Mathematical Model of HYFX Branching type

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Abstract: - In this research, we studied a case for fungal growth when mixing four types of them, study these types consume all the energy. Mathematical models are used as partial differential equations that explain the biological phenomena of each species. It will take some time to get an approximate solution to this system and this is a fact of fungal growth. The solution of this system depends on the numerical solution and this solution gives an approximate solution Therefore, in this solution we need some static steps such as non-dimensionlision, stablility, travlling wave solution and numerical solution. We used some codes (pplane7, Pdepe) in numerical analysis because of the difficulty in the direct mathematical solution, and therefore from all this we get a set of results and conclusions, which are direct and inverse relationships with the growth rate of fungi.

Keywords: - Lateral Branching, Tip death due to overcrowding, Tip-Hypha Anastomosis, Dichotomous branching

1. Introduction

There are many papers of mathematical models that have been proposed by many researchers to explain the mathematical model, for example:

- In (2011) Shuaa [1], it was studied to develop a fungal growth model that can be used to create a source term in a single root model to calculate the absorption of nutrients by fungi. Therefore, focus on loss or death.

- In (2012) Brian Ingalls [2], provided an introduction to the mathematical concepts and techniques needed to construct and interpret models in the biology of Molecular Systems.

-In (2013) Walter [3], studied the independent sections that explain more Important principles of mathematical modeling, a variety of applications, classical models ...

- In (2014) Muzaffar [4], he proposed various modeling procedures, with particular emphasis on their ability to reproduce the biological system and predict measured carried out, highlighting their specific features.

In this paper we have developed new models for the growth of fungi. Partial differential equations reflecting the interaction of biomass with the base substrate are the best option at this scale. Such models have a complex mathematical structure and, as a result, their analytical and numerical features are complex, which is why a set of any number of species can be expressed during the growth stages of a particular fungus. To facilitate the discussion of these species, short codes are used for each species, and several biological species are also illustrated where each species has been mathematically analyzed and given an explanation and description of the parameters [5-10].

2. Mathematical Model

We will study a new type of branching of fungal branching with Tip death due to overcrowding with Tip-Hypha anastomosis with Dichotomous branching with Lateral Branching (HYFX) The table below shows these kinds [1,7]

Biological type	$\sigma(p,n)$	Symbol	Parameters Description
Tip-Hypha Anastomosis	$\delta = -\beta_2 np$	H	β_2 is the rate of tip reconnections per unit length hypha per unit time
Dichotomous branching	$\delta = \alpha_1 n$	Y	α_1 is the number of tips produced per tip per unit time
Lateral Branching	$\delta = \alpha_2 p$	F	α_2 is the number of branches produced per unit length hypha per unit time
Tip death due to overcrowding	$\delta = -\beta_3 p^2$	X	β_3 is the rate at which overcrowding density limitation) eliminates branching

Table 1: Illustrate Biological type, symbol of this type, version and parameters description.

The model system for HYFX is

$$\frac{\partial p}{\partial t} = nv - \gamma p$$

$$\frac{\partial n}{\partial t} = -\frac{\partial (nv)}{\partial t} - \beta_2 np + \alpha_1 n + \alpha_2 p - \beta_3 p^2 \qquad (1)$$

To solve above system as:

2.1 Non- dimensionalities

Let
τ : Time
$\bar{x}$: length scale
$\bar{p}$: hyphal density
$\bar{n}$: tip density
$$p = p^*\bar{p}\ ,\quad n = n^*\bar{n}\quad ,t = t^*\tau\quad and\ x = x^*\bar{x}$$
When put these in (1) we get

$$\frac{\partial p^*}{\partial t^*} = n^* - p^*$$
$$\frac{\partial n^*}{\partial t^*} = -\frac{\partial n^*}{\partial x^*} - \frac{\beta_3\bar{n}v}{\gamma^2}n^*p^* + \frac{\alpha_1}{\gamma}n^* + \frac{\alpha_2 v}{\gamma^2} - \frac{\beta_3 v\bar{p}}{\gamma^2}{p^*}^2 \qquad (2)$$

When put $\bar{x} = \tau v\ and\ \bar{x} = \frac{\bar{p}}{\bar{n}}$ then the system (2) becomes:

$$\frac{\partial p}{\partial t} = n - p$$
$$\frac{\partial n}{\partial t} = -\frac{\partial n}{\partial x} - \alpha n(p-1) + \beta p(1-p) \qquad (3)$$

Where $\alpha = \frac{\alpha_1}{\gamma}$ $and\ \beta = \frac{\alpha_2 v}{\gamma^2}$
To solve above system as stability solution we will show or study the stability of system (3)

2.2 The stability of solution

$$n - p = 0 \qquad \rightarrow \qquad f(p,n)$$
$$-\alpha n(p-1) + \beta p(1-p) = \qquad \rightarrow g(p,n) \qquad (4)$$

The solution of the system (4) we will find the values of (p, n) the steady state is :(0,0), (1,1), we can take Jacobian for these system
$$J(p,n) = \begin{bmatrix} -1 & 1 \\ -\alpha + \beta - 2\beta p & -\alpha n + \alpha \end{bmatrix}$$
Now, determent the eigenvalues as λ_i: $i = 1,2$
In this case we get if $\beta < \alpha$ the point (1,1) stable spiral, and the point (0,0) saddle point see Fig. (1). Using (MATLAB pplane7)

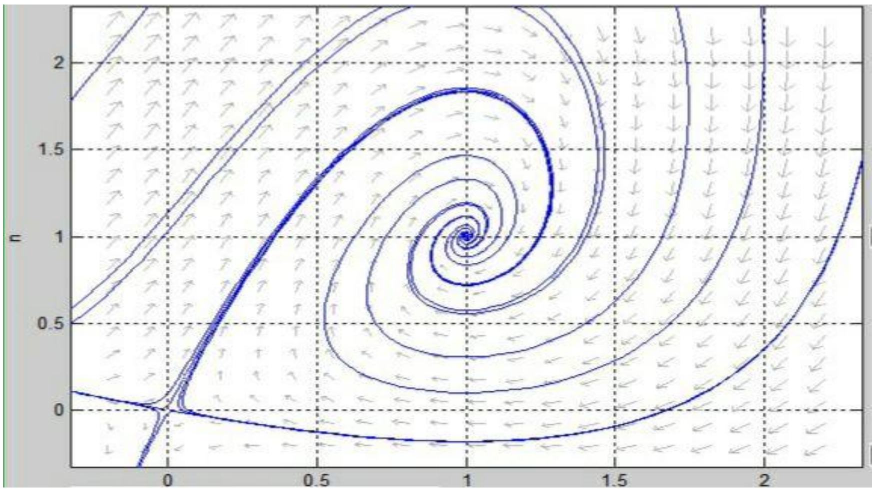

Figure (1): The (p,n) plane note that a trajectory connects in the point (1,1) is the stable spiral and in the point (0,0) saddle point.

2.3 Traveling wave solution

In this part, we will talk about the travelling wave solution, let z = x-ct, and we impose n(x,t)=N(z) and p(x,t)=P(z)

Where P(z) indicate the density profiles, and (c) rate of propagation of colony. N(z) and P(z) positive function for (z) The function N(x,t), p(x,t) are traveling and are moves at constant speed wave c in positive x − direction, where c > 1 and α =2, $\beta = 1$. We appearance the traveling wave solution of the system in t and t in the form [11-16]

$\frac{\partial p}{\partial t} = -c\frac{\partial P}{\partial z}$, $\frac{\partial n}{\partial t} = -c\frac{\partial N}{\partial z}$, $\frac{\partial n}{\partial t} = \frac{\partial N}{\partial z}$ therefore, the above system (3) becomes:

$$\frac{\partial P}{\partial z} = \frac{-1}{c}[N - P]$$

$$\frac{\partial N}{\partial z} = \frac{1}{1-c}[-\alpha n(p-1) + \beta p(1-p)], c \neq 0. -\infty < z < \infty \qquad (5)$$

We notice the steady states of the system (5) we get the point (p,n)=(0,0) is saddle point and (1,1) Unstable spiral for c> 1, by using **(MATLAB 2014)** see Figure (2)

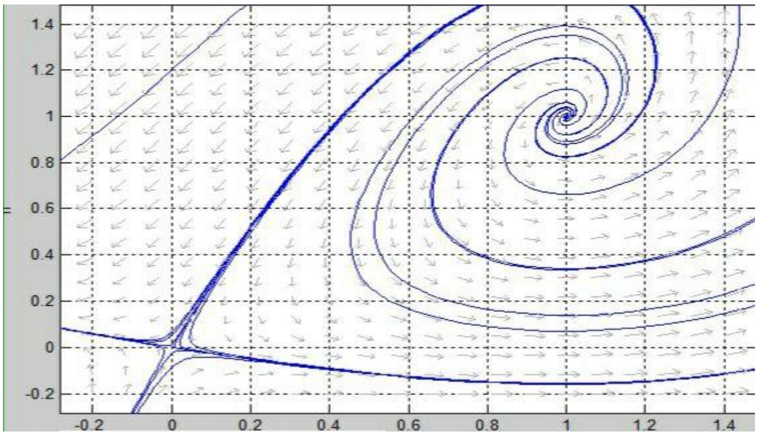

Figure 2: The (p,n) we note that trajectories connect when c= 1.5 ,$\alpha = 2, \beta = 1$ the saddle point (0,0) and unstable spiral (1,1)

2.4 Numerical Solution

To show the system (3**).** we will be using pdepe code in Matlab to show behavior branch and tip.

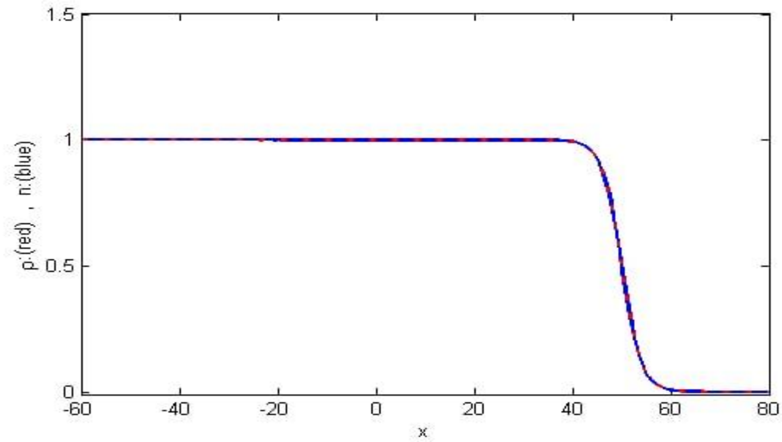

Figure (3): The initial condition of solution to the system (3) with the parameters $\alpha = 0.5 , \beta = 0.4$

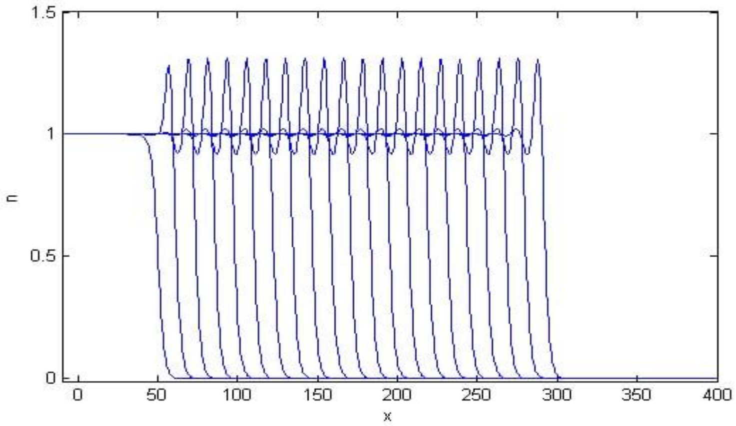

Figure 4 :solution the system (3) with the parameters $\alpha = 0.5$, $\beta = 0.4$ and c=2.5 for time t=1,10,20,………300 where the blue line represented tips (n).

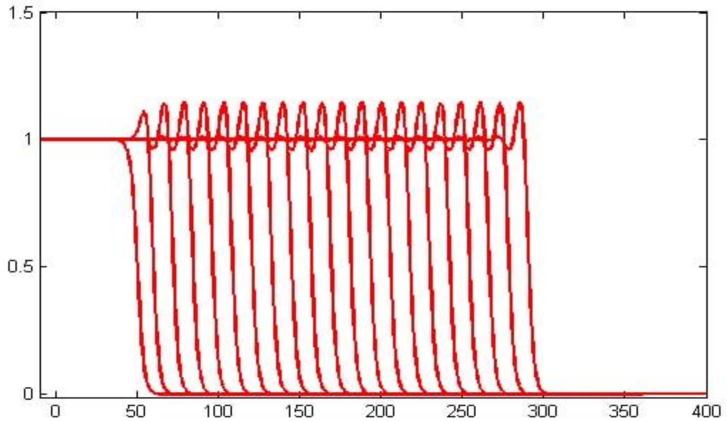

Figure 5: Solution the system (3) with the parameters $\alpha = 0.5$, $\beta = 0.4$ and c= 2.5016 for time t=1,10, 20,……300 where the red line represented branches (p).

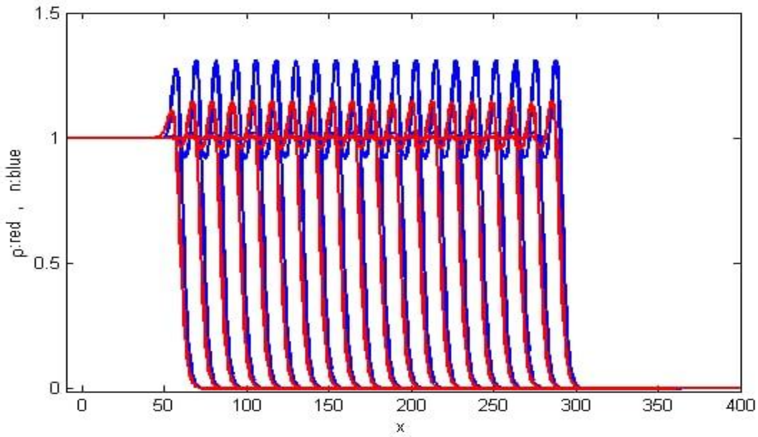

Figure 6: Solution the system (3) with the parameters $\alpha = 0.5$, $\beta = 0.4$ and c=2.5016 for time t=1,10,20,………300 where the blue line represented tips (n) with the red line represented branches(p).

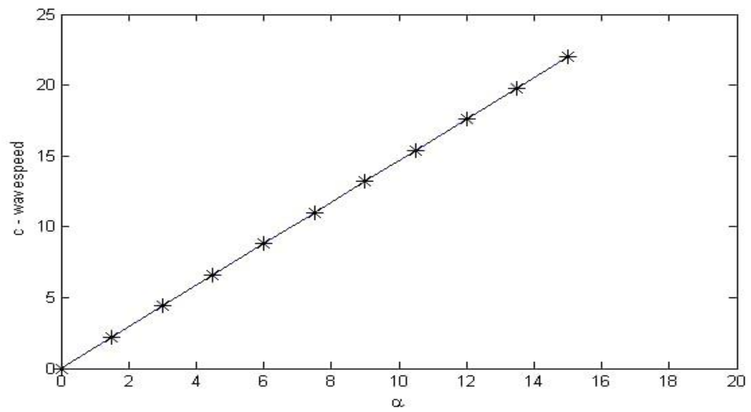

Figure 7: The relation between waves speed c and α values

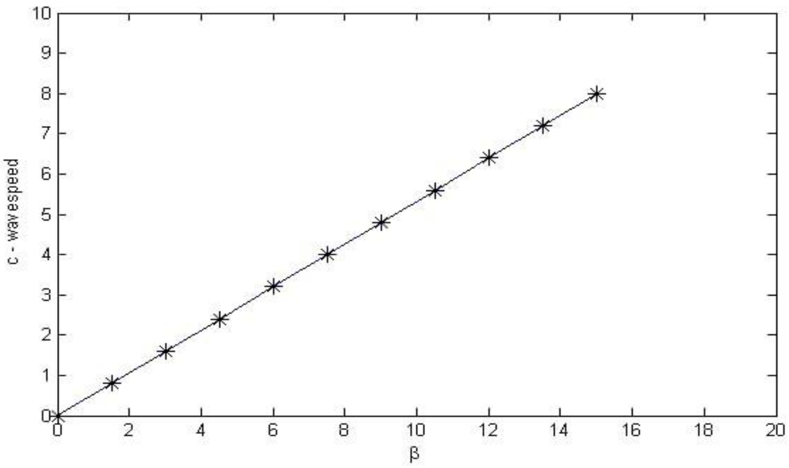

Figure 8: The relation between waves speed c and β values

3. Discussion the Results

From above results, Fig. (7) and Fig. (8) we conclude that the travelling wave c increase whenever the values of α increase at same time β is still constant. So, we know the value of $\alpha = \frac{\alpha_1}{\gamma}$ and we not that α directly proportional with α_1 and v and inversely proportional with γ .Biologically, that is mean the growth increases whenever α increases and finally that means the growth increases according to $\alpha_1$increasing (branches produced per unit length hypha per unit time). From Fig (8) we show that the travelling wave increase whenever the value of β increase at same time α is still constant. So, the value of $\beta = \frac{\alpha_2 v}{\gamma^2}$ and we note that β directly proportional with α_2 and v and inversely proportional with γ^2. Biologically, that is mean the growth increases whenever β increases. Finally, that means the growth decreases according to α_1 increasing (branches produced per unit length hypha per unit time) v increasing γ^2 decreasing.

4. References

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