On Semi pre-generalized-closed sets

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Abstract:- In this paper, a new class of $generalized - closed\ sets$ called $semi\ pre - generalized - closed\ sets$ in $topological\ spaces$ are introduced and studied. Also some of their properties have been investigated. We also introduce $sp - T1/2\ space, submaximal\ space$, and studied the relationship between $spg - closed\ set$ and some space in $topological\ spaces$ as well discuss some properties, theorem, corollary and examples.

Keywords: $sp - closed\ set$, $sp - T1/2\ space$, $submaximal\ space$, $spg - closed\ set$.

1 Introduction

The concepts of $semi - open\ sets$ were introduced and studied by Levin [1] in 1963, and in 1970, Levin [2] began and studied $generalized\ closed\ sets$ and $generalized\ open\ sets$ as a $generalization$ of $closed\ sets$ and $open\ sets$. Dunham [3] came up with the concept of $generalized\ closure$ using Levine $generalized\ closed\ sets$ and explained their properties. The investigation of $generalized\ closed\ sets$ has led to many interesting concepts in topology. Recently, topologists have studied diverse and closed generalized $sets$ in $topological\ spaces$. In 1982, Mashhour, Abdul-Moncef and Deeb [4] identified conquest $pre - sets$ and $pre - continuous\ functions$. The class of previously $generalized\ closed\ sets$ that were used to obtain properties of $pre - T1/2\ spaces$ was introduced by Maki, Umehara and Noiri [5] in 1996. In 1986, D. Andrijevic [6] studied $semi - open\ sets$ in $topology$. Later, many authors studied these previously $semi - open\ sets$ and $semi - closed$ and $generalized\ continuous\ functions$. In 1995, Dontchev [7] identified $generalized\ semi - open\ sets$. Mackey introduced the concepts of $pg - closed\ sets$ and $gp - closed\ sets$ in a similar way in [8]. These concepts are $generalizations$ of $closed\ sets$ and have been studied by Dontchev and Maki [9] leading to a new decomposition of $pre - continuity$. In this paper, our goal is to introduce new types of $generalized\ sets$ in the $topological\ space$ called $semi\ pre - generalized - closed\ sets$ and study the relationship between them and some $sets$ in the $topological\ space$ and verify their basic properties.

2 Preliminaries

Definition 2.1

Let $((X,T)$ be a $topological\ space$, A subset B of X is called:

1. $Semi - open\ set$ (briefly, $s - open$)if there is an $open\ set$ V in X such that $V \subseteq B \subseteq \overline{(V)}$, the complement of $semi - open\ set$ is $semi - closed\ set$ (briefly, $s - closed$).[10]

2. $pre - open$ or $locally\ dense\ set$ (briefly, $p - open$) if $B \subseteq \left(\overline{B}\right)^{o}$, the family of all $pre - open\ sets$ in a space X is denoted by P.O. (X) and the complement of $pre - open\ set$ is a $pre - closed\ set$ and denoted by P.C.(X) .[11]

3. $Semi\ pre - open\ set$ (briefly, $sp - open$) if $B \subseteq \overline{((\overline{B})^{o})}$, The complement of $semi\ pre - open\ set$ is a $semi\ pre - closed\ set$ (briefly, $sp - closed$) and represent that $((\overline{B^{o}}))^{o} \subseteq B$.[12]

4. $generalized - open\ set$ (briefly, $g - open$) if $\overline{(B)} \subseteq V$, whenever $B \subseteq V$ and V is a closed set. The complement of $g - open\ set$ is a $generalized - closed\ set$ (briefly, $g - closed$).[13]

Remark 2.2

1. Every $closed\ set$ ($open\ set$) is a $s - closed$ ($s - open$) set, but the converse does not necessary to be true.[10]

2. Every $closed\ set$ ($open\ set$) is a $p - closed$ ($p - open$) set, but the converse does not necessary to be true.[11]

3. Every $closed\ set$ ($open\ set$) is a $sp - closed$ ($sp - open$) set, but the converse does not necessary to be true.[12]

4. Every $closed\ set$ ($open\ set$) is a $g - closed$ ($g - open$) set, but the converse does not necessary to be true.[13]

Remark 2.3

1. The intersection of all $s - closed\ subsets$ of X which is containing B is said to be a $s - closure$ of B and it is denoted by $\overline{B}^{s}$.[10]

2. The intersection of all $p - closed\ subsets$ of X which is containing B is said to be a $p - closure$ of B and it is denoted by $\overline{B}^{p}$.[14]

3. The intersection of all $sp-closed\ subsets$ of X which is containing B is said to be a $sp-closure$ of B and is denoted by $\overline{B}^{sp}$.[12]
4. We can write the relationship between $p-closure$ of B, $sp-closure$ of B and $closure$ of B : $\overline{B}^{sp} \subseteq \overline{B}^{p} \subseteq \overline{B}$.[12]

Corollary 2.4

Let (X,T) be a $topological\ space$ and let $B,C \subseteq X$, if $B \subseteq C$ then $\overline{(B)}^{sp} \subseteq \overline{(C)}^{sp}$.[16]

Theorem 2.5

Let (X,T) be a $topological\ space$, $B \subseteq X$ then B is a $semi\ pre-closed\ set$ if and only if $B = \overline{(B)}^{sp}$.[16]

Proof:

Let B be a $sp-closed\ subset$ of X

It is clear that $B \subseteq \overline{(B)}^{sp}$ …..(1) [2.2(3)]

Since B is a $sp-closed\ set$

So $\overline{(B)}^{sp} \subseteq B$ …..(2)

From (1) and (2) we get $\overline{(B)}^{sp} = B$

Conversely

Suppose that $\overline{(B)}^{sp} = B$

Since $\overline{(B)}^{sp}$ is a $sp-closed\ set$

So B is a $semi\ pre-closed\ set.$

Theorem 2.6

Let (X,T) be a $topological\ space$, the singleton $\{x\}$ is an $open\ set$ or $semi\ pre-close\ set$.[12]

Proof :

Let $x \subseteq X$

So $\{x\} \subseteq X$ is singleton set

So either $\{x\}^o = \emptyset$ or $\{x\}^o \neq \emptyset$

If $\{x\}^o = \emptyset$ then $\overline{(\{x\}^o)} = \emptyset$ and $(\overline{(\{x\}^o)})^o = \emptyset$

Therefor $(\overline{(\{x\}^o)})^o \subseteq \{x\}$

Hence $\{x\}$ is a $sp - closed\ set$

If $\{x\}^o \neq \emptyset$

Then $\{x\}^o = \{x\}$

Hence $\{x\}$ is an $open\ set$.

Theorem 2.7

Let (X, T) be a $topological\ space$ and let $B \subseteq X$ then B is the intersection of $s - open\ set$ with $dense\ set$ if B is a $sp - open\ set$.[17-22]

Proof :

Suppose that $B = C \cap H$ such that C is a $s - open\ set$ and H is $dense\ set$ such that $\overline{(B)}^s = \overline{(C)}^s$. And since $B \subseteq C \subseteq \overline{(C)}^s = \overline{(B)}^s$

So $B \subseteq C \subseteq \overline{(B)}^s$

Hence B is a $sp - open\ set$.

Definition 2.8

Let (X, T) be a $topological\ space$ and let $B \subseteq X$ then the $set\ B$ is called $semi\ pre\ general\ ized - closed\ set$ $(semi\ pre - generalized - open)$ if $\overline{(B)}^{sp} \subseteq V$ whenever $B \subseteq V$ and V is a $semi\ pre - open\ set$ $(semi\ pre - closed)$ in X and denoted by $spg - closed$ $(spg - open)$ set.

Example 2.9

Let $X = \{1,2,3,4,5\}$ and let $T = \{X, \emptyset, \{1\}, \{3,4\}, \{1,3,4\}\}$ be a $topological\ defined$ on X.

Let $B = \{2,5\}$ and $C = \{3,4\}$ be a $subset$ of X

It is clear that B is a $spg - closed\ set$

since the $semi\ pre - open$ which contain B is:

$X, \{2,3,5\}, \{2,4,5\}, \{1,2,4,5\}, \{2,3,4,5\}, \{1,2,3,5\}, \{1,2,5\}, \{2,5\}$ and also, it contains $\overline{(B)}^{sp} = \{2,5\}$

and C is a $spg - open\ set$ since the $semi\ pre - closed\ set$ which contain C is:

$X, \{3,4\}, \{2,3,4,5\}, \{1,3,4\}$ and it contains $\overline{(C)}^{sp} = \{3,4\}$

Remark 2.10

Every $closed\ set$ ($open\ set$) is a $spg - closed\ \ set(spg - open)$, but the converse does not necessarily to be true for example:

Example 2.11

Let $X = \{1,2,3,4,5\}$ and let $T = \{X, \emptyset, \{1\}, \{3,4\}, \{1,3,4\}\}$ be a $topological$ defined on X.

Let $B = \{1,2,5\}$ and $C = \{3\}$ be a $subset$ of X .

So B is a $spg - closed\ set$ but it is not $closed\ set$ and C is a $spg - open\ set$ but it is not $open\ set$.

Proposition 2.12

Let (X, T) be a $topological\ space$ and let $B \subseteq X$ then B is a $spg - closed\ set$ if B is a $pg - closed\ set$.

Proof:

Let X be a $topological\ space$

And let V is a $p - open\ set$ in X

Such that $(\overline{B}^{p}) \subseteq V$ whenever $B \subseteq V$

So B is a $pg - closed\ set$

Since every $p - open\ set$ is a $sp - open\ set$

So V is sp-open set

And since $\overline{(B)}^{sp} \subseteq (\overline{B}^{p})$ and $(\overline{B}^{p}) \subseteq V$

So $\overline{(B)}^{sp} \subseteq V$ whenever $B \subseteq V$

So B is a $spg - closed\ set$.

Proposition 2.13

Let (X,T) be a $topological\ space$ and let B be a $subset$ of X then B is a $spg-closed\ set$ if B is a $sp-open\ set$ and $pg-closed\ set$.

Proof:

Let (X,T) be a $topological\ space$

And let B be an $open\ set$ in X

So B is a $p-open\ set$ in X

Since every $p-open\ set$ is $sp-open\ set$

So B is a $sp-open\ set$ in X

Hence B is a $sp-open\ set$

let V be an $open\ set$ in X such that $\overline{(B)} \subseteq V$ whenever $B \subseteq V$

Since every $open\ set$ is $p-open\ set$

So V is a $p-open\ set$

And since $(\overline{B}^{p}) \subseteq \overline{(B)}$ [2.2 (4)]

So $(\overline{B}^{p}) \subseteq \overline{(B)} \subseteq V$

Hence $(\overline{B}^{p}) \subseteq V$ whenever $B \subseteq V$ and V is p-open set

Since every p-open set is sp-open set

So V is sp-open set in X

Therefor $(\overline{B}^{p}) \subseteq V$ whenever $B \subseteq V$ and V is a $sp-open\ set$

And since $\overline{(B)}^{sp} \subseteq (\overline{B}^{p}) \subseteq V$ [2.2 (4)]

So $\overline{(B)}^{sp} \subseteq V$ whenever $B \subseteq V$ and V is a $sp-open\ set$

Hence B is a $spg-closed\ set$.

Proposition 2.14

Let (X,T) be a $topological\ space$ and let $B \subseteq X$ if B is a $sp-closed\ set$ then B is a $spg-closed\ set$.

Proof:

Suppose that B is a $sp-closed\ set$ in X

And let V is a $sp-open\ set$ in X such that $B \subseteq V$

Since B is a $sp-closed\ set$, so $\overline{(B)}^{sp}=B$ [2.5]

Hence $\overline{(B)}^{sp}\subseteq V$

So B is a $spg-closed.$

Definition 2.15

Let (X,T) be a $topological\ space$, then X is a $semi\ pre-T_{\mathbf{1/2}}\ space$ if:

1) every $spg-closed\ set$ in X be a $semi\ pre-closed\ set.$

2) every singleton be a $semi\ pre-closed\ set$ or $semi\ pre-open.$

Remark 2.16

In definition (2.15) (1) and (2) are equivalent.

Proof:

Suppose that (1) is a verified

We will prove (2)

Let X be a $semi\ pre-T_{1/2}\ space$ and $x\in X$

Let $\{x\}$ is not $sp-closed\ set$

So $B=X-\{x\}$ is not $sp-open\ set$

Hence X is a $sp-open\ set$ which contain B and also $\overline{(B)}^{sp}\subseteq X$

So B is a $spg-closed\ set$

And by definition [(2.15) (1)], so B is a $sp-closed\ set$

Hence $\{x\}$ is a $sp-open\ set$

And to proof conversely suppose that definition [(2.15) (2)] is verified.

Assume that $B\subseteq X$ be a $spg-closed\ set$

We will prove that B is a $sp-closed\ set$

That means $\overline{(B)}^{sp}=B$

It is clear that $B\subseteq\overline{(B)}^{sp}$ … (*)

Suppose that $x\notin B$ and $x\in\overline{(B)}^{sp}$

So $B\subseteq X-\{x\}$

And by definition [(2.15) (2)] then $\{x\}$ is a $sp-open\ set$ or $sp-closed\ set.$

So if $\{x\}$ is a $sp-open\ set$
Then $X-\{x\}$ is a $sp-closed\ set$
Since $B\ \subseteq\ X-\{x\}$, then $\overline{(B)}^{sp}\ \subseteq\ \overline{(X-\{x\})}^{sp}$ [2.4]
Hence $\overline{(B)}^{sp}\ \subseteq X-\{x\}$
And since $x\in\overline{(B)}^{sp}\ \subseteq X-\{x\}$, this means $x\in X-\{x\}$ and this is contradiction
So $x\in B$ and hence $\overline{(B)}^{sp}\ \subseteq B$ … (**)
From (*) and (**) we get $\overline{(B)}^{sp}=B$
Hence B is a $sp-closed\ set$
Either if $\{x\}$ $sp-closed\ set$
Then $X-\{x\}$ is a $sp-open\ set$
And since $B\ \subseteq X-\{x\}$ and B is a $spg-closed\ set$
So $\overline{(B)}^{sp}\ \subseteq X-\{x\}$
And since $x\in\overline{(B)}^{sp}\ \subseteq X-\{x\}$, this means $x\in X-\{x\}$ and this is contradiction
So $x\in B$ and hence $\overline{(B)}^{sp}\ \subseteq B$ … (***)
And from (*) and (***) we get $\overline{(B)}^{sp}=B$
Hence B is a $sp-closed\ set.$
From theorem 2.6 and definition 2.15(2) and remark 2.2(3) we get the following corollary:

Corollary 2.17
Every $topological\ space$ be a $semi\ pre-\ T_{1/2}-\ space.$
From this corollary and definition 2.15(1) and proposition 2.14 we can be writing the relationship between $spg-closed\ set$ as we will explain it through the following corollary:

Corollary 2.18
Let (X,T) be a $topological\ space$ and let $B\subseteq X$ then B is a $spg-closed\ set$ if and only if B is a $sp-closed\ set.$

Definition 2.19 [15]

Let (X,T) be a $topological\ space$, we say that X is a $submaximal\ space$ if every $dense\ set$ in X is an $open\ set$, we can prove that X is a $submaximal\ space$ if and only if each $sp-open\ set$ in X is an $open\ set$.

Theorem 2.20

Let (X,T) be a $topological\ space$, then X is a $submaximal\ space$ if every $dense\ set$ in X is an $open\ set$.

Proof:

Suppose that (X,T) be a $topological\ space$

And let $B \subseteq X$ such that B is a $sp-open\ set$

So $B = C \cap D$ such that D is a $denes\ set$ and C is an $open\ set$ [2.7]

Since X is a $submaximal\ space$ then D is an $open\ set$

Thus B is an $open\ set$.

Conversely

Assume that B is a $denes\ set$ in X

So $B \subseteq (X)^o$

Since B is a $denes\ set$, so $\overline{(B)} = X$

Hence $B \subseteq (\overline{B})^o$

$(\overline{B}) \subseteq \overline{((\overline{B})^o)}$

So B is a $sp-open\ set$

And by assumption B is an $open\ set$

Hence X is a $submaximal\ space$.

Proposition 2.21

Let (X,T) be a $submaximal\ space$, then every $sp-closed\ set$ in X is a $closed\ set$.

Proof:

Let $B \subseteq X$ is a $sp-closed\ set$

Then B^c is a $sp-open\ set$

And since X be a $submaximal\ space$

So B^c is an $open\ set$ [2.20]

Hence B is a $closed\ set$.

Through the corollary 2.18 we clarify the relationship between $spg-closed\ set$ and $sp-closed\ set$ and from the above proposition 2.21 we can get the following result:

Corollary 2.22

Let (X,T) be a $submaximal\ space$, then every $spg-closed\ set$ in X be a $closed\ set$.

Remark 2.23

The intersection of two of $spg-closed\ sets$ is a $spg-closed\ set$, as we will explain for the following theorem:

Theorem 2.24

Let (X,T) be a $topological\ space$, and let B,C are a $spg-closed\ set$ then $B\cap C$ be a $spg-closed\ set$ in X.

Proof:

Since B is a $spg-closed\ set$

So, for every $sp-open\ set\ V$ in X

Then if $B\subseteq V$ then $\overline{(B)}^{sp} \subseteq V$

And also, for C if $C\subseteq V$ then $\overline{(C)}^{sp} \subseteq V$

Suppose that $B\cap C \subseteq V$

We will prove that $\overline{(B\cap C)}^{sp} \subseteq V$

Since $B\cap C\subseteq V$ so $B\subseteq V\cup C^c$

So V is a $sp-open\ set$ and C^c is a $spg-open\ set$

Also which is a $sp-open\ set$ [2.18]

So $V\cup C^c$ is a $sp-open\ set$

And since B is a $spg-closed\ set$ and $B\subseteq V\cup C^c$

So $\overline{(B)}^{sp} \subseteq V\cup C^c$

And in the same way we can prove that $\overline{(C)}^{sp} \subseteq V\cup B^c$

Since $\overline{(B \cap C)}^{sp} \subseteq \overline{(B)}^{sp} \cap \overline{(C)}^{sp}$ [2.2(4)]

So $\overline{(B \cap C)}^{sp} \subseteq (V \cup C^C) \cup (V \cup B^C)$

Suppose that $x \in \overline{(B \cap C)}^{sp}$

So $x \in (V \cup C^C)$ and $x \in (V \cup B^C)$

So $x \in V$

Hence $\overline{(B \cap C)}^{sp} \subseteq V$

So $B \cap C$ is a $spg - closed\ set$.

Remark 2.25

The union of two $spg - closed\ set$ does not necessarily to be $spg - closed\ set$ for example:

Example 2.26

Let $X = \{1,2,3,4,5\}$ and let $T = \{X, \emptyset, \{1,3,4\}\}$ be a $topological$ defined on X.

And let $B = \{1,3\}$ and $C = \{3,4\}$

So, the $sp - open\ sets$ which contain B and C is: $X, \{1,3,4\}$ and also contain $\overline{(B)}^{sp}$ and $\overline{(C)}^{sp}$.

Hence each of B and C are a $spg - closed\ set$ and since $B \cup C = \{1,3,4\}$

So $\overline{(B \cup C)}^{sp} = X$

Hence $B \cup C \subseteq \{1,3,4\}$ and $\overline{(B \cup C)}^{sp} \nsubseteq \{1,3,4\}$

So $B \cup C$ is not $spg - closed\ set$.

Theorem 2.27

Let (X, T) be a $topological\ space$ and let $B \subseteq X$ is a $spg - closed\ set$ and $C \subseteq X$ is a $closed\ set$ then $B \cup C$ is a $spg - closed\ set$.

Proof:

Suppose that B is a $spg - closed\ set$ and C is a $closed\ set$ in X

We will prove that $B \cup C$ is a $closed\ set$

Let W be a $sp - open\ set$ in X such that $B \cup C \subseteq W$

We will prove that $\overline{(B \cup C)}^{sp} \subseteq W$

Suppose that $x \in \overline{(B \cup C)}^{sp}$

Since $\overline{(B \cup C)}^{sp} = (B \cup C) \cup (\overline{((B \cup C)^{o})})^{o}$ [15]

So $x \in (B \cup C) \cup (\overline{((B \cup C)^{o})})^{o}$

So $x \in (B \cup C)$

Hence $\overline{(B \cup C)}^{sp} \subseteq B \cup C$

But $B \cup C \subseteq W$ [by assumption]

So $\overline{(B \cup C)}^{sp} \subseteq W$, hence $B \cup C$ is a $spg - closed\ set$

Or $x \in (\overline{((B \cup C)^{o})})^{o}$

We will prove that $(\overline{((B \cup C)^{o})})^{o} \subseteq B \cup C$

Since B is a $spg - closed\ set$

So B is a $sp - closed\ set$

Hence $(\overline{((B)^{o})})^{o} \subseteq B$

And since C is a $closed\ set$, so $C = \overline{C}$ [16]

So $(\overline{((B)^{o})})^{o} \cup \overline{C} \subseteq B \cup C$

But $(\overline{((B \cup C)^{o})})^{o} \subseteq (\overline{((B)^{o})})^{o} \cup \overline{C} \subseteq B \cup C$

Since $x \in (\overline{((B \cup C)^{o})})^{o}$

So $x \in B \cup C$

Hence $\overline{(B \cup C)}^{sp} \subseteq B \cup C$

Since $B \cup C \subseteq W$

So $\overline{(B \cup C)}^{sp} \subseteq W$

Thus $B \cup C$ is a $spg - closet\ set.$

Proposition 2.28

Let (X, T) be a $topological\ space$, and let B is a $spg - closed\ set$ in X, if $B \subseteq C \subseteq \overline{(B)}^{sp}$ then C is a $spg - closed\ set$ in X.

Proof:

Suppose that $C \subseteq V$ such that V is a $sp - open\ set$ in X

We will prove that $\overline{(C)}^{sp} \subseteq V$

Since $C \subseteq V$ so $B \subseteq V$ [by assumption $B \subseteq C$]

And also B is a $spg - closed\ set$ in X

So $\overline{(B)}^{sp} \subseteq V$

And since $C \subseteq \overline{(B)}^{sp}$

So $\overline{(C)}^{sp} \subseteq \overline{(B)}^{sp}$ [2.4]

Hence $\overline{(C)}^{sp} \subseteq V$

So C is a $spg - closed\ set.$

Through the remark 2.25 we clarify the union of two $spg - closed\ set$ is not necessarily $spg - closed\ set.$

As for the $spg - open\ set$, this is true as we will show from the following corollary:

Corollary 2.29

Let (X, T) be a topological space, and let each of B, C are a $spg - open\ sets$ in X then $B \cup C$ is a $spg - open\ set$ in X .

proof:

let B, C are a $spg - open\ set$ in X

So B^c, C^c are a $spg - closed\ set$ in X

Hence $B^c \cap C^c$ is a $spg - closed\ set$ in X [2.24]

So $(B^c \cap C^c)^c$ is a $spg - open\ set$ in X

Hence $B \cup C$ is a $spg - open\ set$ in $X.$

Remark 2.30

The intersection of two $spg - open\ sets$ does not necessarily to be $spg - open\ set,$ for example:

Example 2.31

Let $X = \{1,2,3\}$ and let $T = \{X, \emptyset, \{2,3\}\}$ be a $topological$ defined on X

It is clear that $\{1,3\}, \{1,2\}$ is a $sp - open\ set$ in X

but $\{1,3\} \cap \{1,2\} = \{1\}$ is not $sp - open\ set$ in X.

Theorem 2.32

Let (X, T) be a $topological\ space$ and let $B \subseteq X$ is a $spg - open\ set$ and $C \subseteq X$ is an $open\ set$ then $B \cap C$ is a $spg - open\ set$.

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