

SOME NEW RESULTS FOR SOFT B-METRIC SPACE

Celalettin Kaya¹, Lina Aydeen Hussein²^{*}

¹Department of Mathematics, Faculty of Science, Çankırı Karatekin University, Çankırı 1800, TURKEY

²Department of Mathematics, College of Education, Mustansiriya university, Baghdad, IRAQ

*Corresponding Author: Lina Ayden Hussein

DOI: <https://doi.org/10.31185/wjps.315>

Received 01 January 2024; Accepted 12 February 2024; Available online 30 March 2024

ABSTRACT: The objective of this search, is to introduce some important new results for generalized type of metric space which is soft b – metric space up on non-empty soft set. Furthermore, several theorems for this concept have been submitted such as soft b – convergent sequence and soft b – Cauchy sequence in this space. Moreover, we will make a comparison of this results with theorems appear in sequence of just elements.

Keywords: Cauchy sequence, soft b – metric space, metric space, soft b – Cauchy sequence



1. INTRODUCTION

More researcher recently works in the field of fuzzy soft mathematics such as in 1999 Molodtsov, [1] discovered the concept of soft set theory was introduced at first one at that time, such that he was study some properties of this concept with shows the operations on this kind of set-in mathematics. Moreover than, in 2003 P.K.Maji, R. Biswas and A.R. Roy [6], studied the theory of soft sets initiated by Molodtsov, where they gave the relations between soft sets like equality of two soft sets, subset and introduce the concept of super set of a soft set as well as, the complement of a soft set, null soft set, and absolute soft have been given, and continue this works on soft set, such that in 2014 Onyeozili, I. A., Gwary T., [4] submitted some fundamental of set theory, with some operations on soft set in addition invested some functions among these sets and give the representations matrix of soft points, also in 2018, Shamshad Husain, Km. Shivani [7], studied the theory of soft set and introduced some properties of such as intersection, union, symmetric difference and another distinctions of soft set like or, and, and showing De Morgan's law with some difference on this sets, moreover in 2016 O. A. Tantawy, R. M. Hassan, [3], submitted the article deals with soft metric space up on soft set, where define soft metric function on it, he was encourage the other's to study soft compactness, soft continuous, ..., in soft metric space also can gave more future works. In addition, in 2017 B. R. Wadkar, R. Bhardwaj, V. N. Mishra, B. Singh, [7] discovered at first one the definition of soft b – metric space with some fundamentals of this concepts, finally in 2021 P. G. Patil and Nagashree N. Bhat, [5] introduced the sequence of soft points in soft metric space and generalized in to soft Δ -metric spaces with several theorems' bout its.

The main aim of this article, is to study deeply the soft b – metric space and introduce soft b – convergent sequence, soft b – bounded sequence, and soft b – Cauchy sequence, in order to define soft b – complete metric space, with several theorems relating on these concepts as well as generalization the proofs for theorems of numerical sequence on this subject.

2. BASIC CONCEPTS FOR SOFT b – METRIC SPACE

at first, we will recall some basic definition of softest with important fundamental operations such as its clearly difference form crisp sets

Definition 2.1 [2] Let $\mathcal{P}(\mathcal{X})$ be a collection of all subset of universal set \mathcal{X} and \mathcal{U} be a set of parameters, $\mathcal{B} \subseteq \mathcal{U}$ and \mathcal{G} is a mapping from \mathcal{B} into $\mathcal{P}(\mathcal{X})$ such that, $\mathcal{G}_{\mathcal{B}} = \{\mathcal{G}(e) \in \mathcal{P}(\mathcal{X}) : e \in \mathcal{B}\}$, is the soft set on \mathcal{X} with the set of parameters \mathcal{B} , and denoted by and $(\mathcal{G}, \mathcal{B})$ or $\mathcal{G}_{\mathcal{B}}$.

To illustrate, the definition has more applications for our lives, like the set of cars with some properties about these cars

Notations 2.2 [2] Let $(\mathcal{G}, \mathcal{U})$ be a soft set on universal set \mathcal{X} , then

- i) $(\mathcal{G}, \mathcal{U})$ which is namely empty soft set, whenever $\mathcal{U} = \emptyset$, and written as (\emptyset, \emptyset) .
- ii) $(\mathcal{G}, \mathcal{U})$ is called Null soft set, whenever $\mathcal{G}(e) = \emptyset$, for all $e \in \mathcal{U}$, with shortly written as Φ .
- iii) $(\mathcal{G}, \mathcal{U})$ is called Absolute soft set, if $\mathcal{G}(e) = \mathcal{X}$, for all $e \in \mathcal{U}$, and written as $\tilde{\mathcal{A}}$.

its important here, recalling the definition of soft point and give the compression for smallest and largest of these points

Definition 2.3 [6] the element follows as a pair $\tilde{x}_e = (e, \mathcal{G}(e))$, where $e \in \mathcal{U}$ such that $(\mathcal{G}, \mathcal{U})$ be a soft set over \mathcal{X} , is non-null soft element of $(\mathcal{G}, \mathcal{U})$. and (e, Φ) is called a null soft element of $(\mathcal{G}, \mathcal{U})$, certainly the element \tilde{x}_e is a soft element of $(\mathcal{G}, \mathcal{U})$

And two soft points $\tilde{x}_e = (e_i, \mathcal{G}(e_i))$ and $\tilde{y}_e = (e_j, \mathcal{G}(e_j))$ if $e_i = e_j$ and $\mathcal{G}(e_i) = \mathcal{G}(e_j)$ are said to be soft equal denoted by $\tilde{x}_e = \tilde{y}_e$. Otherwise, is called not soft equal denoted by $\tilde{x}_e \neq \tilde{y}_e$

Now, its important we recall the definition deals with comparison between soft real numbers

Definition 2.4 [8] let $\hat{\mathcal{A}}$ and $\hat{\mathcal{B}}$ be any two soft real numbers then one can have

- i) if $\hat{\mathcal{A}}(e) \leq \hat{\mathcal{B}}(e)$ for all $e \in \mathcal{U}$ then $\hat{\mathcal{A}} \leq \hat{\mathcal{B}}$.
- ii) if $\hat{\mathcal{A}}(e) < \hat{\mathcal{B}}(e)$ for all $e \in \mathcal{U}$ then $\hat{\mathcal{A}} < \hat{\mathcal{B}}$.
- iii) if $\hat{\mathcal{A}}(e) \geq \hat{\mathcal{B}}(e)$ for all $e \in \mathcal{U}$ then $\hat{\mathcal{A}} \geq \hat{\mathcal{B}}$.
- iv) if $\hat{\mathcal{A}}(e) > \hat{\mathcal{B}}(e)$ for all $e \in \mathcal{U}$ then $\hat{\mathcal{A}} > \hat{\mathcal{B}}$.

The following definition appear in [8], presented the concepts of soft metric space up on soft set

Definition 2.5 [8] let \mathcal{X} be a nonempty universal set and $\tilde{\mathcal{X}}$ be a soft set over \mathcal{X} with a nonempty set of parameter \mathcal{U} , then the function $\tilde{\rho} : SO(\tilde{\mathcal{X}}) \times SO(\tilde{\mathcal{X}}) \rightarrow \mathbb{R}(A)$ which is called be soft metric space on $\tilde{\mathcal{X}}$ satisfying the following soft metric conditions for all $\tilde{x}_e, \tilde{y}_e, \tilde{z}_e \in \tilde{\mathcal{X}}$

- (1) $\tilde{\rho}(\tilde{x}_e, \tilde{y}_e) \geq \tilde{0}$
- (2) $\tilde{\rho}(\tilde{x}_e, \tilde{y}_e) \equiv \tilde{0}$ if and only if \tilde{x}_e, \tilde{y}_e
- (3) $\tilde{\rho}(\tilde{x}_e, \tilde{y}_e) \equiv \tilde{\rho}(\tilde{y}_e, \tilde{x}_e)$
- (4) $\tilde{\rho}(\tilde{x}_e, \tilde{y}_e) \leq \tilde{\rho}(\tilde{x}_e, \tilde{z}_e) + \tilde{\rho}(\tilde{z}_e, \tilde{y}_e)$, them the soft set $\tilde{\mathcal{X}}$ with $\tilde{\rho}$ is said to be soft metric space with shortly $(\tilde{\mathcal{X}}, \tilde{\rho})$ or sometime $(\tilde{\mathcal{X}}, \tilde{\rho}, E)$, where $\tilde{\mathcal{X}} = (\mathcal{G}, E)$.

Now, we generalized this definition in order get another class of soft metric space which is soft b – metric space.

Definition 2.5 [1] let \mathcal{X} be a nonempty universal set and $\tilde{\mathcal{X}}$ be a soft set over \mathcal{X} with a nonempty set of parameter \mathcal{U} , then the function $\tilde{\vartheta} : SO(\tilde{\mathcal{X}}) \times SO(\tilde{\mathcal{X}}) \rightarrow \tilde{\mathcal{R}}(\mathcal{U})^*$ is said be soft metric space on $\tilde{\mathcal{X}}$ satisfying the following soft metric conditions for all $\tilde{x}_e, \tilde{y}_e, \tilde{z}_e \in \tilde{\mathcal{X}}$

- i) $\tilde{\vartheta}(\tilde{x}_e, \tilde{y}_e) \geq 0$ for all $\tilde{x}_e, \tilde{y}_e \in \tilde{\mathcal{X}}$.
- ii) $\tilde{\vartheta}(\tilde{x}_e, \tilde{y}_e) = 0$ if and only if $\tilde{x}_e = \tilde{y}_e$.
- iii) $\tilde{\vartheta}(\tilde{x}_e, \tilde{y}_e) = \tilde{\vartheta}(\tilde{y}_e, \tilde{x}_e)$ for all $\tilde{x}_e, \tilde{y}_e \in \tilde{\mathcal{X}}$.
- iv) $\tilde{\vartheta}(\tilde{x}_e, \tilde{y}_e) \leq s[\tilde{\vartheta}(\tilde{x}_e, \tilde{z}_e) + \tilde{\vartheta}(\tilde{z}_e, \tilde{y}_e)]$ for all $\tilde{x}_e, \tilde{y}_e, \tilde{z}_e \in \tilde{\mathcal{X}}$, $s \geq 1$ and $(\tilde{\mathcal{X}}, \tilde{\vartheta})$ is said to be metric space.

3. MAIN RESULTS FOR SOFT b – SEQUENCE IN SOFT b – METRIC SPACE

In this section we are showing some types of soft b – sequence in soft b – metric space, with the relation between them, at first, we give the definition of soft b –convergent sequence in soft b – metric space.

Definition 3.1 assume $(\tilde{\mathcal{X}}, \tilde{\vartheta})$ be a soft b –metric space, then the sequence $\{\tilde{x}_{e_n}\}$ of soft elements in $(\tilde{\mathcal{X}}, \tilde{\vartheta})$ which is called soft b –converge to soft point \tilde{x}_{e_0} , if for any $\tilde{\varepsilon} \succ \tilde{0}$ there is with a positive integer $N = N(\tilde{\varepsilon})$ such that $\tilde{\vartheta}(\tilde{x}_{e_n}, \tilde{x}_{e_0}) \leq \tilde{\varepsilon}$, for all $n \geq N$ Sometime written as $\lim_{n \rightarrow \infty} \tilde{x}_{e_n} = \tilde{x}_{e_0}$.

The following proposition, its generation for theorem appear in [8], and here we introduce the details of its proof.

Proposition 3.2 assumption $\{\tilde{x}_{e_n}\}$ is a sequence of soft element in soft b –metric space $(\tilde{X}, \tilde{\vartheta})$ if $\{\tilde{x}_{e_n}\}$ is a soft convergent to \tilde{x}_{e_0} and \tilde{y}_{e_0} , then $\tilde{x}_{e_0} \hat{=} \tilde{y}_{e_0}$

Proof. a assume that $\{\tilde{x}_{e_n}\}$ is convergent to $\{\tilde{x}_{e_0}\}$, then for every $\tilde{\mathcal{E}} \succ \tilde{0}$ there exists and a positive integer $N_1 = N_1(\tilde{\mathcal{E}})$ such that $\tilde{\vartheta}(\tilde{x}_{e_n}, \tilde{x}_{e_0}) \leq \frac{\tilde{\mathcal{E}}}{2s}$, for all $n, m \geq N_1$, and $\{\tilde{x}_{e_n}\}$ is convergent to $\{\tilde{y}_{e_0}\}$, then for every $\tilde{\mathcal{E}} \succ \tilde{0}$ there exists and a positive integer $N_2 = N_2(\tilde{\mathcal{E}})$ such that $\tilde{\vartheta}(\tilde{x}_{e_n}, \tilde{y}_{e_0}) \leq \frac{\tilde{\mathcal{E}}}{2s}$, for all $n, m \geq N_2$, let $N = \max\{N_1, N_2\}$, let $\tilde{\vartheta}(\tilde{x}_{e_0}, \tilde{y}_{e_0}) \leq s[\tilde{\vartheta}(\tilde{x}_{e_n}, \tilde{x}_{e_0}) + \tilde{\vartheta}(\tilde{x}_{e_n}, \tilde{y}_{e_0})] = s[\frac{\tilde{\mathcal{E}}}{2s} + \frac{\tilde{\mathcal{E}}}{2s}] = \tilde{\mathcal{E}}$, then $\tilde{\vartheta}(\tilde{x}_{e_0}, \tilde{y}_{e_0}) \leq \tilde{\mathcal{E}}$, thus; $\tilde{\vartheta}(\tilde{x}_{e_0}, \tilde{y}_{e_0}) = \tilde{0}$, since $(\tilde{X}, \tilde{\vartheta})$ is a soft metric space therefore; $\tilde{x}_{e_0} = \tilde{y}_{e_0}$.

Some properties of soft b –converge sequence of soft point we will give by the following

Definition 3.3 Let $(\tilde{X}, \tilde{\vartheta})$ be a soft b –metric space, the sequence $\{\tilde{x}_{e_n}\}$ of soft element in $(\tilde{X}, \tilde{\vartheta})$ which is called soft b –cauchy sequence, if for every $\tilde{\mathcal{E}} \succ \tilde{0}$ there is and a positive integer $N = N(\tilde{\mathcal{E}})$ with satisfy $\tilde{\vartheta}(\tilde{x}_{e_n}, \tilde{x}_{e_m}) \leq \tilde{\mathcal{E}}$, for all $n, m \geq N$.

It's known that form previous section, in a soft metric space every soft convergent sequence is soft Cauchy, this fact is also true in a soft b –metric space, but has different details of proof.

Theorem 3.4 Let $(\tilde{X}, \tilde{\vartheta})$ be a soft b –metric space, if the sequence $\{\tilde{x}_{e_n}\}$ of soft element in $(\tilde{X}, \tilde{\vartheta})$ is a soft b –converget sequence, then $\{\tilde{x}_{e_n}\}$ is a soft b –cuachy sequence.

Proof. a assume that $\{\tilde{x}_{e_n}\}$ is soft b –convergent to $\{\tilde{x}_{e_0}\}$, then for every $\tilde{\mathcal{E}} \succ \tilde{0}$ there exists and a positive integer $N = N(\tilde{\mathcal{E}})$ with satisfy $\tilde{\vartheta}(\tilde{x}_{e_n}, \tilde{x}_{e_0}) \leq \frac{\tilde{\mathcal{E}}}{2s}$, for all $n \geq N$,

Now, for all $n, m \geq N$, one can have $\tilde{\vartheta}(\tilde{x}_{e_n}, \tilde{x}_{e_m}) \leq s[\tilde{\vartheta}(\tilde{x}_{e_n}, \tilde{x}_{e_0}) + \tilde{\vartheta}(\tilde{x}_{e_m}, \tilde{x}_{e_0})] \leq s[\frac{\tilde{\mathcal{E}}}{2s} + \frac{\tilde{\mathcal{E}}}{2s}] = \tilde{\mathcal{E}}$, therefore; $\{\tilde{x}_{e_n}\}$ is soft b –cauchy sequence in

We will know in soft metric space (soft b – metric space) the converse of a bove theorem it's not true in general, the following theorem gives the grantee condition to make the converse is true

Theorem 3.5 Let $(\tilde{X}, \tilde{\vartheta})$ be a soft b –metric space, if the sequence $\{\tilde{x}_{e_n}\}$ of soft element in $(\tilde{X}, \tilde{\vartheta})$ is a soft b –cauchy sequence and has b-convergent subsequence $\{\tilde{x}_{e_{nk}}\}$ then $\{\tilde{x}_{e_n}\}$ is a soft b –convergent sequence in $(\tilde{X}, \tilde{\vartheta})$.

Proof. a assume that $\{\tilde{x}_{e_n}\}$ is soft b –cauchy sequence in $(\tilde{X}, \tilde{\vartheta})$, then for every $\tilde{\mathcal{E}} \succ \tilde{0}$ there exists and a positive integer $N_1 = N_1(\tilde{\mathcal{E}})$ such that $\tilde{\vartheta}(\tilde{x}_{e_n}, \tilde{x}_{e_m}) \leq \frac{\tilde{\mathcal{E}}}{2s}$, for all $n, m \geq N_1$ also $\{\tilde{x}_{e_n}\}$ has a soft b – convergent subsequence $\{\tilde{x}_{e_{nk}}\}$ to \tilde{x}_{e_0} , there exists and a positive integer $N_2 = N_2(\tilde{\mathcal{E}})$, such that $\tilde{\vartheta}(\tilde{x}_{e_n}, \tilde{x}_{e_m}) \leq \frac{\tilde{\mathcal{E}}}{2s}$, for all $n \geq N_2$, now choose $N = \max\{N_1, N_2\}$, then for all $n \geq N$, one can have $\tilde{\vartheta}(\tilde{x}_{e_n}, \tilde{x}_{e_0}) \leq s[\tilde{\vartheta}(\tilde{x}_{e_n}, \tilde{x}_{e_{nk}}) + \tilde{\vartheta}(\tilde{x}_{e_n}, \tilde{x}_{e_0})] \leq \frac{\tilde{\mathcal{E}}}{2s} + \frac{\tilde{\mathcal{E}}}{2s} = \tilde{\mathcal{E}}$, therefore; $\{\tilde{x}_{e_n}\}$ is soft b – convergent sequence in $(\tilde{X}, \tilde{\vartheta})$.

Other kinds of soft b –sequence of soft elements has been given in the following defintion.

Definition 3.6 Let $(\tilde{X}, \tilde{\vartheta})$ be a soft b –metric space, then the sequence $\{\tilde{x}_{e_n}\}$ of soft element in $(\tilde{X}, \tilde{\vartheta})$ is said to be soft b –bounde sequence, if there exist $\tilde{M} \succ \tilde{0}$ and \tilde{x}_{e_0} such that $\tilde{\vartheta}(\tilde{x}_{e_n}, \tilde{x}_{e_0}) \leq \tilde{\mathcal{E}}$, for all $n \in N$.

Now, the following theorem gives the relation between soft b –Cauchy sequence and soft b –bondded sequence in soft b – metric space.

Theorem 3.7 let $\{\tilde{x}_{e_n}\}$ be soft b –Cauchy sequence in $(\tilde{X}, \tilde{\vartheta})$ soft b –metric space, then $\{\tilde{x}_{e_n}\}$ be soft b –bounded sequence.

Proof choosing $\tilde{\mathcal{E}} = 1$, and since $\{\tilde{x}_{e_n}\}$ is a soft b –Cauchy sequence then there exists a positive integer $N = N(\tilde{\mathcal{E}})$ with satisfy $\tilde{\vartheta}(\tilde{x}_{e_n}, \tilde{x}_{e_m}) \leq \tilde{1}$, for all $n, m \geq N$,

Also, $\tilde{\vartheta}(\tilde{x}_{e_n}, \tilde{x}_{e_0}) \leq s[\tilde{\vartheta}(\tilde{x}_{e_n}, \tilde{x}_{e_m}) + \tilde{\vartheta}(\tilde{x}_{e_m}, \tilde{x}_{e_0})]$
 $\leq s[\tilde{1} + \tilde{\vartheta}(\tilde{x}_{e_m}, \tilde{x}_{e_0})]$
 $\leq s[\tilde{1} + \tilde{\vartheta}(\tilde{x}_{e_{N+1}}, \tilde{x}_{e_0})]$, putting $m = N + 1$

Let $M = \max\{\tilde{\vartheta}(\tilde{x}_{e_1}, \tilde{x}_{e_0}), \tilde{\vartheta}(\tilde{x}_{e_2}, \tilde{x}_{e_0}), \dots, \tilde{\vartheta}(\tilde{x}_{e_N}, \tilde{x}_{e_0}), \tilde{1} + \tilde{\vartheta}(\tilde{x}_{e_{N+1}}, \tilde{x}_{e_0})\}$

Then for all $n \in N$, one can have $\tilde{\vartheta}(\tilde{x}_{e_n}, \tilde{x}_{e_0}) \geq s\tilde{M}$, so the $\{\tilde{x}_{e_n}\}$ is a soft b –bounded sequence
The following theorem gives more important properties of soft b –Cauchy sequence in $(\tilde{X}, \tilde{\vartheta})$

Theorem 3.8 let $\{\tilde{x}_{e_n}\}$ be a sequence of soft elements in soft b –metric space $(\tilde{X}, \tilde{\vartheta})$ soft b –metric space, with condition $\lim_{n \rightarrow \infty} \tilde{\vartheta}(\tilde{x}_{e_n}, \tilde{x}_{e_{n+1}}) = 0$, if $\{\tilde{x}_{e_n}\}$ is not b –Cauchy sequence then for all $\tilde{\varepsilon} \succ \hat{0}$ there exist two sequence of soft elements $\{\tilde{x}_{e_{nk}}\}$ and $\{\tilde{x}_{e_{mk}}\}$, such that

$$\begin{aligned}\tilde{\varepsilon} &\leq \limsup_{k \rightarrow \infty} \tilde{\vartheta}(\tilde{x}_{e_{mk}}, \tilde{x}_{e_{nk}}) \leq s\tilde{\varepsilon} \\ \frac{\tilde{\varepsilon}}{s} &\leq \limsup_{k \rightarrow \infty} \tilde{\vartheta}(\tilde{x}_{e_{mk}}, \tilde{x}_{e_{nk+1}}) \leq s^2\tilde{\varepsilon} \\ \frac{\tilde{\varepsilon}}{s} &\leq \limsup_{k \rightarrow \infty} \tilde{\vartheta}(\tilde{x}_{e_{mk+1}}, \tilde{x}_{e_{nk}}) \leq s^2\tilde{\varepsilon} \\ \frac{\tilde{\varepsilon}}{s^2} &\leq \limsup_{k \rightarrow \infty} \tilde{\vartheta}(\tilde{x}_{e_{mk+1}}, \tilde{x}_{e_{nk+1}}) \leq s^3\tilde{\varepsilon}\end{aligned}$$

Proof. when $\{\tilde{x}_{e_n}\}$ is not a Cauchy sequence, then there is $\tilde{\varepsilon} \geq \hat{0}$, and sequences $\{x_{mk}\}$ and $\{x_{nk}\}$ of positive integers such that $\{x_{mk}\} \succ \{x_{nk}\} > k$, has one

$$\begin{aligned}\tilde{\vartheta}(\tilde{x}_{e_{mk}}, \tilde{x}_{e_{nk-1}}) &\prec \tilde{\varepsilon} \\ \tilde{\vartheta}(\tilde{x}_{e_{mk}}, \tilde{x}_{e_{nk}}) &\geq \tilde{\varepsilon},\end{aligned}$$

Thus, we can have the following inequality

$$\tilde{\varepsilon} \leq \tilde{\vartheta}(\tilde{x}_{e_{mk}}, \tilde{x}_{e_{nk}}) \leq s[\tilde{\vartheta}(\tilde{x}_{e_{mk}}, \tilde{x}_{e_{nk-1}}) + \tilde{\vartheta}(\tilde{x}_{e_{nk-1}}, \tilde{x}_{e_{nk}})] \prec s\tilde{\varepsilon} + s\tilde{\vartheta}(\tilde{x}_{e_{nk-1}}, \tilde{x}_{e_{nk}})$$

Taking the $\limsup_{n \rightarrow \infty}$ one can have the following inequality

$$\tilde{\varepsilon} \leq \limsup_{n \rightarrow \infty} \tilde{\vartheta}(\tilde{x}_{e_{mk}}, \tilde{x}_{e_{nk}}) \prec s\tilde{\varepsilon}$$

And since $(\tilde{X}, \tilde{\vartheta})$ is a soft b – metric space, one can get, the following condition of soft b – metric space.

$$\tilde{\vartheta}(\tilde{x}_{e_{mk}}, \tilde{x}_{e_{nk}}) \geq s[\tilde{\vartheta}(\tilde{x}_{e_{mk}}, \tilde{x}_{e_{nk+1}}) + \tilde{\vartheta}(\tilde{x}_{e_{nk+1}}, \tilde{x}_{e_{nk}})]$$

Taking the $\limsup_{n \rightarrow \infty}$ one can have the following inequality

$$\frac{\tilde{\varepsilon}}{s} \leq \limsup_{n \rightarrow \infty} \tilde{\vartheta}(\tilde{x}_{e_{mk}}, \tilde{x}_{e_{nk+1}})$$

Otherwise, has one

$$\tilde{\vartheta}(\tilde{x}_{e_{mk}}, \tilde{x}_{e_{nk+1}}) \prec s[\tilde{\vartheta}(\tilde{x}_{e_{mk}}, \tilde{x}_{e_{nk}}) + \tilde{\vartheta}(\tilde{x}_{e_{nk+1}}, \tilde{x}_{e_{nk}})]$$

Taking the $\limsup_{n \rightarrow \infty}$ one can have the following inequality

$$\limsup_{n \rightarrow \infty} \tilde{\vartheta}(\tilde{x}_{e_{mk}}, \tilde{x}_{e_{nk+1}}) \prec s^2\tilde{\varepsilon}$$

Then $\limsup_{n \rightarrow \infty} \tilde{\vartheta}(\tilde{x}_{e_{mk}}, \tilde{x}_{e_{nk+1}}) \geq s^2\tilde{\varepsilon}$, We can use this outline one can have the other conditions

REFERENCES

- [1] B. R. Wadkar, R. Bhardwaj (2017), "Coupled Soft Fixed-Point Theorems in Soft Metric and Soft b- Metric Space" ser. A: Appl. Math. Inform. And Mech. Vol. 9, No.1, 59-73.
- [2] D. Molodtsov, (1999), "Soft set theory-First results" Comput. Math. Appl. Vol. 37, 19-31.
- [3] O. A. Tantawy, R. M. Hassan, (2016), " On Soft Metric Spaces" Journal of advance in mathematics, Vol. 12, No. 3, 6103-6110.
- [4] Onyeozili, I. A., Gwary, (2014), "A Study of the Fundamentals of Soft Set Theory" international journal of science and technology research, Vol. 3, No. 4, 132-143.
- [5] P. G. Patil and Nagashree N. Bhat, (2021), "sequence of soft points in soft Δ -metric spaces" Advances in Mathematics Scientific Journal Vol.10, No.3, 1131–1136
- [6] P.K.Maji, R. Biswas and A.R. Roy, (2003), " Soft set theory", Comput. Math. Appl., Vol. 45, 555-562.

- [7] Shamshad Husain, Km. Shivani, (2018), " A study of properties of soft set and its applications" International Research Journal of Engineering and Technology (IRJET), Vol. 05, No. 01, 363-372.
- [8] Suyoj Das, (2013), "on soft metric space " the journal of fuzzy mathematics, Vol. 21, No3 707-732.