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A New Hybrid Conjugate Gradient Method with Global Convergence Properties

Rahma F. Aziz[1]*, Maha S. Younis[2]

[1,2]Department of Mathematics, College of Education for Pure Sciences, Mosul University, IRAQ.

*Corresponding Author: Rahma F. Aziz

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ABSTRACT: This work introduces a novel hybrid conjugate gradient (CG) technique for tackling unconstrained optimization problems with improved efficiency and effectiveness. The parameter θ_k is computed as a convex combination of the standard conjugate gradient techniques using β_k^{LS} and $\beta_k^{Ha_2}$. Our proposed method has shown that when using the strong Wolfe-line-search (SWC) under specific conditions, it achieves global theoretical convergence. In addition, the new hybrid CG approach has the ability to generate a search direction that moves downward with each iteration. The quantitative findings obtained by applying the recommended technique about 30 functions with varying dimensions clearly illustrate its effectiveness and potential.

Keywords: Unconstraint Optimization, Hybrid Conjugate Gradient, Line Search, Global Convergence, Strong Wolfe Conditions.

1. INTRODUCTION

Assuming we have a general function $f: R^n \to R$ that is continuously differentiable. Also, examine the subsequent un-constrained optimization issue

$$min\ (or\ max)\ \{f(x)\} \in R^n \tag{1}$$

The space R^n represent an "n-dimensional Euclidean space".

To solve Equation (1), we begin by selecting an initial guess $x_0 \in R^n$. We then employ a non-linear conjugate gradient approach to produce a series $\{x_k\}$

$$x_{k+1} = x_k + \alpha_k d_k \qquad , k = 0, \quad 1, \quad 2, \ldots \tag{2}$$

The value of $\alpha_k > 0$ is determined by a process called "line search". The direction denoted d_k are formed using a specific design as

$$d(x) = \begin{cases} -g_k, & k = 0 \\ -g_k + \beta_k d_k, & k > 0 \end{cases} \tag{3}$$

Let g_k be the gradient of $f(x_k)$, and β_k be a scalar parameter that defines the properties of conjugate gradient techniques.

In the field of computing, the step-size α_k is considered to meet any of the line search conditions during the procedure. This work focuses on the robust Wolfe line search Equation (4) [1].

*Corresponding author: rahma.21esp32@student.uomosul.edu.iq

$$f(x_k + \alpha_k d_k) \le f(x) + \delta \alpha_k g_k^T d_k \ , \qquad 0 \le \delta \le \frac{1}{2} \tag{4}$$
$$|d_k^T g(x_k + \alpha_k d_k)| \le -\sigma g_k^T d_k, \qquad 0 \le \delta \le 1$$

The search direction, denoted as d_k is explicitly defined in Equation (3).

Researchers have devoted significant attention to CG approaches for a long time. The result of such investigations is the development of numerous formulas with variations in the CG coefficient (β_k) for addressing unconstrained optimisation issues[2].

Here are several typical formulas for β_k:

$$\beta_k^{ER} = \frac{g_{k+1}^T g_{k+1}}{g_k^T g_k}, \qquad \text{FR(Fletcher – Reeves) [3]}$$

$$\beta_k^{PR} = \frac{g_{k+1}^T y_k}{g_k^T g_k}, \qquad \text{PR (Polak – Ribiere) [4]}$$

$$\beta_k^{DY} = \frac{g_k^T g_k}{d_{k-1}^T y_{k-1}}, \qquad \text{DY (Dai – Yuan) [5]}$$

$$\beta_k^{CD} = \frac{-g_k^T g_k}{d_{k-1}^T g_{k-1}}, \qquad \text{CD (conjugate descent) [6]}$$

$$\beta_k^{LS} = \frac{-g_k^T y_{k-1}}{d_{k-1}^T g_{k-1}} \qquad \text{LS (Liu – Storey) [7]}$$

$$\beta_k^{HS} = \frac{g_k^T y_{k-1}}{d_{k-1}^T y_{k-1}} \qquad \text{HS (Hestenes- Stiefel) [8]}$$

Were

$$y_{k-1} = g_k - g_{k-1}, \tag{5}$$

Despite their excellent global theoretic convergence, the computational performance of the CG approaches β_k^{LS} *and* β_k^{CD} is lower. however, superior computing performance is generally achieved by the $\beta_k^{PR}, \beta_k^{LS}$ and $\beta_k^{Ha_2}$, despite the fact that they haven't demonstrated convergence[9]. "In most cases, hybrid conjugate gradient methods are more efficient than basic conjugate gradient methods. The hybrid conjugates gradient techniques discussed in this study are of particular importance. These algorithms are a mixture of a number of different conjugate gradient techniques."[10]

The main principle behind their strategy is to capitalize on projected outcomes. They are frequently promoted as a means to prevent congestion. We have presented a novel hybrid CG approach that relies on the Ha_2 and LS methods. The method incorporates the parameters $\beta_k^{Ha_2}$ and β_k^{LS}.

$$\beta_k^{LS} = \frac{g_{k+1}^T y_k}{-d_k^T g_k} \quad , \quad \beta_k^{Ha_2} = \frac{\|g_{k+1}\|^2}{(f_{k+1} - f_k/\alpha k - 3/2) d_k^T g_k} \quad [11].$$

To solve the unconstrained optimization problems with $\beta_k^{MR_2}$

The parameter $\beta_k^{MR_2}$ in our proposed method is computed as a convex combination of $\beta_k^{Ha_2}$ *and* β_k^{LS} such that

$$\beta_k^{MR_2} = (1 - \theta_k)\beta_k^{LS} + \theta_k \beta_k^{Ha_2} \tag{6}$$

We have $0 \leq \theta_k \leq 1$

The rest of the document is structured in the following manner. In section 2, we outline our suggested approach for acquiring the parameter θ_k using several methodologies. We further analyse the adequate descent property of our approach under certain suitable conditions, and moreover establish the parameter constraint in the form of $0 \leq \theta_k \leq 1$. Section 3 encompasses several assumptions, whereas section 4 defines the global convergence of the proposed approach. Section 5 concludes by presenting the results of numerical experiments that were conducted.

2. GRADIENT METHOD CONJUGATES WITH THE NEW HYBRID

2.1 The New θ_k Parameter Derivation

The recurrence is utilized to determine the iterates $x_0, x_1, x_2, \dots\dots$ of our method (2.2). The strong Wolfe requirements (4) determine the step size $\alpha_k > 0$, whereas the rule generates the directions.

$$\begin{Bmatrix} d_0 = -g_0 \\ d_{k+1} = -g_{k+1} + \beta_k^{MR_2} d_k \end{Bmatrix} \tag{7}$$

and the parameter $\beta_k^{MR_2}$ in the form Eq. (6), where $0 \leq \theta_k \leq 1$ and we derived hybrid parameter by, derivation of the new parameter

$$B_k^{MR_2} = (1-\theta_k)B_k^{LS} + \theta_k B_k^{Ha_2} = (1-\theta_k)\frac{g_{k+1}^T y_k}{-g_k^T d_k} + \theta_k \frac{\|g_{k+1}\|^2}{(f_{k+1} - f_k)/\alpha_k + 3/2\, d_k^T y_k}$$

$$d_{k+1} = -g_{k+1} B_k^{MR_2} d_k$$

$$= -g_{k+1} + (1-\theta_k)\frac{g_{k+1}^T\ y_k}{-g_k^T\ d_k}\ d_k + \theta_k \frac{\|g_{k+1}\|^2}{(f_{k+1} - f_k)/\alpha_k\ +\ 3/2\ d_k^T\ y_k}\ d_k$$

Multiply both sides of the equation y_k^T

$$y^T d_{k+1} = -y^T g_{k+1} + (1-\theta_k)\frac{g_{k+1}^T\ y_k}{-g_k^T\ d_k}\ y^T d_k + \theta_k \frac{\|g_{k+1}\|^2}{\delta\ g_k^T d_k\ +\ 3/2\, d_k^T\ y_k}\ y^T d_k$$

$$0 = -y_k^T\ g_{k+1} - \frac{g_{k+1}^T\ y_k}{g_k^T\ d_k}\ y_k^T d_k + \theta_k \frac{g_{k+1}^T\ y_k}{g_k^T\ d_k}\ y_k^T d_k + \theta_k \frac{\|g_{k+1}\|^2}{\delta\ g_k^T d_k\ +\ 3/2\, d_k^T\ y_k}\ y_k^T d_k$$

$$y_k^T\ g_{k+1} + \frac{g_{k+1}^T\ y_k}{g_k^T\ d_k}\ y_k^T d_k = \theta_k \left[\frac{g_{k+1}^T\ y_k}{g_k^T\ d_k} y_k^T d_k + \frac{\|g_{k+1}\|^2}{\delta\ g_k^T d_k\ +\ 3/2\, d_k^T\ y_k}\ y_k^T d_k\right]$$

$$\therefore \theta_k = \frac{y_k^T\ g_{k+1} + \dfrac{g_{k+1}^T\ y_k}{g_k^T\ d_k}\ y_k^T d_k}{\dfrac{g_{k+1}^T\ y_k}{g_k^T\ d_k} y_k^T d_k + \dfrac{\|g_{k+1}\|^2}{\delta\ g_k^T d_k\ +\ 3/2\, d_k^T\ y_k}\ y_k^T d_k} \tag{8}$$

In order to Prove that $0 \leq \theta_k \leq 1$ we will use the following mathematical lemma.

Lemma:

The hybridization parameter in the convex structure is limited to 0 and 1. And so is our hybridization scalar Eq (8).

Proof: -

1-The first possibility: - we take the case of a fraction that is less than zero. we get a contradiction because a positive value + a positive value is impossible for a negative result to appear.

$$\frac{Numerator + denominator}{Numerator + the\ \ vule}$$

$$\frac{\frac{y_k\ g_{k+1}}{y_k^T\ d_k} + \frac{g_{k+1}^T\ y_k}{d_k^T\ g_k}}{[\frac{g_{k+1}^T\ y_k}{d_k^T g_k} + \frac{\|g_{k+1}\|^2}{(S\ g_k^T\ d_k + 3/2\ d_k^T\ y_k)}]} \quad < \frac{0}{1}$$

The product of two sides multiplied by two means.

$$\frac{y_k\ g_{k+1}}{y_k^T\ d_k} + \frac{g_{k+1}^T\ y_k}{d_k^T\ g_k}$$

A contradiction because a positive value + a positive value

2-The second possibility: we take, the fraction greater than zero and obtain that the denominator is greater than zero

$$0 < \frac{\frac{y_k\ g_{k+1}}{y_k^T\ d_k} + \frac{g_{k+1}^T\ y_k}{d_k^T\ g_k}}{\left[\frac{g_{k+1}^T\ y_k}{d_k^T g_k} + \frac{\|g_{k+1}\|^2}{(S\ g_k^T\ d_k + 3/2\ d_k^T\ y_k)}\right]}$$

3-The third possibility:- we take the fraction less than one and obtain that the denominator is the greater than the numerator

$$\frac{\frac{y_k\ g_{k+1}}{y_k^T\ d_k} + \frac{g_{k+1}^T\ y_k}{d_k^T\ g_k}}{[\frac{g_{k+1}^T\ y_k}{d_k^T g_k} + \frac{\|g_{k+1}\|^2}{(S\ g_k^T\ d_k + 3/2\ d_k^T\ y_k)}]} < 1$$

$$\frac{\cancel{the\ vlue} + denominator}{Numerator + \cancel{the\ vlue}} < 1$$

$$the\ vlue + denominator\ \ < the\ vlue\ \ + Numerator$$

$$denominator\ < Numerator$$

$$\frac{y_k\ g_{k+1}}{y_k^T\ d_k}+\frac{g_{k+1}^T\ \cancel{y_k}}{d_k^T \cancel{g_k}}<\frac{g_{k+1}^T\ \cancel{y_k}}{\cancel{d_k^T}\ g_k}+\frac{\|g_{k+1}\|^2}{(S\ g_k^T\ d_k+3/2\ d_k^T\ y_k)}$$

$$\frac{y_k\ g_{k+1}}{y_k^T\ d_k}<\frac{\|g_{k+1}\|^2}{(S\ g_k^T\ d_k+3/2\ d_k^T\ y_k)}$$

4- The fourth possibility:- we take the fraction greater than one and obtain a contradiction due to the presence of the squared swelling. By simplifying , this result appears (a contradiction)

$$1<\frac{\frac{y_k\ g_{k+1}}{y_k^T\ d_k}+\frac{g_{k+1}^T\ y_k}{d_k^T\ g_k}}{[\frac{g_{k+1}^T\ y_k}{d_k^T g_k}+\frac{\|g_{k+1}\|^2}{(S\ g_k^T\ d_k+3/2\ d_k^T\ y_k)}]}$$

(contradiction) Due to the presence of $\|g_{k+1}\|^2$ square swelling, by simplifying we will obtain a (contradiction)

3. THE DESCENDING CHARACTERIZED

Assume that the function fulfills the hypotheses, β^{MR_2} and when α_k it satisfies the strong wolf line, then $\beta_k^{MR_2}$ it satisfies the formula, then the following is holding.

$$g_{k+1}^T d_{k+1}<0\ ,\quad \forall\ k$$

Proof: using the principle of mathematical induction

$$g_{k+1}^T\ d_{k+1}=-g_{k+1}^T\ g_{k+1}+(1-\theta_k)\frac{g_{k+1}^T\ g_{k+1}}{g_k^T\ d_k}g_{k+1}^T\ d_k+\theta_k\frac{\|g_k+1\|^2}{(\ f_{k+1}-f_k)\ /\alpha_k\ +\ 3/2\ d_k^T\ y_k}\ g_{k+1}^T\ d_k$$

$$=-g_{k+1}^T\ g_{k+1}+(1-\theta_k)\frac{g_{k+1}^T(\ g_{k+1}-g_k)}{-g_k^T\ d_k}g_{k+1}^T\ d_k+\theta_k\frac{\|g_k+1\|^2}{\delta\ g_k^T d_k+3/2 d_k^T\ y_k}\ g_{k+1}^T\ d_k$$

$$=-g_{k+1}^T\ g_{k+1}+(1-\theta_k)[\frac{g_{k+1}^T\ g_{k+1}-g_{k+1}\ g_k}{-\cancel{g_k^T d_k}}\ (\cancel{-}\sigma g_k^T \cancel{d_k})+\theta_k\frac{\|g_k+1\|^2}{\delta\ g_k^T d_k+3/2 d_k^T\ (g_{k+1}-g_k)}(-\sigma g_k^T\ d_k)$$

$$\leq-\|g_k+1\|^2+(1-\theta_k)[\|g_k+1\|^2-\|g_k+1\|^2]\sigma+\theta_k\frac{\|g_k+1\|^2}{\delta\ g_k^T d_k\ +\ 3/2\ d_k^T\ g_{k+1}-3/2\ d_k^T\ g_k}(-\sigma g_k^T\ d_k)$$

$$\leq-\|g_k+1\|^2+\theta_k\frac{\|g_k+1\|^2}{\delta\ g_k^T d_k\ +\ 3/2\,\sigma\ g_k^T d_k\ \ -3/2\ g_k^T\ d_k}(-\sigma g_k^T\ d_k)$$

$$=-\|g_k+1\|^2+\theta_k\frac{\|g_k+1\|^2}{(\delta\ +\ 3/2\ \sigma-3/2)\ \cancel{g_k^T}\ d_k}(-\sigma g_k^T \cancel{d_k})$$

$$=-\left(1+\frac{\theta_k\ \sigma}{\delta\ +\ 3/2\ \sigma-3/2}\right)\|g_k+1\|^2$$

$$=-C_1\ \|g_k+1\|^2$$

4. GLOBAL CONVERGENCE

f A uniformly convex function

$$\lim_{k\to\infty} inf \|g_k\| = 0$$

$\because 0 \le \theta \le 1$

it is obtained from the strong wolff line $\|\delta_k\|$ approaching zero, as there is an on-negative number $x_k > 0$ so that

$$\|g_k\|^2 \ge \eta_2 \|\delta_k\|$$

$$\|g_{k+1}\|^2 \ge \eta_1 \|\delta_k\|^2$$

Since $d_k$the convergence vector and obtain $\alpha k > 0$ is the step length, we through the strong wolff line if

$$\sum_{k\ge 1} \frac{1}{\|d_{k+1}\|^2} < \infty$$

$$\therefore \lim_{k\to\infty} inf \ \|g_k\| = 0$$

Proof:

$$B_k^{MR_2} = (1-\theta_k)B_k^{LS} + \theta_k B_k^{Ha_2}$$

$$= (1-\theta_k)\frac{g_{k+1}^T y_k}{-g_k^T d_k} + \theta_k \frac{\|g_{k+1}\|^2}{(f_{k+1}-f_k)/\alpha_k + 3/2 d_k^T y_k}$$

$$= (1-\theta_k)\frac{g_{k+1}^T (g_{k+1}-g_k)}{-d_k^T \ g_k} + \theta_k \frac{\|g_{k+1}\|^2}{\delta \ g_k^T d_k + 3/2 d_k^T (g_{k+1}-g_k)}$$

$$= (1-\theta_k)\frac{g_{k+1}^T \ g_{k+1} - g_{k+1}^T \ g_k}{-d_k^T \ g_k} + \theta_k \frac{\|g_{k+1}\|^2}{\delta \ g_k^T d_k + 3/2 d_k^T g_{k+1} - 3/2 \ d_k^T g_k}$$

$\because 0 < \theta < 1$ we get

$$\le \frac{\|g_{k+1}\|^2 - \cancel{|g_{k+1}^T \ g_k|}}{d_k^T \ g_k} + \frac{\|g_{k+1}\|^2}{\delta \ g_k^T d_k + 3/2 \ \sigma d_k^T g_k - 3/2 \ d_k^T \ g_k}$$

From the hypothesis $\|g_{k+1}\|^2 > |g_{k+1}^T \ g_k|$ we get

$$= \frac{\|g_{k+1}\|^2 - \|g_{k+1}\|^2}{-d_k^T \ g_k} + \frac{\|g_{k+1}\|^2}{(\delta + 3/2 \ \sigma - 3/2) \ d_k^T \ g_k}$$

$$= \frac{\|g_{k+1}\|^2}{\sigma_1} \text{ where } \sigma_1 = \delta + 3/2 \ \sigma - 3/2 \qquad \ldots(*)$$

$$d_{k+1} = -g_{k+1} + B_k^{MR_2}\, d_k$$

$$\|d_{k+1}\| = \left\|-g_{k+1} + B_k^{MR_2}\, d_k\right\|$$

$$\|d_{k+1}\|^2 = \left\|-g_{k+1} + B_k^{MR_2}\, d_k\right\|^2 \leq\ [\, \|g_{k+1}\| +\ \left\|B_k^{MR_2}\, d_k\right\|]^2$$

$$= \|g_{k+1}\|^2 + B_k^{MR_2}\, \|g_{k+1}\|\, \|d_k\| +\ (B_k^{MR_2})^2\, \|d_k\|^2$$

From the relationship (*) we get

$$= \|g_{k+1}\|^2 + 2\, \frac{\|g_{k+1}\|^2}{\sigma_1\, g_k^T\, d_k}\ \|g_{k+1}\|\, \|d_k\| + (\frac{\|g_{k+1}\|^2}{\sigma_1\, g_k^T\, d_k})^2 \|d_k\|^2$$

$$\leq\ \eta_2\, \|S_k\| + \frac{2\,\eta_2\, \|S_k\|}{\sigma_1\, g_k^T\, d_k}\ |g_{k+1}^T\ \ d_k| + \left[\frac{\eta_2\, \|S_k\|}{\sigma_1\, g_k^T\, d_k}\right]^2\ \|d_k\|^2$$

$$\leq\ \eta_2\, \|S_k\| + \frac{2\,\eta_2\, \|S_k\|}{\sigma_1\, g_k^T\, d_k}\ (-\sigma\, g_k^T\, d_k) + \left[\frac{\eta_2\, \|S_k\|}{\sigma_1\, g_k^T\, d_k}\right]^2\ \|d_k\|^2$$

$$\leq\ \eta_2\, \|S_k\| - 2\,\frac{\sigma}{-\sigma_1}\eta_2\, \|S_k\|\ + \left[\frac{\eta_2\, \|S_k\|^2}{-c\, \sigma_1\, \|d_k\|^2}\right]^2\ \|d_k\|^2$$

$$\leq\ \eta_2\, \|S_k\| - \frac{2\,\sigma}{-\sigma_1}\eta_2\, \|S_k\|\ + \left[\frac{\eta_2\, \|S_k\|^2}{-c\, \sigma_1\, \eta_1\, \|S_k\|^2}\right]^2\ \frac{\|S_k\|^2}{\alpha_k^2}$$

Let

$$L = max\, \{x_{k+1}\ - x_k\}$$

$$\left.\begin{array}{c} x_{k+1} = x_k + \alpha_k d_k \\ x_{k+1} =\ -x_k = \alpha_k d_k \\ S_k = \alpha_k d_k \\ \therefore\, d_k = \frac{S_k}{\alpha_k} \end{array}\right\} ** \qquad \|S_k\| \leq D$$

Since the function is restricted, then

$$\therefore\, \|g_{k+1}\|^2 \leq\ \eta_2\, D - \frac{2\,\sigma}{\sigma_1}\, \eta_2\, D + \frac{\eta_2^2}{c^2\sigma_1^2\, \eta_1^2}\, \frac{D^2}{\alpha_k^2}$$

$$\therefore\, \|d_{k+1}\|^2 \leq \phi$$

$$\frac{1}{\|d_{k+1}\|^2} \geq \frac{1}{\phi}$$

$$\sum_{k\leq 1} \frac{1}{\|d_{k+1}\|^2}\ \geq \sum_{k\geq 1} \frac{1}{\phi}\ \ \geq \frac{1}{\phi}\ \sum 1 = \frac{1}{\phi} * \infty = \infty$$

$$\lim_{k\to\infty} inf \|g_k\| = 0$$

5. NUMERICAL TESTS

The following part will analyse the results of our numerical experiments that employed the hybrid MR method. In addition, we will compare these results with the numerical outputs of two other algorithms (Hanen 2, LS) that utilise the Wolfe line search method. The comparison will be conducted using the metrics of iteration count (NoI) and function evaluation count (NoF). The iterations will stop when the magnitude of the gradient norm is less than or equal to 10−6.

Furthermore, we utilized 31 unconstrained optimization problem functions with a variable count of either 100 or 1000. All the graphs in this work were generated using Fortran software. The discussion the results by Dolan and More miner to compare based on MATLAB program.

Table 1. -The details of testing results using 31 functions with different dimension.

Function	Dim	Ha2		LS		MR2	
		Ni	NF	Ni	NF	Ni	NF
Extended Trigonometric	100	18	32	32	73	18	33
	1000	41	66	90	611	30	54
Extended Rosenbrock SROSENBR	100	1001	1056	1001	1888	35	74
	1000	1001	1031	1001	2093	35	79
Extended White & Holst	100	1001	1018	1001	1712	31	67
	1000	1001	1024	1001	1586	29	57
Extended Beale BEALE (CUTE)	100	889	928	35	352	15	30
	1000	985	1027	160	4269	15	29
Extended Penalty	100	20	40	229	1134	8	15
	1000	21	43	1001	3975	10	20
Perturbed Quadratic	100	585	625	206	581	117	227
	1000	1001	1024	713	2006	429	751
Raydan 1	100	439	472	4	9	72	117
	1000	1001	1024	4	9	359	556
Generalized Tridiagonal 1	100	42	67	63	875	21	39
	1000	42	71	135	3530	25	46
Extended Tridiagonal 1	100	1001	1022	85	386	13	26
	1000	1001	1021	149	957	13	26
Extended Three Expo Terms	100	11	18	4	9	11	19
	1000	12	21	4	9	12	19
Generalized Tridiagonal 2	100	86	122	36	235	38	63
	1000	202	244	47	193	58	91
Diagonal 5	100	25	51	29	427	15	27
	1000	27	55	764	63	18	32
Generalized PSC1	100	16	33	57	93	8	17
	1000	8	17	65	105	6	13
Extended PSC1	100	1001	1021	1001	1615	82	159
	1000	1001	1021	1001	1313	112	211
Extended Block-Diagonal BD1	100	1001	1032	64	1514	68	160
	1000	1001	1244	18	48	63	147
Extended Maratos	100	1001	2000	212	515	9	22
	1000	1001	1996	100	2158	12	27
Extended Cliff CLIFF (CUTE)	100	15	34	2	5	9	22
	1000	15	34	2	5	8	21
Quadratic Diagonal Perturbed	100	1001	1683	187	2489	23	54
	1000	1001	1056	113	1596	13	27
Quadratic QF1	100	101	123	99	419	28	46
	1000	95	115	413	1978	32	50

Extended Quadratic Penalty QP1	100	1001	1025	1001	1935	113	213
	1000	1001	1022	1001	1621	1001	27767
Extended EP1	100	1001	1999	7	14	14	27
	1000	1001	1999	14	38	14	29
Extended Tridiagonal 2	100	1001	1014	10	18	1001	1862
	1000	1001	1018	11	19	1001	1872
BDQRTIC (CUTE)	100	1001	2001	13	24	99	203
	1000	1001	2001	14	25	205	542
TRIDIA (CUTE)	100	7	14	594	1664	7	14
	1000	7	14	1001	2497	6	12
ARWHEAD (CUTE)	100	10	18	75	282	10	18
	1000	10	17	90	351	10	17
NONDQUAR (CUTE)	100	418	449	70	745	57	88
	1000	1001	1023	66	513	221	342
DQDRTIC (CUTE)	100	68	103	11	29	30	51
	1000	74	114	16	43	37	64
DIXMAANA (CUTE)	100	55	88	216	638	19	36
	1000	58	93	1001	1680	18	41
Partial Perturbed Quadratic	100	4	9	19	86	4	9
	1000	4	9	18	79	4	9
Tridiagonal Perturbed Quadratic	100	10	21	78	318	7	15
	1000	10	21	101	587	7	15
LIARWHD (CUTE)	100	3	9	29	427	3	9
	1000	4	11	35	764	4	11
Total		29462	37603	16619	55256	5792	36739

We conclude the practical aspect by reviewing the graphics related to the policy of comparisons by researchers Dolan and More. The first drawing showed the differences between the new algorithm and the classic algorithms regarding the number of iterations. The second drawing was for standard function calculations.

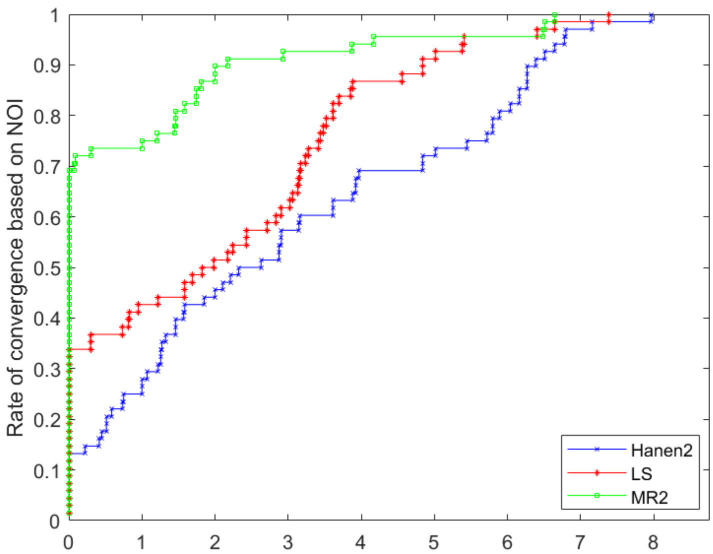

FIGURE 1. - NoI comparsion.

We have observed that our strategy has transitioned to an ascending trajectory in contrast to the previous method.

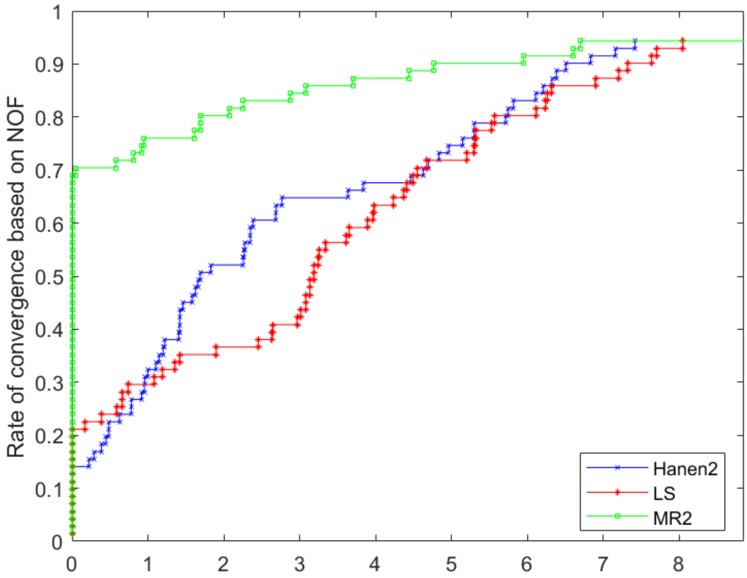

FIGURE 2. - NoF comparison.

6. CONCLUSION

In this study, a novel strategy called $\beta_k^{MR_2}$ was created for unconstrained optimization. It combines the algorithms β_k^{LS} and $\beta_k^{Ha_2}$ to create a hybrid conjugate gradient method. The suggested method has undergone thorough theoretical and practical analysis. The algorithm's features of adequate descent and global convergence have been confirmed by applying certain hypotheses.

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