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Mathematical Model of Fungal growth with Hyphae death and Substrate

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ABSTRACT: One of the most crucial subjects in mathematics education is mathematical modeling, which involves connecting situations from the real world to mathematical formulas and vice versa. Our study will focus on how substrate affects fungal growth and lateral branching behaviors, hyphal death, and tip death due to overcrowding. In order to solve a mathematical model, we will employ the system a three-dimensional of equations (PDEs). using numerical solutions, traveling wave theory, stability, and non-dimensionality in our mathematical approach as well as we use trace and determine to find stable states. Computer programs, such as the Maple program and Pdep code in MATLAB, will also be used to aid in the solution process.

Keywords: Mathematical Model, Fungal Growth, Lateral branching, Tip death due to overcrowding, Substrate, Hyphal death.

1. INTRODUCTION

Mathematical modeling is a technique for explaining real-world systems and Events using mathematical concepts, descriptions and methods. Previously, professionals used mathematical models to analyze and predict behavior and events, as well as to solve complex problems and ask questions [5], In [7], they divided the mathematical model into discrete models, where variables change occasionally, and continuous models, where variables vary continuously in space and time.

Even though it has significant ecological implications, the development of fungi in colonies on the surface of a solid food medium is a frequently used experimental method.

The law of growth, $\frac{dM}{dt} = \alpha M$, which takes into account the mass of the organism per unit volume (M), time (t), and a constant called the specific growth rate (α) forms the foundation for the idea of fungal growth.where the organism develops exponentially in water-based homogenous liquid cultures, Without an excess of all nutrients and the accumulation of growth inhibitors [9].

Leah Kishet, a person who converted biological phenomena into a mathematical form. we study in this paper new types of fungi FXDS where F: lateral branching, X: Tip death due to overcrowding D: Hyphal death and S: substrate (represent energy), it's a biological model with complex analytical properties where Table (1) illustrates the mathematical form, biological kind, and clarification of each branch's parameters and version.

The phenomenon of fungal growth has been reduced to mathematical formulas and equations.

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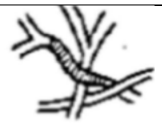
2. FXDS: LATERAL BRANCHING, TIP DEATH DUE TO OVERCROWDING WITH HYPHAL DEATH AND SUBSTRATE

Table (1): Show branching, the biological type, this type's symbol, and its version [10]

Branching	Biological type	Symbol	Version	Parameters description
	Lateral branching	F	$\delta = \alpha_2 p$	The number of branches formed in a unit of time per unit length of hypha is represented by α_2.
	Tip death due to overcrowding	X	$\delta = -\beta_3 \rho^2$	The rate at which branching is eliminated due to overcrowded density limits is β_3
	Hyphal death	D	d = dρ	The hyphal loss rate (constant for hyphal death) is denoted by d.

we will look at how substrate inclusion affects colony growth modeling using d(ρ) ≠0 .[8]

2.1 Mathematical Model

The behavior of the species in terms of ρ = density of the hypha in unit length per unit area and n = tip density and s = interaction of substrate is what mathematicians observed when they transformed the fungi's branches into letters [1,5]. We can use the following system to explain hyphal growth:

$$\frac{\partial \rho}{\partial t} = nvs - d\rho$$
$$\frac{\partial n}{\partial t} = -\frac{\partial (nvs)}{\partial x} + \delta(\rho,n,s) \qquad (1)$$
$$\frac{\partial s}{\partial t} = -kn$$

Where: $\delta(\rho,n,s) = \alpha_2 \rho s - \beta_3 \rho^2$. So, system (1) becomes:[1]

$$\frac{\partial \rho}{\partial t} = nvs - d\rho$$
$$\frac{\partial n}{\partial t} = -\frac{\partial (nvs)}{\partial x} + \alpha_2 \rho s - \beta_3 \rho^2 \qquad (2)$$
$$\frac{\partial s}{\partial t} = -kn$$

2.2 Non – dimensionlision and Stability

Keshet (1982) [2] and Ali H. (2011) [1] demonstrate how these parameters can be set to lower dimensions, at which point the system (2) becomes:

$$\frac{\partial \rho}{\partial t} = ns - \rho$$
$$\frac{\partial n}{\partial t} = -\frac{\partial ns}{\partial x} + \alpha\rho(s - \rho) \qquad (3)$$
$$\frac{\partial s}{\partial t} = -kn$$

Where $\alpha = \frac{\alpha_2 v}{\gamma^2}$

the α parameter The hyphenated branching rate per unit length per unit time is represented by it , $\alpha p\,(1-p)$ denotes the number of branches generated per unit time per unit hyphen length and k is a non-negative.[1,2] to find steady states of the system(3):

$$ns-\rho=0 \quad , \quad \alpha\rho(s-\rho)=0 \quad , \quad -kn=0$$

we get two steady state points $(\rho_0, 0, 0)$, $(0, 0, S_0)$ for (nonnegative) constants ρ_0 and S_0.

By using Jacobin and eigenvalues of these equations [3,4], we can obtain three values of (λ)

$$J(\rho\ , n\ , s)=\begin{bmatrix} -1 & s & n \\ \alpha s - 2\alpha\rho & 0 & \alpha\rho \\ 0 & -k & 0 \end{bmatrix}$$

$$J(\rho\ , 0\ , 0)=\begin{bmatrix} -1 & 0 & 0 \\ -2\alpha\rho & 0 & \alpha\rho \\ 0 & -k & 0 \end{bmatrix}$$, by use B= |A - λI| = 0 , then :

$$\lambda_1=-1,\ \lambda_{2,3}=\pm(\sqrt{-k\alpha\rho})$$

since we get λ_i , s.t i = 1,2,3, then we can use the trace and determinate of matrix (B) to find stability of the steady state [2,4]:

Trace(B) = $-1-3\lambda$, determinate (B) = $(-1+\lambda)(K\alpha\rho+\lambda^2)$

if $\lambda>0$, then trace(B) is negative, det(B) is negative, we obtain steady state is saddle point.

if $\lambda<0$, then trace(B) is positive, det(B) is positive, we obtain steady state is unstable node.

$$J(0\ , 0\ , s)=\begin{bmatrix} -1 & s & 0 \\ \alpha s & 0 & 0 \\ 0 & -k & 0 \end{bmatrix}$$, by use B= |A - λI| = 0 , then :

$$\lambda_1=0,\ \lambda_{2,3}=\frac{-1\pm\sqrt{4\alpha s^2+1}}{2}$$

since we get λ_i , s.t i = 1,2,3, then we can use the trace and determinate of matrix (B) to find stability of the steady state [2,4] :

Trace (B) = $-1-3\lambda$, determinate (B) = $(\alpha S^2\lambda-\lambda^2-\lambda^3)$

if $\lambda>0$, then trace(B) is negative, det(B) is negative, we obtain steady state is saddle point.

if $\lambda<0$, then trace(B) is positive, det(B) is positive, we obtain steady state is unstable node.

2.3 Traveling Wave Solution

The functions p(x,t), n(x,t), and s(x,t) move at a constant speed c in the positive x direction. We seek solutions for P(z), N(z), and S(z), where z = x – ct, in order to discover the traveling wave solution to the system's (3) equations in x and t. Three ordinary differential equations can be used to simplify the system (3). [1,5,8]:

$$\frac{dP}{dZ}=\frac{-1}{C}NS-P$$
$$\frac{dN}{dZ}=\frac{1}{S-C}[\frac{-K}{c}N^2+\alpha P(S-P) \qquad S\neq c \ \text{ and } \ c\neq 0 \qquad (4)$$
$$\frac{dS}{dZ}=\frac{K}{C}N$$

As noted before, we are able to determine steady states for the system (4). The two lines of steady states are (0; 0; S) and (P ; 0; 0).

In order to assess the stability of the framework we use, such as The earlier actions, we get (ρ, n, s) = (P,0,0) saddle point, (0,0,S) stable node of c > 0 , λ > 0 , c ⩾ P and c > S

2.4 Numerical Solution

Since a precise solution is not possible for the system (3), We are forced to use numerical fixes. To do this, we use the **pdepe** code in **MATLAB**. [1,3]

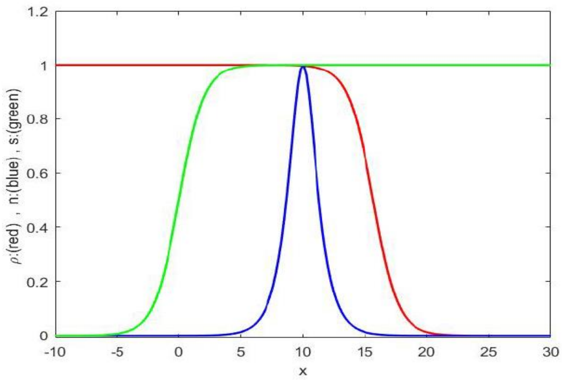


Figure (1): The first prerequisite for the system's solution (3) : (red) is branching, (blue) is tips and (green) is substrate.

As anticipated, autocatalytic coupling results in interaction between the ρ, n, and s wave fronts. These fronts hold for suitable parameter selections. The parameters α, υ, d, and k determine the solution to system (3). Figure 2 shows the solution type derived for α = 0.1, υ = 0.01, d = 4.5, and k = 0.4 illustrates the profiles of the substrate, tip, and branching. The wave that is traveling moves from left to right. [1,4,6]

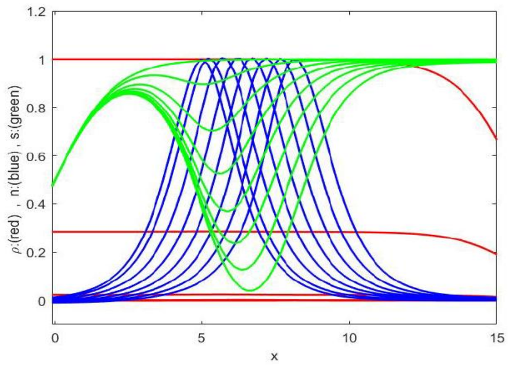


Figure (2): The solution PDEs system (3.33) for FXDS with the parameters (α = 0.1, υ = 0.01, d = 4.5, k = 0.4) . The time spacings: (40,… ,100.)

2.5 The Effect α of Solution

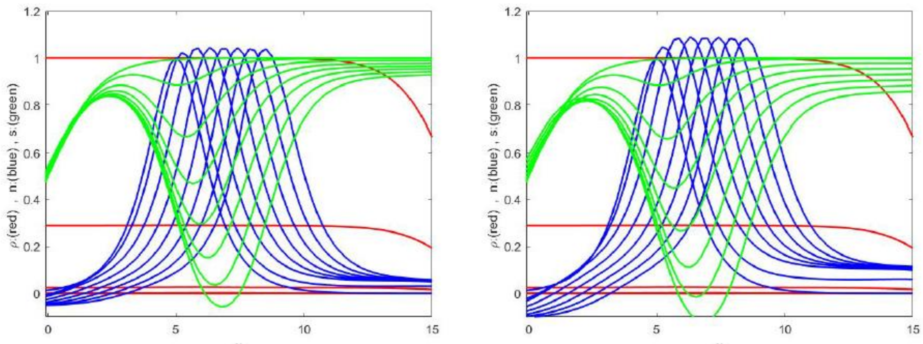


Figure (3,4): The solution PDEs system (3) for FXDS with the parameters α, υ, d, k. taking values of fig (3) α = 0.5, υ = 0.01, d=4.5, k = 0.4 and fig (4) α =1, υ= 0.01, d = 4.5, k = 0.4.

By utilizing υ = 0.01, d = 4.5, k = 0.4, we can obtain the relationship between traveling waves solution c and α values, as demonstrated in table (1). In this scenario, traveling waves solution c is growing as α increases [1, 4, 6]. Looking at Fig. (5)

Table (1): the values of α and c for solution of FXDS, when υ = 0.01, d =4.5, k = 0.4

α	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6
c	0.0667	0.0673	0.0679	0.0687	0.0696	0.0706	0.0717	0.0729	0.0742	0.0756	0.0773

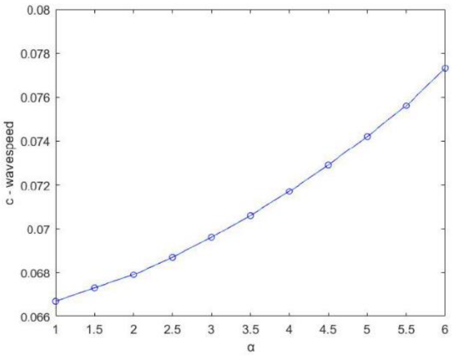


Figure (5): The relationship that exists between α values and wave speed (c)

2.6 The Effect υ of Solution

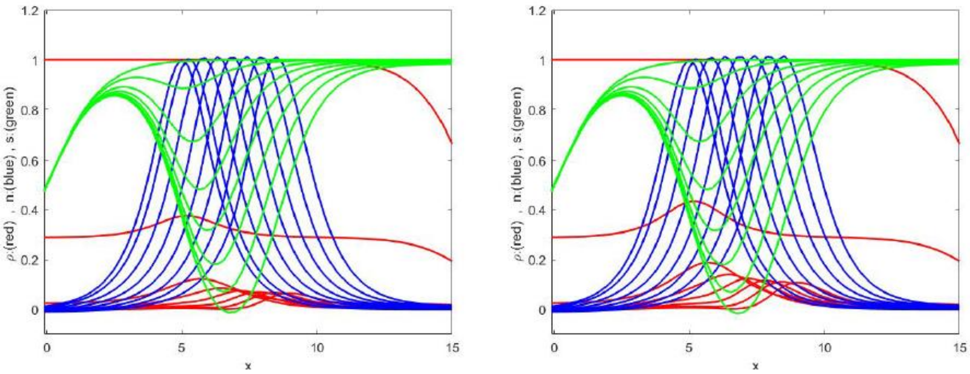


Figure (6,7): The solution PDEs system (3) for FXDS with the parameters α, υ, d, k. taking values of fig (6) α =0.1, υ = 0.6, d=4.5 , k = 0.4 and fig(7) α =0.1, υ = 1, d = 4.5, k = 0.4.

By utilizing α = 0.1, d = 4.5, k = 0.4 , we can obtain the relationship between traveling waves solution c and υ values, as demonstrated in table (2). In this scenario, traveling waves solution c is growing as υ increases [1, 4, 6]. Looking at Fig. (8)

Table (2): the values of υ and c for solution of FXDS, when α = 0.1, d = 4.5, k = 0.4

υ	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6
c	0.1554	0.1999	0.2438	0.2868	0.3286	0.3692	0.4083	0.4458	0.4815	0.5154	0.5474

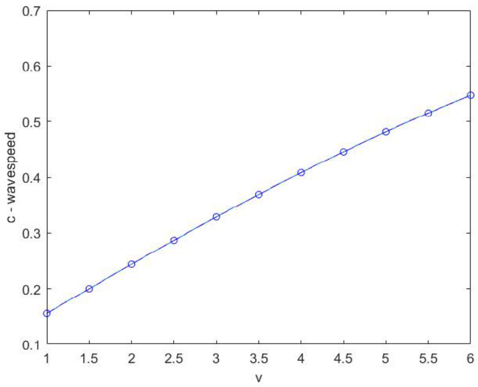


Figure (8): The relationship that exists between υ values and wave speed (c)

2.7 The Effect d of Solution

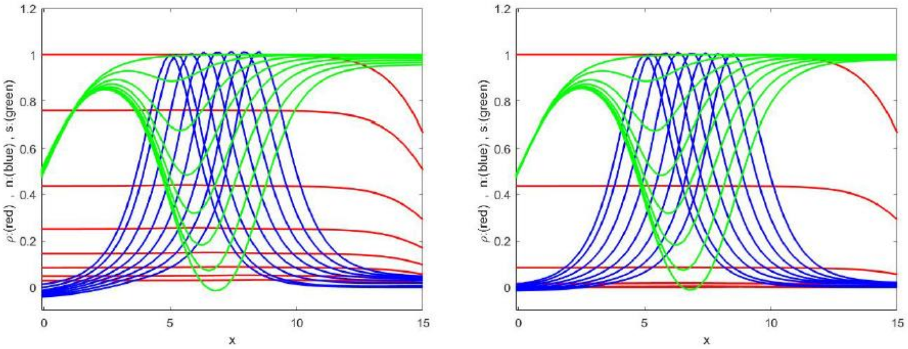


Figure (9,10): The solution PDEs system (3) for FXDS with the parameters α, υ, d, k. taking values of fig (9) α =0.1, υ = 0.01, d=1, k = 0.4 and fig (10) α =0.1, υ = 0.01, d = 3, k = 0.4.

By utilizing α = 0.1, υ = 0.01, k = 0.4, we can obtain the relationship between traveling waves solution c and d values, as demonstrated in table (3). In this scenario, traveling waves solution c is decreasing as d increases [1, 4, 6]. Looking at Fig. (11)

Table (3): the values of d and c for solution of FXDS, when α = 0.1, υ =0.01, k = 0.4

d	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6
c	2.8974	1.7553	1.0243	0.5924	0.3420	0.1976	0.1140	0.0659	0.0383	0.0223	0.0131

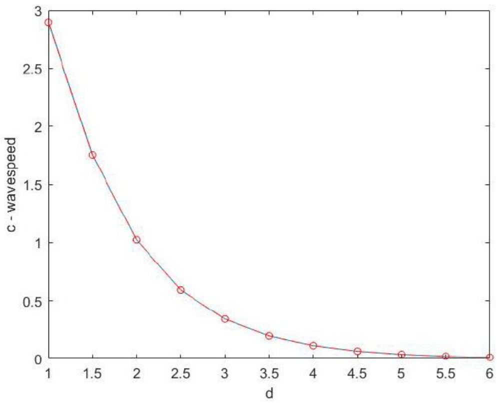


Figure (11): The relationship that exists between d values and wave speed (c)

2.8 The Effect k of Solution

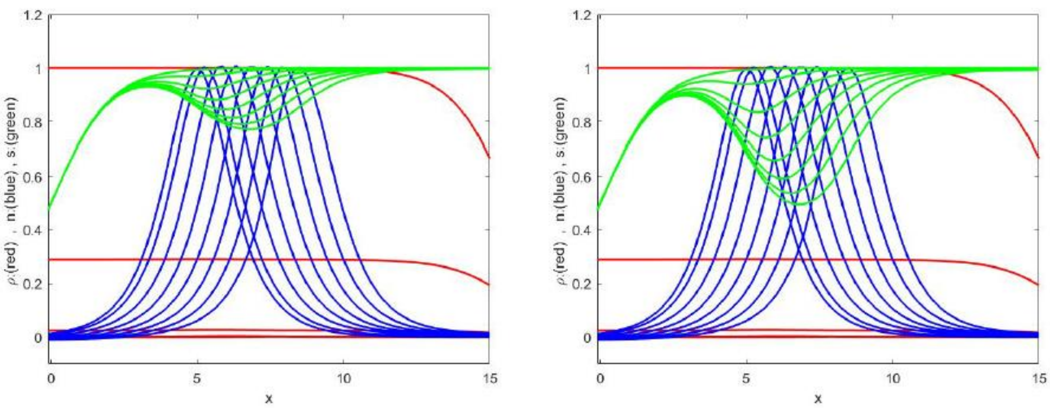


Figure (12,13): The solution PDEs system (3) for FXDS with the parameters α, υ, d, k. taking values of fig (12) α =0.1, υ = 0.01, d=4.5, k = 0.09 and fig (13) α =0.1, υ = 0.01, d = 4.5, k = 0.2.

By utilizing α = 0.1, υ = 0.01, d = 4.5, we can obtain the relationship between traveling waves solution c and k values, as demonstrated in table (4). In this scenario, traveling waves solution c is growing as k increases [1, 4, 6].Looking at Fig. (14)

Table (4): the values of k and c for solution of FXDS, when α = 0.1, υ =0.01, d = 4.5

k	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6
c	0.0673	0.0685	0.0696	0.0708	0.0719	0.0732	0.0745	0.0758	0.0775	0.0791	0.0809

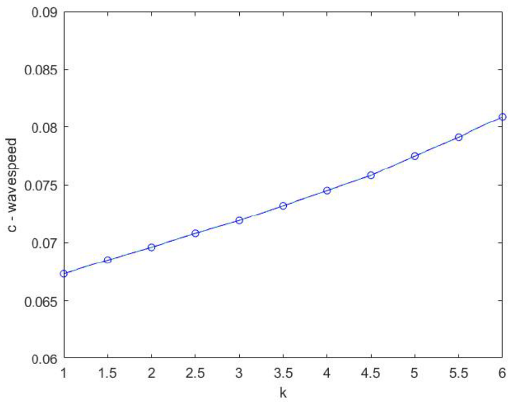


Figure (14): The relationship that exists between k values and wave speed (c)

Results Using Matlab.

3. CONCLUSION

We have clarified the effects of the substrate and parameters α, υ, d and k on the development of fungal networks it is clear that relationship between c and α, looking at Figure (5), which shows the wave speed c increases when α is an increasing function. We draw the relationship between c and υ, looking at Figure (8), and this is clear the speed of wave c increases when υ is an increasing function. Also drawing the relationship between c and d, looking at Figure (11), this is clear the speed of wave c decreases when d is an increasing function and drawing the relationship between c and k, looking at Figure (14) and this is clear the speed of wave c increases when k is an increasing function.
While maintaining constant values for υ and γ^2, the growth rate (α = α_2υ/γ^2) rises with α_2, and conversely, the growth rate increases with υ.
In contrast, as γ^2 increases, the growth rate falls while α_2 and υ stay unchanged. As we observe The colony growth rate remains unaffected by modifications to the anastomosis rate β_3, since α is not dependent on β_3. Density accumulation inside would be decreased by increasing β_3.

The main objective of our research is to ascertain the role of the substrate in the model.
In a biological sense, fungi should require energy to grow in order to branch or form tips.

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