with the results of students using the traditional method

a. METHODOLOGICAL DETAIL

- **Reading Images:** The captured images are read into the MATLAB environment for analysis.
- **Fourier Transform Calculation:** The Fourier Transform is computed for each image, converting spatial data into frequency data.
- **Dynamic Range Computation:** The dynamic range of the Fast Fourier Transform (FFT) is calculated, representing the difference between the maximum and minimum values.
- **PSF Metric Calculation:** The PSF metric, derived from the dynamic range, quantifies image clarity.
- **PSF Comparison:** The PSF values of the images are compared to identify the image with the lowest PSF, indicating the least distortion and highest clarity.
- **Displaying the Best Image:** The image with the lowest PSF value is displayed as the optimal image.
- **Distance Determination:** The corresponding distance for the lowest PSF image is identified, which is crucial for accurate focal length measurement.
- **Displacement Method Application:** The displacement method is applied to refine the measurements further and validate the experimental results.

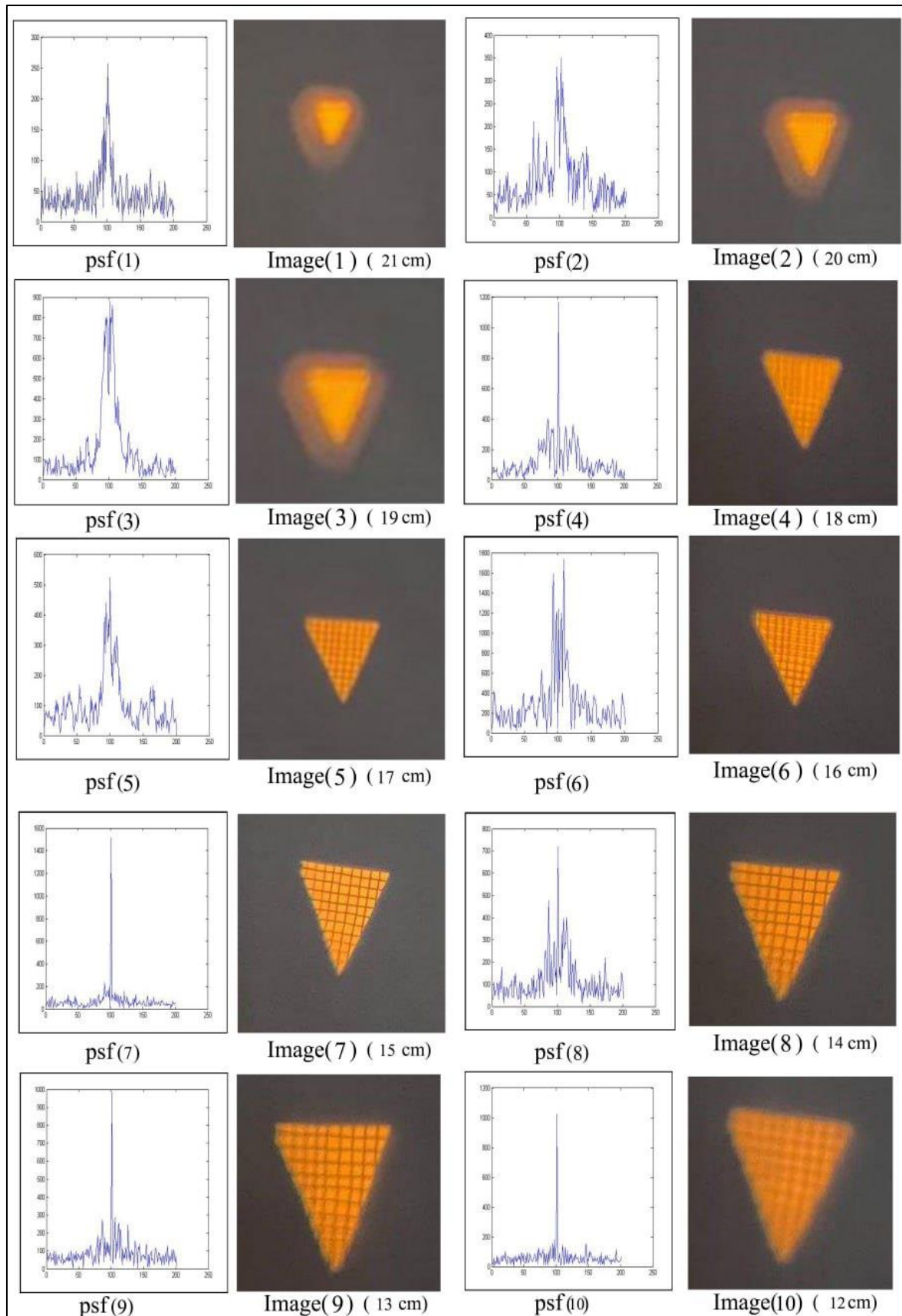


Figure (3): Point Spread Function (PSF) for each image.

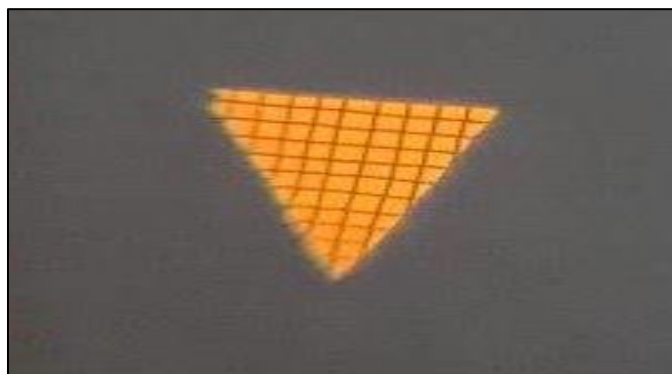


Figure (4) Minimum PSF image

Image No.7 was chosen as the best computer image through the equations used in the Fourier transform, as well as the dynamic range of contrast in the images, as mentioned above.

7. STATISTICAL ANALYSIS

The table provides a comparison between focal length measurements made by different students using the traditional method. The computer recorded the "focal length" as 10 cm, with Student (1) scoring 13.2 cm, Student (2) scoring 10.5 cm, Student (3) scoring 10.6, Student (4) scoring 10.5, Student (5) scoring 10.3 cm, Student 6 scoring 10.7 cm, Student 7 scoring 12.2 cm, and Student 8 scoring 11.2 cm. The Statistical Test column confirms the execution of a statistical analysis, yielding a value of 3.195. The "P-value" column indicates the statistical test's significance level, with a value of 0.023, denoted by "S" to denote a significant difference between groups at a p-value threshold of less than 0.05. This shows that there is significant variation in focal length measurements between students, with the p-value indicating statistical significance. These differences in students' calculations are due to estimating the best picture using the eye. Students may use different methods to find the best picture, and then record the distance to the best picture. Next, we take measurements to determine the focal length. The improvement that has been made over this traditional method is that the decision to calculate the best image is computerized.

Table 1: Compare the Focal Length for concave mirror for different students

Computers	Focal Length (cm)	Statistical Test (ANOVA)	P value
Control	10	4.029	0.018 *
Student 1	10.7		
Student 2	10.5		
Student 3	10.4		
Student 4	9.8		
Student 5	11		
Student 6	9.5		
Student 7	10.2		
Student 8	10.15		
Student 9	10.6		
Student 10	10.9		

* Significant difference between groups (p value < 0.05)

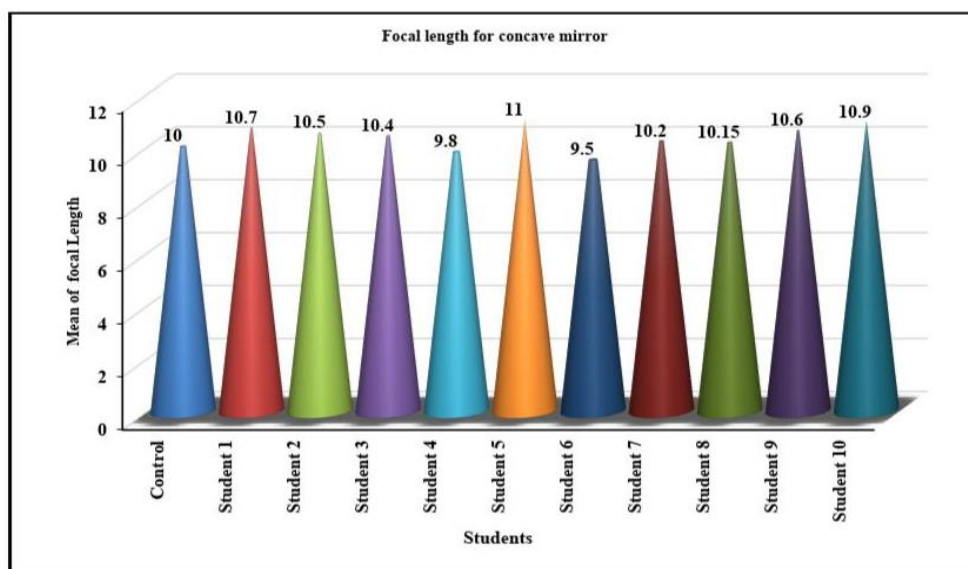


Figure (5) Compare the focal length for different students

8. RESULT AND DISCUSSION

In this research, we found a simplified method to calculate the focal length of a concave mirror using digital image processing techniques. We used a modern camera to take pictures at variable distances (1 cm), along with simple laboratory tools. The pictures are taken and then transferred to the computer for processing via the MATLAB program using a special code and mathematical equations. This code loads a set of images and then calculates a PSF scale (point spread function) for each image. The PSF function describes the path of light from a point in the image to a point in the recorded image. The mathematical equations used in this code are as follows:

Below is a detailed explanation of the mathematical formulas used in the code:

1. Reading Images

Steps to Read Images Using MATLAB:

I. Load Image Files:

o We use MATLAB's `imread` function to load the image files into the workspace. This function reads the image from the specified file path and stores it as a matrix.

```
img = imread('image_file_path');
```

II. Display the Image:

o We display the loaded image using the `imshow` function to ensure it has been correctly loaded and to visually inspect it for any obvious issues.

```
imshow(img);
```

III. Convert to Grayscale (if necessary):

o If the image is in colour, we convert it to grayscale using the `rgb2gray` function. Grayscale images are simpler to process and often sufficient for analysis.

```
gray_img = rgb2gray(img);
```

IV. Store Images in an Array:

o For batch processing, we store multiple images in an array. This allows for easy iteration and processing of each image.

```
images{i} = gray_img;
```

V. Iterate Over Multiple Images:

o We use a loop to read and store multiple images for analysis.

```
for i = 1:num_images
```

```
    images{i} = imread(sprintf('image_%d.png', i));
```

```
    if size(images{i}, 3) == 3
```

```
        images{i} = rgb2gray(images{i});
```

```
    end
```

```
end
```

2. Fourier Transform Calculation

Calculating the Fourier Transform for Each Image:

a. Apply Fourier Transform:

o We compute the 2D Fourier Transform of the image using the `fft2` function. This transforms the image from the spatial domain to the frequency domain.

```
fft_img = fft2(gray_img);
```

b. Shift Zero Frequency Component:

o We use the `fftshift` function to shift the zero-frequency component to the centre of the spectrum. This makes the frequency analysis more intuitive.

```
fft_shifted = fftshift(fft_img);
```

The formula for 2D Fourier Transform:

The 2D Fourier Transform of an image $f(x,y)$ is given by:

Where:

- $F(u,v)$ is the Fourier Transform of the image.
- $f(x,y)$ is the original image in the spatial domain.
- M and N are the dimensions of the image.
- u and v are the frequency coordinates.
- j is the imaginary unit.

This transformation allows us to analyze the image's frequency components, which is essential for understanding the distribution of light and identifying patterns that contribute to the Point Spread Function (PSF).

3. Dynamic Range Calculation

Calculating the Dynamic Range of the FFT:

a) Compute Magnitude Spectrum:

o We calculate the magnitude of the Fourier Transform using the `abs` function. The magnitude spectrum represents the amplitude of each frequency component.

```
magnitude_spectrum = abs(fft_shifted);
```

b) Calculate Dynamic Range:

o We determine the dynamic range by finding the difference between the maximum and minimum values of the magnitude spectrum. The dynamic range is an indicator of the contrast in the frequency domain.

```
dynamic_range = max(magnitude_spectrum(:)) - min(magnitude_spectrum(:));
```

The dynamic range of the FFT spectrum is a critical measure in evaluating image quality. A higher dynamic range indicates greater contrast and potentially clearer images, while a lower dynamic range may suggest noise or blurriness.

4. Point Spread Function (PSF) Metric Calculation

Using the Dynamic Range to Calculate the PSF Metric:

□ Calculate PSF Metric:

o The PSF metric is derived from the dynamic range of the FFT. The PSF describes how a point source of light is represented in the image, influencing the clarity and sharpness of the image.

```
psf_metric = dynamic_range;
```

The PSF metric helps us identify the image with the least distortion. A lower PSF value indicates better image quality, as it represents a more accurate spread of light from a point source.

5. Comparison and Selection

Comparing the PSF for Each Image to Find the One with the Lowest PSF:

A. Initialize Minimum PSF:

o We initialize variables to store the minimum PSF value and the corresponding image index.

```
min_psf = inf;
```

```
min_psf_index = 0;
```

B. Compare PSF Values:

o We iterate through all images to compare PSF values and identify the image with the lowest PSF.

```
for i = 1:length(images)
```

```
    if psf_metric(i) < min_psf
```

```
        min_psf = psf_metric(i);
```

```
        min_psf_index = i;
```

```
    end
```

```
end
```

C. Store and Display Minimum PSF Image:

o We store the image with the lowest PSF for further analysis and display it using `imshow`.

```
best_image = images{min_psf_index};
```

```
imshow(best_image);
```

1. Finding the Distance Corresponding to the Lowest PSF Image

□ Record Distance:

o We record the distance corresponding to the image with the lowest PSF. This distance is critical for calculating the focal length accurately.

```
best_distance = distances(min_psf_index);
```

6. Applying the Displacement Method:

❖ Calculate Focal Length:

- We use the displacement method and the recorded distance to calculate the focal length. The displacement method involves using the object distance (U) and the image distance (V) to find the focal length (f) using the lens formula:

```
focal_length = calculate_focal_length(U, V);
```

Minimum PSF Image:

❖ Display or Save Minimum PSF Image:

- We display the image with the minimum PSF or save it for documentation and further analysis.

```
imshow(best_image);
imwrite(best_image, 'min_psf_image.png');
```

This extended methodology provides a detailed and comprehensive approach to reading images, performing Fourier transform calculations, calculating dynamic range, computing the PSF metric, and selecting the image with the lowest PSF for accurate focal length determination using MATLAB. By following these steps meticulously.

9. SIMULATION RESULTS: BEST IMAGE SELECTION

We employed an iterative process to evaluate each image's PSF metric, comparing it against the current best metric. We identified the image with the lowest PSF metric as the best. Specifically, we ensure precision, reliability, and reproducibility in measuring the focal length of lenses. This rigorous analysis identified Image No. 7 as the optimal image. The PSF metric for Image No. 7 was the lowest among the evaluated images, indicating superior clarity and sharpness. The Fourier effectiveness of the Fourier transform and dynamic range analysis in quantifying image quality led to the selection of the best possible image for subsequent computational. Now, we apply the general law of lenses by measuring the distance from the source to the object, which is equal to 15 cm, and the distance from the image, which is equal to 32 cm. This allows us to determine the focal length of the lens and demonstrates the effectiveness of the developed method in determining focal lengths.

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} = \frac{1}{30} + \frac{1}{15} = 10\text{cm} \quad \text{the focal length of the lens}$$

10. CONCLUSION

Digital image processing is one of the important processes that are applied for the purpose of improving images and reducing noise levels in them. In this research, this technology was applied to find the focal length of the concave mirror. This was done by developing algorithms and mathematical equations for the purpose of calculating the focal length. Excellent and satisfactory results were obtained, as the best image (image 7) was obtained, through which the distance of this image is taken, which is equal to (30 cm), in addition to the distance from the light source to the concave mirror, which was recorded as (15 cm). Then the general law of lenses is applied for the purpose of finding the focal length. These results were also compared with the results of students using the traditional method (at Wasit University). We noticed an increase and decrease in these results, which proves the effectiveness and validity of the method that was proposed and developed.

REFERENCES

- [1] F. J. Torcal-Milla and L. M. Sanchez-Brea, "Near-field diffraction-based focal length determination technique," *Opt. Lasers Eng.*, vol. 92, pp. 105-109, 2017, doi: 10.1016/j.optlaseng.2016.09.008.
- [2] B. J. DeBoo and J. M. Sasian, "Novel method for precise focal length measurement," in *Proc. International Optical Design Conference 2002*, vol. 4832, SPIE, 2002
- [3] Z. Yang, H. Zhang, C. Chen, and Y. Zhang, "Measurement of focal length of lenses and mirrors using Fiesta interferometer," *Optics Express*, vol. 22, no. 7, pp. 7854-7859, Apr. 2014.
- [4] J. Xu and S. Zhuang, "Measurement of lens focal length with Hartmann–Shack wavefront sensor based on 4F system," *Optik*, vol. 126, no. 13, pp. 1303-1306, 2015.

- [5] Z. Lu and L. Cai, "Paraxial focal length measurement method with a simple apparatus," *Optics Express*, vol. 27, no. 3, pp. 2044-2055, 2019
- [6] Y. Mejía and R. Díaz-Urbe, "Measuring the principal planes and effective focal length of an optical imaging system," *Optical Engineering*, vol. 60, no. 10, pp. 104105-1–104105-7, 2021
- [7] J. Dou, X. He, Y. Hu, X. Chen, and Z. Yang, "Focal length measurement based on vortex beam interference," *Optics and Lasers in Engineering*, vol. 178, p. 108197, 2024
- [8] M. I. Malik, Z. H. Idan, and K. A. Assaf, "Study the magnetization of water using a digital camera and laser beam," in *IOP Conference Series: Materials Science and Engineering*, vol. 928, no. 7, p. 072099, Nov. 2020.
- [9] S. H. Shahad, M. I. Malik, and H. A. Al-Dabbagh, "Land use and land cover study of Al-Kut city, Iraq using Sentinel-2 images by supervised classification techniques," in *AIP Conference Proceedings*, vol. 2834, no. 1, Dec. 2023.
- [10] R. C. Gonzalez and R. E. Woods, *Digital Image Processing*, Global Edition. Pearson Education Canada, 2017.