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Mathematical Model of FWE Branching Type with Effect on Energy

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ABSTRACT: Mathematical modeling is one of the most important topics in teaching mathematics and converting real world problems into mathematical formulas in order to solve them. The mathematical model shows the behavior of the lateral branching, tip-tip anastomosis with the effect of energy on it, although there is an error rate, but the mathematical model reduces the ratio amount of work, time and money. To obtain the best result required through this work, we will explain that the branching mathematical model depends on the solution of the system of partial equations (PDEs) through which we get results that represent the success or failure of some types of fungi in terms of growth. We will use mathematical techniques in order to reach the solution using non-dimensionality, stability, traveling wave, and numerical solution and use computer programs to facilitate the solution, including the Maple program, the MATLAB program, use Pdepe code and the pplane7 code.

Keywords: Lateral branching, Tip-tip anastomose, effect on the Energy

1. INTRODUCTION

In this model, new types of fungi have been developed, which are one of the biological models and have complex analytical properties, from people who have transformed biological phenomena into a mathematical form Leah Kishit, The mathematical model has been illustrated by several previous researchers:

In (2011) shuaa [1], it was studied developed a model for the growth of fungi that is used to create a source in one root model to calculate the absorption of nutrients through fungi, so focus on loss or death. In (2014) Muzaffar [2], has developed a type of modeling with the substrate and its ability to reproduce a biological system in particular and predict measurement. In (2022) Zainab [3], has have developed some types of models for the growth of fungi and how to consume energy. We have some symbols that describe the biological type of fungi and show how they work and analyze. [4]

Table 1: Clarifying the biological type for each branch and issuing each case

Branching	Biological type	Symbol	Version	Parameters description
	Lateral branching	F	$\delta = \alpha_2 p$	The value of α_2 represents the rate of branch formation, measured as the number of branches produced per unit length of hypha over a given time period.
	Tip-tip anastomosis	W	$\delta = -\beta_1 n^2$	β_1 is the tip reconstitution rate per unit of time.

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2. MATHEMATICAL MODEL

In this model, we will consider a new type of fungal growth branching. The type we will consider is lateral branching, Tip-tip anastomosis with effect on energy, where p represents junction density and n represents tip density. This growth model can be described by the following system:[1][4][5]

$$\frac{\partial p}{\partial t} = nv - d(p)$$

$$\frac{\partial n}{\partial t} = -\frac{\partial (nv)}{\partial (x)} + \sigma(p, n) \tag{1}$$

When we take:$\sigma(p, n) = \alpha_2 p - \beta_1 n^2$
Then this system(1) and $0 \leq \Psi \leq 1$ becomes:

$$\frac{\partial p}{\partial t} = nv$$

$$\frac{\partial n}{\partial t} = -\frac{\partial (nv)}{\partial (x)} + \alpha_2 p - \beta_1 n^2 + \Psi \tag{2}$$

when take : : $\alpha = \frac{\alpha_2 v}{\gamma^2}$

3- NON-DIMENSIONLISION

In this system, we will use the non-iterative form of equations to assist in numerical solution as explained to us by Leah Keshed (1982) Dr.Ali Hussein Shuaa(2011) that these parameters are fewer dimensions as in the following system:[1][6]

$$\frac{\partial p}{\partial t} = n$$

$$\frac{\partial n}{\partial t} = -\frac{\partial n}{\partial x} + \alpha(p - n^2) + \Psi \tag{3}$$

Where:$\alpha = \frac{\alpha_2 v}{\gamma^2}$

4- THE STABILITY OF SOLUTION, WHEN $\Psi = 0.5$

We find stability when we discuss the next system(3): [7]

$$\frac{\partial p}{\partial t} = n = 0 \Longrightarrow n = 0 \tag{4}$$

$$\frac{\partial n}{\partial t} = \alpha(p - n^2) + 0.5 \Longrightarrow \alpha(p - n^2) + 0.5 = 0 \tag{5}$$

$\alpha p = -0.5 \Longrightarrow (p_1, n_1) = (\frac{-0.5}{\alpha}, 0)$

$$(\alpha p - \alpha n^2) = -0.5, n = 1 \Longrightarrow (p_2, n_2) = (\frac{-0.5 + \alpha}{\alpha}, 1)$$

Therefor we take the Jacobin of these equation(4),(5): [8]
Now, the Jacobian for the system :

$$J_{(p,n)} = \begin{bmatrix} 0 & 1 \\ \alpha & -2\alpha n \end{bmatrix}$$

Jacobian at $(\frac{-0.5}{\alpha}, 0)$

$$J_{(\frac{-0.5}{\alpha}, 0)} = \begin{bmatrix} 0 & 1 \\ \alpha & 0 \end{bmatrix}$$

Jacobian at $(\frac{-0.5+\alpha}{\alpha}, 1)$

$$J_{(\frac{-0.5+\alpha}{\alpha}, 1)} = \begin{bmatrix} 0 & 1 \\ \alpha & -2\alpha \end{bmatrix}$$

Now,determent the eigenvalues as $\lambda_i; i = 1,2$

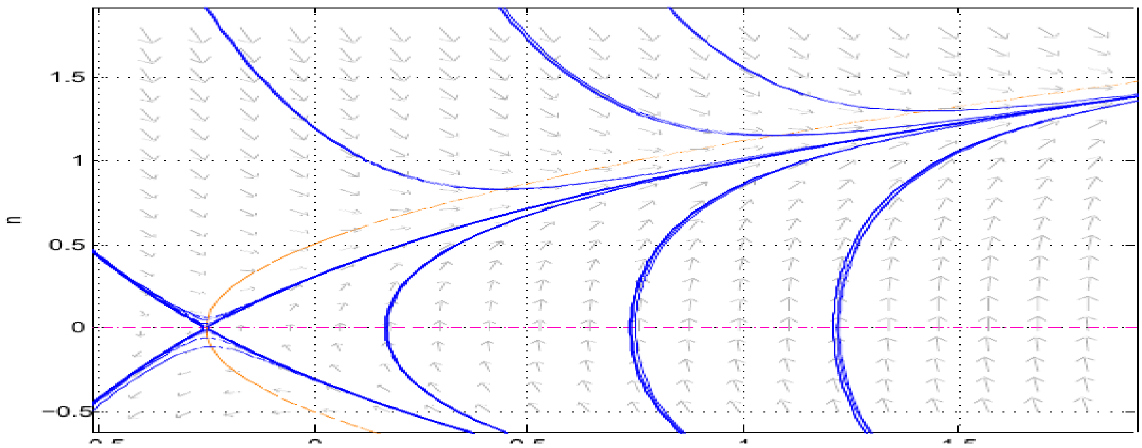


Figure 1: If $\alpha = 2$ we note that a trajectory connects the saddle point in $p_1(\frac{-0.5}{\alpha}, 0)$ to the stable node in $p_2(\frac{-0.5+\alpha}{\alpha}, 1)$

5- TRAVELING WAVE SOLUTION

In this section, we will talk about the traveling wave solution. This section shows that the traveling speed is in the same direction as F(z) when we let f(x,t) be a function that depicts a moving wave to the right at a constant rate. (F(z)=f(x,t) provided z = x-ct): [9,10,11]

$$n(x,t) = N(z),\ p(x,t) = P(z)$$

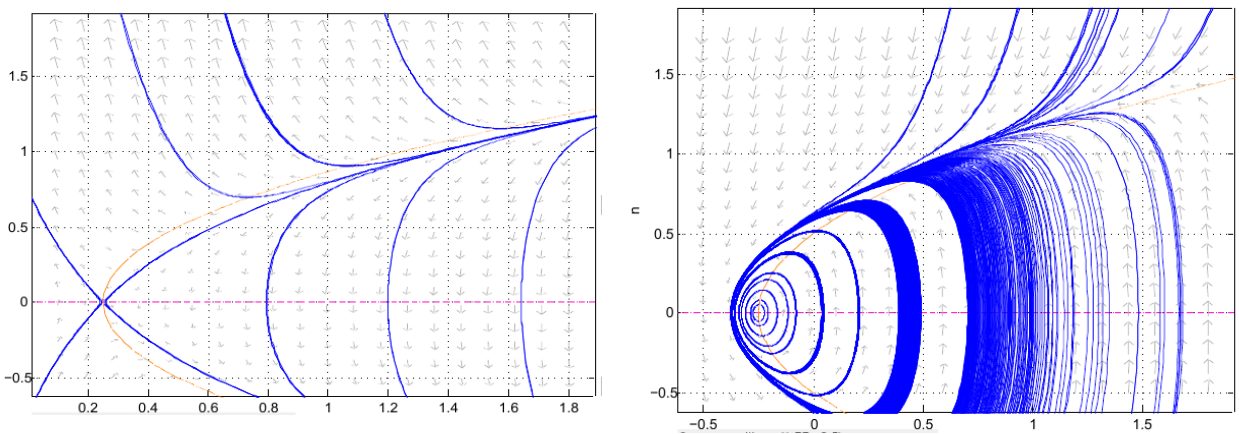
$$\frac{\partial p}{\partial t} = -c\frac{dP}{dz},\ \frac{\partial n}{\partial t} = -c\frac{dN}{dz}$$

$$\frac{\partial n}{\partial x} = \frac{dN}{dz}$$

We find the solution of stability when discussing the following equation:

$$\frac{dP}{dz} = \frac{-1}{c}[N] \Longrightarrow \frac{-1}{c}[N] = 0, c \neq 1, -\infty < z < \infty$$

$$\frac{dN}{dz} = \frac{1}{1-c}\alpha(P - N^2) \Longrightarrow \frac{1}{1-c}[\alpha(P - N^2) + 0.5] = 0 \quad (6)$$



(a) (b)

Figure 4: (In figure 4a), when if $\alpha = 2$ and $c = 2$ we note a trajectory connects the (saddle point) in $p_1(\frac{0.5c-0.5}{\alpha}, 0)$ to the (saddle point) in $p_2(\frac{0.5c-0.5+\alpha}{\alpha}, 1)$. (In figure 4b), when if $\alpha = -2$ and $c = 2$ we see a trajectory connects the (center) in $p_1(\frac{0.5c-0.5}{\alpha}, 0)$ to the(unstable node) in $p_2(\frac{0.5c-0.5+\alpha}{\alpha}, 1)$

6- NUMERICAL SOLUTION

We will discuss in this section numerical solution because this system can not be solved,and we need MATLAB in this case .pdepe code will used to solve the system(3):[9,10,11]

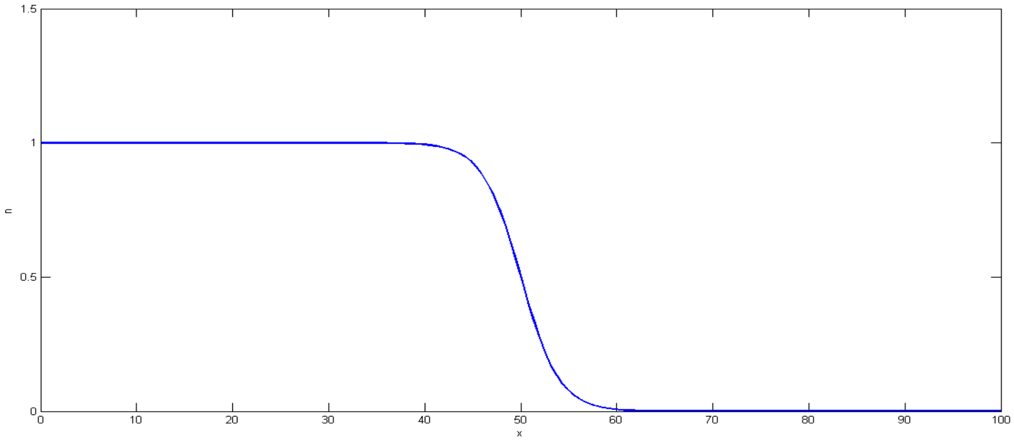


Figure 5: Solve the system (3) through the initial condition with the parameter value $\alpha = 2$

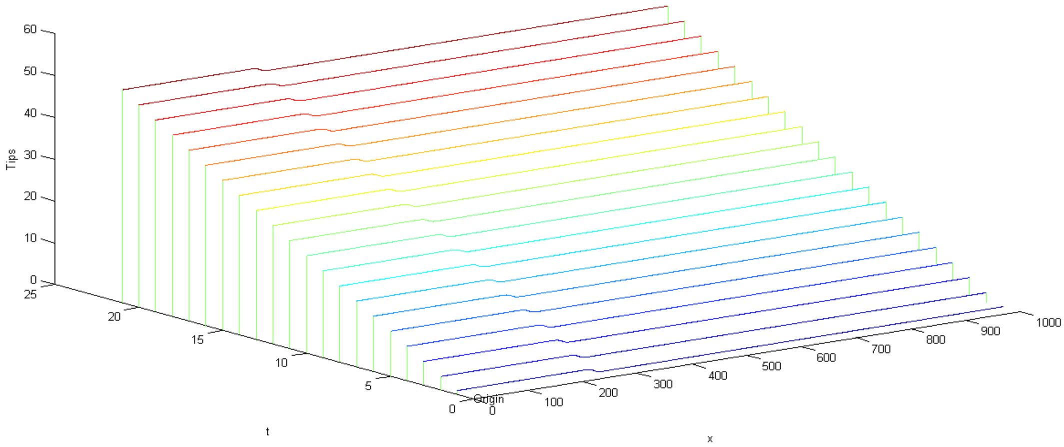


Figure 6: Growth the tips (n) and the branches (p) for FWE (3D) let α = 2, c = 298.4 for time t = 1, 10, 20,30,......, 200 where the red line represent branches (p) and the blue line represented tips(n), solution are produced using MATLAB pdepe.

We define the correlation between a values α and wave speed (c) ,where traveling wave increasing anytime the values of α increase:

Table 2: The relation between values (α) and wave speed (c) for type FWE

α	0.5	1	2	3	4	5	6	7	8	9	10
c	298	298.4	298.8	299.2	299.6	300	300.4	300.8	301.2	301.6	302

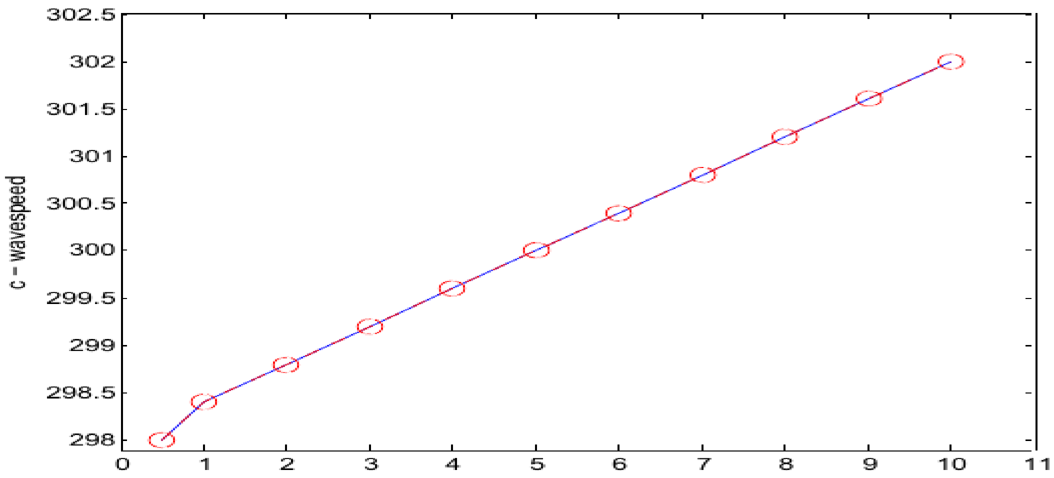


Figure 7: The relation between (α) values and wave speed (c)

We define the correlation between a values v and wave speed (c) ,where traveling wave increasing anytime the values of v increase:

Table 3: The relation between values (v) and wave speed (c) for type FWE

v	0.5	1	2	3	4	5	6	7	8	9	10
c	150	298.4	600	899	1200	1499	1800	2100	2400	2699	2999

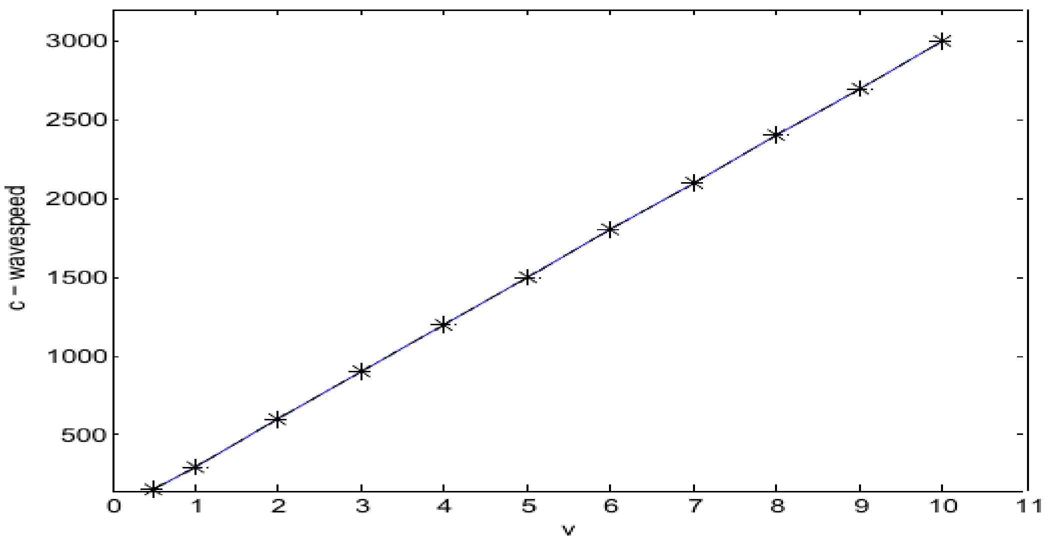


Figure 8: The relation between (v) values and wave speed (c)

7- THE STABILITY OF SOLUTION, WHEN $\Psi = 1$

We find stability when we discuss the next system(3):

$$\frac{\partial p}{\partial t} = n = 0 \Longrightarrow n = 0 \tag{7}$$

$$\frac{\partial n}{\partial t} = \alpha(p - n^2) + 1 \Longrightarrow \alpha(p - n^2) + 1 = 0 \quad (8)$$

$\alpha p = -1 \Longrightarrow (p_1, n_1) = (\frac{-1}{\alpha}, 0)$

$$(\alpha p - \alpha n^2) = -1, n = 1 \Longrightarrow (p_2, n_2) = (\frac{\alpha - 1}{\alpha}, 1)$$

Therefor we take the Jacobin of these equation(7),(8):[8]

Now, the Jacobian for the system :

$$J_{(p,n)} = \begin{bmatrix} 0 & 1 \\ \alpha & -2\alpha n \end{bmatrix}$$

Jacobian at $(\frac{-1}{\alpha}, 0)$

$$J_{(\frac{-1}{\alpha},0)} = \begin{bmatrix} 0 & 1 \\ \alpha & 0 \end{bmatrix}$$

take the Jacobian at $(\frac{\alpha-1}{\alpha}, 1)$

$$J_{(\frac{\alpha-1}{\alpha},1)} = \begin{bmatrix} 0 & 1 \\ \alpha & -2\alpha \end{bmatrix}$$

Now,determent the eigenvalues as $\lambda_i; i = 1,2$

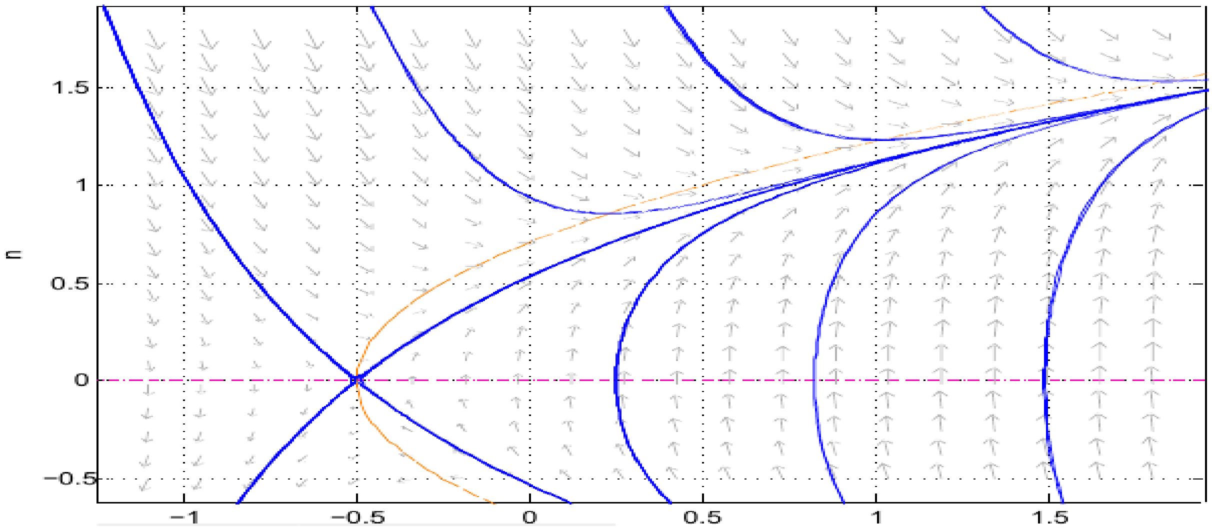


Figure 9: If($\alpha = 2$) we note that a trajectory connects the saddle point in $p_1(\frac{-1}{\alpha}, 0)$ to the saddle point in $p_2(\frac{\alpha-1}{\alpha}, 1)$

8- TRAVELING WAVE SOLUTION

In this section, we will talk about the traveling wave solution. This section shows that the traveling speed is in the same direction as F(z) when we let f(x,t) be a function that depicts a moving wave to the right at a constant rate. (F(z)=f(x,t) provided z = x-ct): [9,10,11]

$$n(x,t) = N(z), \; p(x,t) = P(z)$$

$$\frac{\partial p}{\partial t} = -c\frac{dP}{dZ}, \; \frac{\partial n}{\partial t} = -c\frac{dN}{dZ}$$

$$\frac{\partial n}{\partial x} = \frac{dN}{dZ}$$

We find the solution of stability when discussing the following equation:

$$\frac{dP}{dz} = \frac{-1}{c}[N] \Longrightarrow \frac{-1}{c}[N] = 0, c \neq 1, -\infty < z < \infty$$

$$\frac{dN}{dz} = \frac{1}{1-c}\alpha(P - N^2) \Longrightarrow \frac{1}{1-c}[\alpha(P - N^2) + 1] = 0 \tag{9}$$

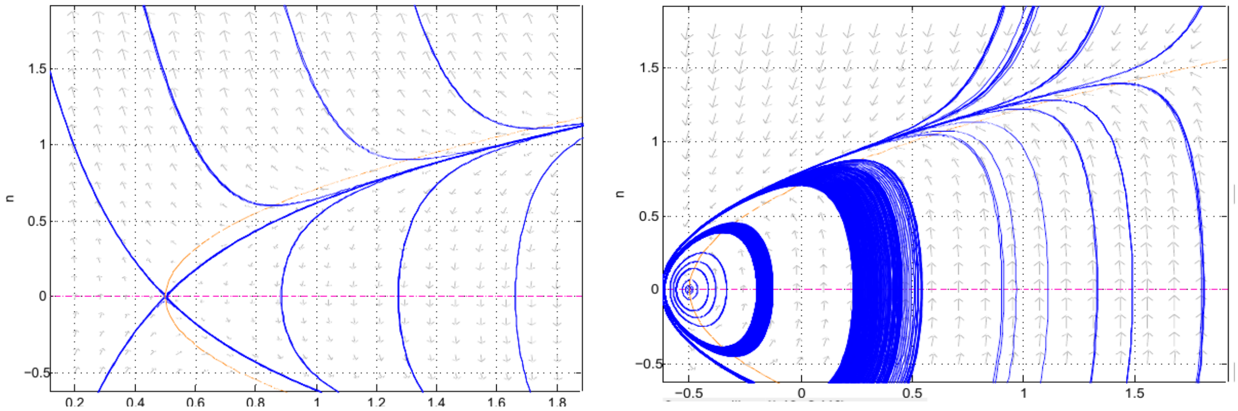


Figure 12: In (a), when if $\alpha = 2$ and $c = 2$ we see a trajectory connects the (saddle point) in $p_1(\frac{c-1}{\alpha}, 0)$ to the (saddle point) in $p_2(\frac{c-1+\alpha}{\alpha}, 1)$, In (b), when if $\alpha = -2$ and $c = 2$ we see a trajectory connects the (center) in $p_1(\frac{c-1}{\alpha}, 0)$ to the(unstable node) in $p_2(\frac{c-1+\alpha}{\alpha}, 1)$

9- NUMERICAL SOLUTION

We will discuss in this section numerical solution because this system can not be solved,and we need MATLAB in this case .pdepe code will used to solve the system(3):[9,10,11]

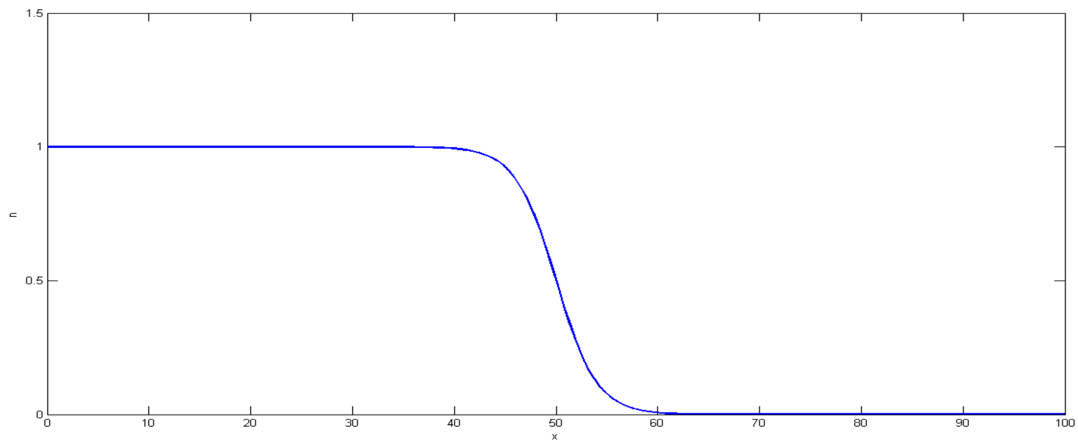


Figure 13: Solve the system (3) through the initial condition with the parameter value $\alpha = 2$

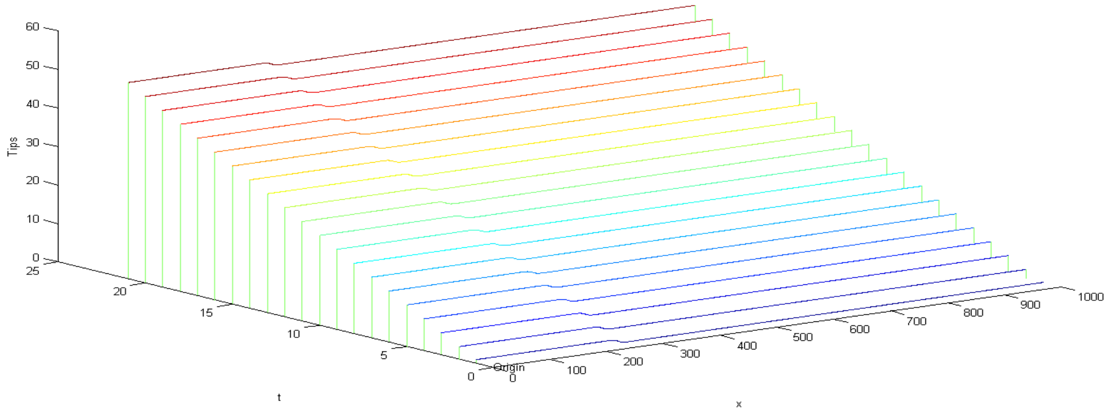


Figure 14: Growth the tips (n) and the branches (p) for FWE (3D). Let α = 2, c = 298 for time t = 1, 10, 20,30,......, 200, where the red line represent branches (p), and the blue line represented tips(n). solution are produced using MATLAB pdepe.

We define the correlation between a values α and wave speed (c) ,where traveling wave increasing anytime the values of α increase

Table 4: The relation between values (α) and wave speed (c),take value υ=1 for type FWE

α	0.5	1	2	3	4	5	6	7	8	9	10
c	298	298.4	298.8	299.2	299.6	300	300.4	300.8	301.2	301.6	302

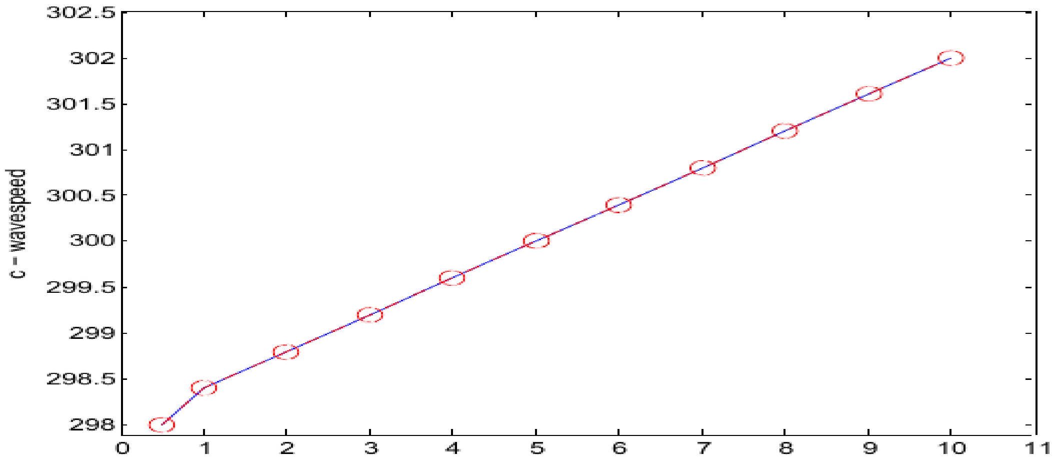


Figure 15: The relation between (α) values and wave speed (c)

We define the correlation between a values υ and wave speed (c) ,where traveling wave increasing anytime the values of υ increase:

Table 5: The relation between values (v) and wave speed (c),α=1 for type FWE

v	0.5	1	2	3	4	5	6	7	8	9	10
c	150	298.4	600	899	1200	1499	1800	2100	2400	2699	2999

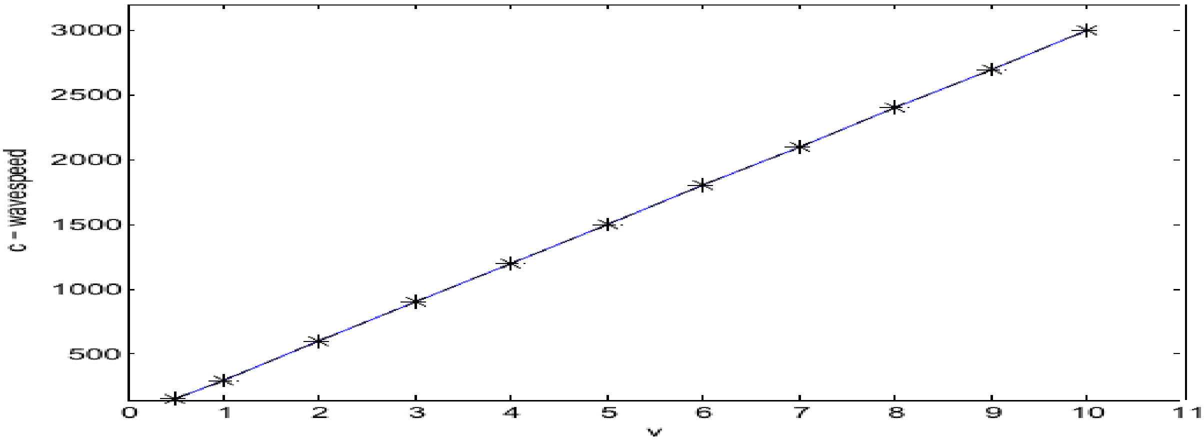


Figure 16: The relation between (v) values and wave speed (c)

10- CONCLUSION

We inferred that the speed of the vector (c) increases when the values of a grow α , as shown in figure (8). We also conclude that the traveling wave increases with the increase in v values, as shown in figure (9).Since $\alpha = \frac{\alpha_2 v}{\gamma^2}$ Rate of growth of an increases α_2 while keeping γ constant.

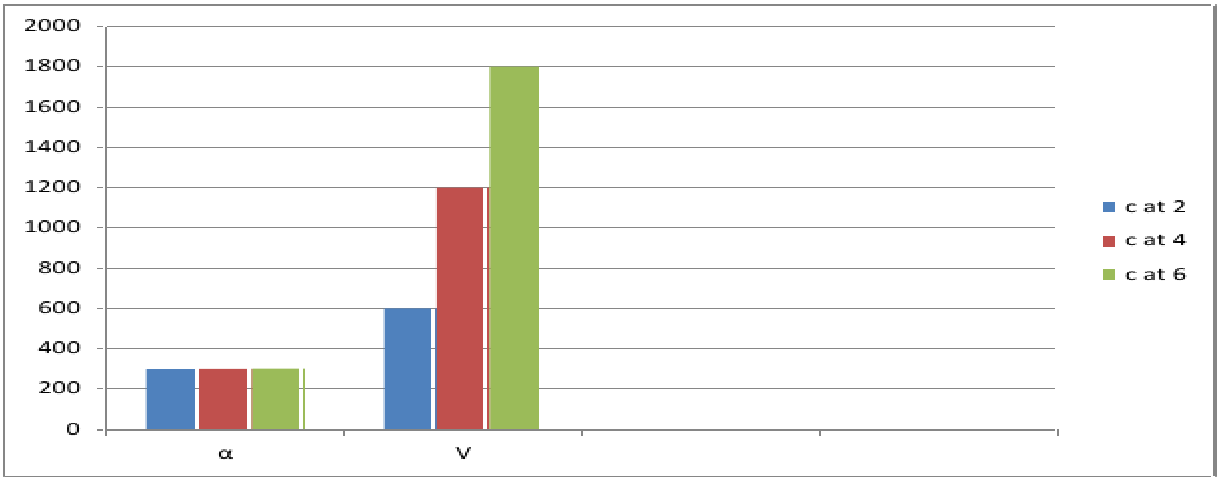


Figure17: Illustration of the speed difference at 2, 4 and 6 at the values of α, v

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