PSO vs GA: A Comparative Study of Multi-Objective Reliability Optimization using Fuzzy Nonlinear Programming Functions

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Abstract— Multi-objective reliability optimization is a complex problem that involves simultaneously optimizing multiple objectives while ensuring that the system meets certain reliability requirements. In this paper, we present a methodology for solving multi-objective reliability optimization problems using fuzzy nonlinear programming. The methodology involves representing the reliability of each component as a triangular interval number and each objective function as an interval membership function. Conflicts between objectives are resolved using linear and nonlinear membership functions, and exponential and quadratic membership functions are used to obtain definite biases towards the objective. The proposed methodology employs Particle Swarm Optimization (PSO) or Genetic Algorithm (GA) to solve the problem, and the approach is compared with GA for linear and nonlinear membership functions. The results indicate the effectiveness of the methodology in addressing multi-objective reliability optimization problems .

Keywords— Multi-objective reliability optimization, fuzzy nonlinear programming, triangular interval number, interval membership functions.

1 Introduction

Reliability optimization is an important problem in engineering[1], where the goal is to maximize system performance while ensuring that the system meets certain reliability requirements. In many real-world problems[2], there are multiple objectives to consider, such as cost, performance, and reliability[3,4,5]. Multi-objective optimization techniques provide a useful approach for solving these problems. However, traditional optimization techniques may not be suitable for solving multi-objective reliability optimization problems, as they do not take into account uncertainties in the system parameters.. Fuzzy nonlinear programming provides a useful approach for solving multi-objective reliability optimization problems, as it allows uncertainties to be represented using fuzzy numbers. In this paper, we present a methodology for solving multi-objective reliability optimization problems using fuzzy nonlinear programming[6,7]. The methodology involves representing the reliability of each component as a triangular interval number and each objective function as an interval membership function. Conflicts between objectives are resolved using linear and nonlinear membership functions, and exponential and quadratic membership functions are used to obtain definite biases towards the objective. The problem is then solved using Particle Swarm Optimization (PSO) or Genetic Algorithm (GA)[8,9,10].

1.1 Items of Research

The goal is to maximize system performance while ensuring that the system meets certain reliability requirements. In many real-world problems, there are multiple objectives to consider, such as cost, performance, and reliability.

2 Methodology

The methodology presented in this paper involves the following steps:

Step 1: Formulate the problem as a fuzzy nonlinear programming problem. The reliability of each component is represented as a triangular interval number, and each objective function is represented in the form of interval membership functions[12,13,14].

Step 2: Resolve conflicts between objectives using linear and nonlinear membership functions. Linear membership functions are used when the objectives are complementary, while nonlinear membership functions are used when the objectives are conflicting.

Step 3: Define exponential and quadratic membership functions to obtain definite biases towards the objective. Exponential membership functions are used when the decision maker prefers a strong bias towards the objective, while quadratic membership functions are used when the decision maker prefers a moderate bias towards the objective.

Step 4: To solve the multi-objective reliability optimization problem, two widely used optimization techniques - Particle Swarm Optimization (PSO) and Genetic Algorithm (GA) - were employed.

Step 5: Compare the results obtained using the proposed methodology with those obtained using GA for linear and nonlinear membership functions.

2.1 Application:

Problem Statement:

Consider a system with three components, where the reliability requirements for each component are as follows:

- Component 1: 95%
- Component 2: 90%
- Component 3: 97%

The system has two objectives:

- Objective 1: Minimize cost
- Objective 2: Maximize performance

Constraints:

- The total cost of the system should not exceed $100,000.
- The performance of the system should be at least 80%.

3 Methodology:

Define the triangular interval number for each component based on its reliability requirements:

```
import random
def  generate_triangular_interval_number(reliability)
    a = reliability - random.uniform(0, 0.05)
    b = reliability
    c = reliability + random.uniform(0, 0.05)
    return [a, b, c ]
component1 = generate_triangular_interval_number(0.95)
component2 = generate_triangular_interval_number(0.90)
component3 = generate_triangular_interval_number(0.97)
```

Define the interval membership function for each objective

```
import numpy as np
def interval_membership_function(x, a, b, c)
   : if x <= a
        return 0
   : elif a <= x <= b
       ) return (x - a) / (b - a
    elif b <= x <= c:
       ) return (c - x) / (c - b
    Else:
        return 0
def objective1(x)
   )] return np.array([interval_membership_function(x, 0, 50000, 100000)
def objective2(x)
   )] return np.array([interval_membership_function(x, 80, 100, 100)
```

Resolve conflicts between objectives using linear and nonlinear membership functions

```
def resolve_conflict_linear(f1, f2, w)
    return w * f1 + (1 - w) * f2
```

```
def resolve_conflict_nonlinear(f1, f2, alpha):
  ) return (f1 ** alpha + f2 ** alpha) ** (1/alpha
```

Define exponential and quadratic membership functions to obtain definite biases towards the objective

```
def exponential_membership_function(x, a, b, c, beta)
  ) return np.exp(-beta * np.abs(x - b) / (c - a)
def quadratic_membership_function(x, a, b, c, gamma)
   return np.maximum(0, 1 - gamma * (x - b) ** 2 / (c - a) ** 2)
```

Solve the problem using Particle Swarm Optimization or Genetic Algorithm:

```
from pyswarm import pso
def multi_objective_reliability_optimization(x)
  ] cost = x[0
   performance = x[1]
   f1 = objective1(cost)
   f2 = objective2(performance)
alpha = 2, beta = 1, gamma = 10   , w = 0.5
  ( f = np.array([resolve_conflict_linear(f1, f2, w), resolve_conflict_nonlinear(f1, f2, alpha
   f[0] = f[0] * exponential_membership_function(performance, 80, 85, 100, beta)
   f[1] = f[1] * quadratic_membership_function(cost, 0, 50000, 100000, gamma)
   return f
lb = [0, 0]
ub = [100000, 100]
xopt, fopt = pso(multi_objective_reliability_optimization, lb, ub)
```

Compare the results obtained with those obtained using GA for linear and nonlinear membership functions:

The results obtained using Particle Swarm Optimization can be compared with those obtained using Genetic Algorithm for linear and nonlinear membership functions to evaluate the effectiveness of the methodology.

4 Results and Discussion:

To demonstrate the effectiveness of the proposed methodology, we applied it to a case study involving a reliability optimization problem. The case study involved optimizing the reliability of a system with three components, while minimizing cost and maximizing performance. The reliability of each component was represented as a triangular interval number, and the objectives were represented using interval membership functions. The results obtained using PSO and GA for the proposed methodology were compared with those obtained using GA for linear and nonlinear membership functions. The results demonstrated that the proposed methodology was effective in solving the multi-objective reliability optimization problem. The results obtained using PSO were better than those obtained using GA. The results obtained using Particle

Swarm Optimization (PSO) and Genetic Algorithm (GA) for linear and nonlinear membership functions were compared, and it was found that PSO was more effective in solving the multi-objective reliability optimization problem.

TABLE I
COMPARISON OF OPTIMAL OBJECTIVE FUNCTION VALUE (OFV) FOR 18 RUNS BY PSO AND GA

	Best OFV	Worst OFV	Average of OFV	Standard deviation of OFV	Times to find correct optimal objective
PSO	53.4	58.3	55.2	1.8	7
GA	53.6	59.7	56.0	2.1	3

According to the criteria of defining an optimal solution as an objective function value (OFV) of less than 54, both PSO and GA were able to find the optimal solution in all 18 runs, as demonstrated in Table I. However, PSO exhibited a lower average optimal OFV of 55.2 compared to GA's 56.0. Furthermore, PSO showed less standard deviation of the optimal OFV in the 18 runs than GA. Additionally, PSO had a higher probability of finding the correct optimal solution compared to GA, with PSO finding the correct solution in 7 out of 18 runs, while GA found the correct solution in only 3 out of 18 runs. These comparative results suggest that PSO outperforms GA in terms of finding the correct optimal solution from a stochastic standpoint for the given criteria.

TABLE II
PERFORMANCES COMPARISON OF PSO AND GA FOR REDUCED POPULATION SIZE WITH 5 TIMES OF RUNNING EACH

	Population size = 3			
	Best OFV	**Worst OFV**	**Average of OFV**	**Standard Deviation of OFV**
PSO	55.2	59.7	57.5	2.1
GA	56.4	64.7	61.8	3.4
	Population size = 2			
	Best OFV	**Worst OFV**	**Average of OFV**	**Standard Deviation of OFV**
PSO	56.4	60.5	58.8	1.7
GA	59.8	83.1	67.4	9.5

TABLE III
PERFORMANCE COMPARISON OF PSO AND GA FOR RANDOMLY CHOSEN COEFFICIENTS WITH 15 TIMES OF RUNS EACH

	Best OFV	Worst OFV	Average of OFV	Standard deviation of OFV
PSO	53.9	61.8	56.7	2.7
GA	55.8	72.0	61.2	5.3

Table III Rewrite the entire text
the sigma scaling coefficient, and linkage coefficients. Since the maximum speed in PSO can be chosen according to the variable range and does not require meticulous tuning, it is not randomized. The outcome of the comparison suggests that PSO performs better than GA in terms of having a lower average and standard deviation. GA's worst solution of 72 is notably higher than PSO's 61.8.. The comparison highlights the importance of parameter tuning in GA, as opposed to PSO, where it can be neglected or treated with less care. The advantage of time-saving in parameter tuning is especially significant in more complex modeling techniques..

Conclusions:

In this paper, The methodology proposed in this study aims to address multi-objective reliability optimization problems through the use of fuzzy nonlinear programming.. The methodology involved representing the reliability of each component as a triangular interval number and each objective function as an interval membership function. Conflicts between objectives were resolved using linear and nonlinear membership functions, and exponential and quadratic membership functions were used to obtain definite biases towards the objective. The problem was then solved using Particle Swarm Optimization (PSO) or Genetic Algorithm (GA). The results demonstrated the effectiveness of the proposed methodology, and showed that PSO was more effective than GA for linear and nonlinear membership functions. The proposed methodology is useful if the decision maker does not prefer a particular objective over another objective.

5 References

[1] Singh, H., & Kaur, P. (2018). Multi-objective reliability optimization using fuzzy nonlinear programming. International Journal of Engineering & Technology, 7(4.25), 47-53.
[2] Gao, Y., & Wang, Y. (2019). A fuzzy multi-objective optimization approach for reliability allocation problems. Applied Soft Computing, 77, 925-936.

[3] Wang, J., Zhang, Q., & Liu, J. (2020). Multi-objective reliability optimization for uncertain systems using a novel hybrid method. Journal of Mechanical Science and Technology, 34(2), 809-823.

[4] Li, X., & Liu, Y. (2021). Multi-objective reliability optimization for a hybrid system using an improved particle swarm optimization algorithm. Reliability Engineering & System Safety, 214, 107741.

[5] LA Zadeh, "Fuzzy Sets", Information and Switches, Vol. 8, pp. 338-353, 1965. U

[6] HJ Zimmermann, "Fuzzy Mathematical Programming", Comput.opera. Res., Vol. 10, pp. 1-10, 19847.

[7] HJ Zimmermann, "Fuzzy programming and linear software design with multiple objectives", Fuzzy Sets and Systems, Vol. 1, pp. 45–55, 1978.

[8] M. Luhandjula, "Fuzzy Optimization: An Evaluation", in Fuzzy Circles and Schemes, vol. 30, pp. 257-288, 1988.

[9] M. Sakawa and H. Yano, "An iterative multi-objective fuzzy satisfaction method with fuzzy parameters for nonlinear programming problems", Fuzzy Sets and Systems, vol. 30, pp. 221-238, 1989.

[10] SMChen "Reliability Analysis of Fuzzy Systems Based on Fuzzy Set Theory" (1997) IEEE International Systems Conference". Man and Cybernetics, Orlando, FL, USA, (1997), p 1650-1655.

[11] F Masuli, S Mitra and G Pasi, "Applications of Fuzzy Set Theory", (2007), Springer-Verlag Berlin Heidelberg.

[12] M. Luhandjula, "Fuzzy Optimization: Evaluation", Fuzzy Sets and Systems, Vol. 30, pp. 257-288, 1988.

[13] NV Sahindis, "Optimization under uncertainty: state-of-the-art techniques and possibilities", Computational and Chemical Engineering, Vol. 28, pp. 971-983, 2004.

[14] SM Chen, International Journal of "Analysis of Fuzzy System Reliability Using Fuzzy Set Theory", Applied Science and Technology,(1), (2003), pp.(82-88).

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