Certain Types of Function Via Alpha -Open Sets

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Abstract—
In this effort, some new types of (alpha-open, alpha-star-open, alpha-star star-open) function. And addition to the previous study, there are some concepts associated with what we have taken to be ready relationships, semi (alpha-open, alpha-star-open, alpha-star star-open) function. All of these ideas introduce the topological space, we prove that this class (alpha-open and alpha-star[open) function. Also we find some basic properties and application of alpha-star star-open function, we as well lead and study a new class of space between concepts, and learning a new class of space, that is to say(semi-alpha-open, semi-alpha-star-open and semi-alpha-star-star-open) by function. We will need some theorems and observations to achieve the results between these mathematical concepts, while taking the test by renting the presentations necessary to solve mathematical problems.

Keywords—
α-open, α^*-open, α^{**}-open, $semi\ \alpha$-open, $semi\ \alpha^*$-open, and **$semi\ \alpha^{**}$-open**

1 Introduction

The open set is the main component of the topological space. In 2000, G. B. Novalage introduce provide a detailed study of the definitions you need. It also added continuity properties by In 1981, Takashi Noiri, and he did another in-depth study on that information that is looking at this topic referred to. In 1987, Takashi Noiri, through the properties of some weak continuity, our study in this research focuses on open sets of type alpha its structure and saturating it with research through definition, proof, examples, and its application.

2 Notions

Definition 2.1. [1]

i- Let $(X, T_X) and\ (Y, T_Y)$ be two topological spaces , if $f: X \rightarrow Y$, then f *is* called ***α-open* function**. If and only if each M *is open set* of X , then $f(M)$ an ***α-open set*** in Y.

ii- Let $(X, T_X) and\ (Y, T_Y)$ be two topological spaces if $f: X \rightarrow Y$, then f is named ***semi α*-open function,** if and only if, for each M *open set in* X. thus $f(M)$ is *semi α-open* in Y,

Theorem 2.2. [2]

For (X, T_X) and (Y, T_Y) be two topological spaces if $f : X \rightarrow Y$, is α-open function then the following properties are equivalent :

1. f^{-1} ($cl\ Int\ cl\ N$) $\subseteq cl$ every $N \in Y$.

2. $Int\ T^{\alpha}\ f^{-1}\ M \subseteq f^{-1}\ (Int$ M) for each M $\in Y$.

Theorem 2.3. [2]

Let $(X, T_X)$and (Y, T_Y)be two topological spaces if $f: X \rightarrow Y$ be named α-open function , $f\ (Int\ M) \subset Int\ T^{\alpha}\ f(M)$, Every $M \in X$.

Remark 2. 4.

Each *open* function be α-open function, however the opposite is false of Overall, for example shows.

Example 2. 5.

If $X = \{0,2,5,8\}$, $T_x = \{\emptyset, \{0\}, \{2\}, \{0,2\}, \{0,2,5\}, X\}$,
Then, the α-open sets in X space ; $T_x^{\alpha} = T_x \cup \{\{0,2,8\}\}$,
The f is identity $function$,
$f(0) = 0, f(2) = 2, f(5) = f(8) = 8$.
Therefore f **is *α*-open,**
on the other hand not *open* function. because
$\{0,2,5\}$, be *open* of space X, and $f\ (\{0,2,5\}) = \{0,2,8\}$,
However $\{0,2,8\}$, be not *open* set in X space.

Remark 2. 6.

For each open function be ***α-open*** function, thus f be ***semi α-open*** function, Then a converse be false of overall.

Example 2. 7.

Given $X = \{1,0,3,4\}$, $T_X = \{\emptyset, \{1\}, \{0\}, \{1,0\}, \{1,0,3\}, X\}$,
then α-open in X space, $T_{(X)}^{\alpha} = T_X \cup \{1,0,4\}$,

thus $semi\ \alpha$-open in X space ;
$S\alpha O((x) = T^{\alpha}_{(X)} \cup \{\{0,3,4\},\{1,3,4\},\{1,3\},\{1,0\},\{0,3\},\{0,4\}\}$,
And $f\ is$ Identity function. $f(1) = f(0) = 1, f(3) = f(4) = 3$.
So f is semi α-open function. But is not α-open function,
Because $\{1,0,3\}$ is open in space X. However $f(\{1,0,4\}) = \{1,3\} \notin T^{\alpha}_{x}$.
Therefore f is $\boldsymbol{semi\ \alpha}$**-open**, then is not α-**open** function.

Example 2. 8.
Give $X = \{1,9,6,3\}, T_x = \{\emptyset,\{1\},\{9\},\{1,9\},\{1,9,6\},X\}$.
$T^{\alpha}_{X} = T_x \cup \{1,9,3\}$.
$S\alpha O(x) = T^{\alpha}_{x} \cup \{\{9,6,3\},\{1,6,3\},\{1,6\},,\{9,6\},\{9,3\}\}$.
Define $f: X \to X$ (Identity function) , $f(1) = f(9) = 1, f(6) = f(3) = 6$,
And f is semi α-open function, so f is not open because $\{1,9,6\}$ be $open$ Set in X . But $f\{1,9,6\} = \{1,6\}$ not $open\ set$.
Thus f a $\boldsymbol{semi\ \alpha\text{-}open}$ functi.on, that is not $\boldsymbol{open}$ function in X.

Remark 2.9. [5]
Each α^*-open function is $\boldsymbol{\alpha}$**-open** and $\boldsymbol{semi}$ **α-open**.Then the convers is not Right of overall, thus the following sample display.

Example 2.10.
If $X = \{0,2,6,8\}, T_x = \{\emptyset,\{0\},\{2\},\{0,2\},\{2,6,0\},X\}$, $T^{\alpha}_{x} = T_x \cup \{0,2,8\}$,
And**,** $S\alpha O(X) = T^{\alpha}_{x} \cup \{\{2,6,8\},\{0,6,8\},\{2,6\},\{2,8\},\{0,8\},\{0,6\}\}$,
A function $f: X \to X$. (Identity function)
By $f(0) = f(2) = 0,\ f(6) = 2, f(8) = 6$,
We get f is α-open function,
Then it's not α^*-open function ,
Because $\{0,2,8\} \in T^{\alpha}_{x}$,
however $f\{0,2,8\} = \{0,6\} \notin T^{\alpha}_{x}$.
Thus f is $\boldsymbol{\alpha}$**-open** function,
But f it's **not** $\boldsymbol{\alpha^*}$**-open** function .

Through the previous observations we get ;
open function $\to$ $\boldsymbol{\alpha}$**-open function** $\to$ $\boldsymbol{semi\ \alpha}$**-open function**

3 Concepts and Their Relationship Via Function

Definition 3.1 [1]
I- Let $(X, T_X)$and (Y, T_Y) be two topological spaces, if $f : X \to Y$, then f is called $\boldsymbol{\alpha^*\text{-}open}$, if and only if each $\boldsymbol{A}$ is $\boldsymbol{\alpha\text{-}open}$ $set$of X, thus $\boldsymbol{f}(\boldsymbol{A})$ be $\alpha\text{-}open$ set of Y.

II- Let $(X, T_X) and\ (Y, T_Y)$ be two topological spaces, if $f : X \to Y$ be a semi α^*-open. If and only if , for each $M$is **semi $\boldsymbol{\alpha^*}$-*open* *set*** in X, as $f(M)$ is ***semi α-open*** *set* of Y.

Remark 3.2.
for ideas are open functions as well as α^*-open functions are inde pendent. By way of the resulting example shows.

Example 3.3
Give $X = \{0,3,5,7\}$, $T_X = \{ \emptyset , \{0\}, \{0\} , \{0,3\}, \{0,3,5\}, X \}$,
$T^{\alpha}_{(X)} = T_X \cup \{ 0,3,7\}$,
$S\alpha O$(X)= $T^{\alpha}_{(X)} \cup \{\{3,5,7\}, \{0,5,7\}, \{3.7\}, \{0,7\}, \{0,5\}\}$.
Suppose f : X →Y by $f(0) = f(3) = 0$, $f(5) = 3$ and $f(7) = 5$.
Therefore f is open function and f is not α^*-open,
Because $\{0,3,7\} \in T^{\alpha}_{x}$, but $f(\{0,3,7\}) = \{0,5\} \notin T^{\alpha}_{(X)}$.
As a result f is **open** function, however f it is **$\boldsymbol{\alpha^*}$-open** function.

Example 3.4
Give $X = \{ 5,6,7,8 \}$. $T_X = \{ \emptyset, \{5\}, \{6\}, \{5,6\}, \{5,6,7\} , X \}$,
$T^{\alpha}_{(X)} = T_X \cup \{ 5,6,8 \}$, If $f : X \to X$ (Identity function),
By $f\ (5) = 5$, $f\ (6) = 9$, $f\ (7) = f\ (8 = 8$.
As sure as that f be α^*-*open*, function. on the other hand it be not *open* Function. Because $\{ 5,6,7 \} \in T_X$ other than $f(\{5,6,7\}) = \{5,6,8\} \notin T_X$.
Wherefore f is **$\boldsymbol{\alpha^*}$-open** function. but f is **not open** function .

Proposition 3.5 [3]

I. Let $(X , T_X)$and (Y , T_Y) be two topological space and $f: X \to Y$ is open and continuous function , Then α^*-open function .

II. If (X , T_X) and(Y , T_Y) be two topological space and $f: X \to Y\ is\ \alpha^*$-open $function$, $f: \left(X , T^{\alpha}_{(X)}\right) \to (Y , T^{\alpha}_{(Y)})$ is open .

Proof :
Given $f : X \to Y$ is *open* as well as continuous *function*,
To prove f is α^*-open.
Let $N \in T^{\alpha}_{(X)}$ so M be *open set* of X thus,
$M \subseteq N \subseteq Int\ cl\ M$ " by theorem : for any subset $N \in T^{\alpha}_{(X)}$ of a space X ,
N iff $\exists$ an *open set* M then $M \subseteq N \subseteq Int\ Cl\ M$".
Then f(M) ⊆ f(N) ⊆ f ($Int\ Cl$ M).
However f ($Int\ Cl$ M) ⊆ Int (f Cl M) , (by f is open function).
Thus f (M) ⊆ f (N) ⊆ f ($Int\ Cl\ M$) ⊆ Int (f Cl M).
But Int (f Cl M)⊆ Int (cl (f (M)), (by f is open continuous function),

So we obtain f (M) ⊆ f (N) ⊆ Int (Cl f (M)).
Also f (M) be $open\ set$ of Y , (by f be $open$ function),
Therefore $f(\ N\ )\ \in\ T^{\alpha}_{(Y)}$, " by theorem : for any subset $N \in T^{\alpha}_{(X)}$, N iff,
∃an open set M, thus M ⊆ N ⊆ $Int\ Cl\ M$ ".
Hence f is α^*-open. And the proof of part (II) simply.

Definition : 3.6

I- Let $(X\ ,T_X\ )$and $(Y\ ,T_Y)$ be two topological space , $and\ f\ :X\ \to Y$ be a function, then f is called $\boldsymbol{\alpha^{**}}$**-open** if And only if each M $\boldsymbol{\alpha}$-$\boldsymbol{open}$ set of X, then $f(M)$ be $\boldsymbol{open}$ set of y.

II- If $(X\ ,T_X\ )$and $(Y\ ,T_Y)$be two topological space and $f{:}\,X \to Y$ is a function, then f is termed $\boldsymbol{semi\ \alpha^{**}}$**-open** if and only if. Every M $\boldsymbol{semi\ \alpha}$**-open set** in X, therefore $f(M)$ is **open set** in Y.

Using the definition is possible to get. Two examples of application definitio

Example 3.7.

Let $X = \{a, b, c, d\}\ , T_x = \{\emptyset, \{a, b\}, \{a, b, c\}\ , X\ \}$,
$T^{\alpha}_x =\ T_x\ \cup \{a, b, d\}$
$S\alpha O\ (X) =\ T^{\alpha}_x\ \cup \{\ \{b, c, d\}, \{a, c, d\}\ , \{a\ , c\}\ , \{a, b\}\ , \{b, c\}, \{b, d\}\ \}$.
Define $f\ : X \to X$ (Identity function). By $f(a) = a\ \ , f(b) = b\ , f(c) = f(d) =\ c$. As a result f is $\boldsymbol{\alpha}$**-open** function and **open** function ,
Therefor $\{a, b, c\}$be $open$ of X, so $f\ (\{a, b, c\}) = \{a, b, c\}$ be $open$ of X.
Thus f is $\boldsymbol{\alpha^{**}}$**-open** because $\boldsymbol{f}$ ($\boldsymbol{\alpha}$**-open** and **open**).

Example 3.8

If $X = \{a, b, c, d\}, T_x = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, d\}, X\}$,$T^{\alpha}_x = T_x \cup \{a, b, c\}$,
$S\alpha O(X) = T^{\alpha}_x \cup \{\ \{b, c, d\}, \{a, c, d\}, \{a, c\}, \{b, c\}, \{b, d\}\ \}$.
A function $f{:}\,X \to X\ , f(a) = a\ , f(b) = b\ , f(c) = f(d) = d$.
We observe f is $\boldsymbol{semi\ \alpha}$ **-open** function ,and **open** function ,
Since $\{a, b, c\}$is open in X, and $f(\{a, b, c\})\ = \{a, b, d\}$,
Thus f is $\boldsymbol{semi\ \alpha^{**}}$**-open** (by $\boldsymbol{f}$ **are** $\boldsymbol{semi\ \alpha}$ **-open and open**) .

Theorem 3.9 [4]

if $(X\ , T_X\ )$ and $(Y\ , T_Y\ )$ be two topological space. A $f\ : X \to\ Y$Thus f be $semi\ \alpha$-$continuous$ if f every $x \in X$ and every open set A $f(x) \in A$
there exists a $semi\ \alpha$-open $set\ M$ having x then $f(M)\ \subset\ A$.

proposition 3.10

A function $f{:}\,X\ \to Y$ is an α^*-open and continuous , then f is $semi\ \alpha^*$-open

Proof :

Give $f{:}\,X \to Y$ be α^*-open and continuous,
The set A is a $semi\ \alpha$-open in X.
Then, $M \in T^{\alpha}_x$. Such that $M \subseteq A \subseteq Cl\ M$. Thus $f(M) \subseteq\ f(A) \subseteq\ f(Cl\ M)$,
However , f (Cl M) $\subseteq$ Cl (f (M)) , (by f is continuous).

So, f (M)⊆f (A)⊆ Cl $(f$ $(M))$.
But f (M) $\in$ T_x^{α}, (by f is α^*-open)
Therefore, $f(A)$ $\in S\alpha O(Y)$. As a result f is $semi$ α^*-open.

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