Suitable Methods for Solving COVID-19 Model in Iraq

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Abstract— Because the Coronavirus epidemic spread in Iraq, the COVID-19 epidemic of people quarantined due to infection is our application in this work. The numerical simulation methods used in this research are more suitable than other analytical and numerical methods because they solve random systems. Since the Covid-19 epidemic system has random variables coefficients, these methods are used. Suitable numerical simulation methods have been applied to solve the COVID-19 epidemic model in Iraq. The analytical results of the Variation iteration method (VIM) are executed to compare the results. One numerical method which is the Finite difference method (FD) has been used to solve the Coronavirus model and for comparison purposes. The numerical simulation methods which are Mean Monte Carlo Finite difference (MMC_FD) and Mean Latin Hypercube Finite difference (MLH_FD), are also used to solve the proposed epidemic model under study. The obtained results are discussed, tabulated, and represented graphically. Finally, the absolute error is the tool used to compare the numerical simulation solutions from 2020 to 2024 years. The behavior of the Coronavirus in Iraq has been expected for 4 years from 2020 to 2024 using the proposed numerical simulation methods.

Keywords—COVID-19 epidemic model, Variation iteration method, Finite difference method, Mean Monte Carlo finite difference method, Mean Latin Hypercube Finite difference method.

1 Introduction:

History has seen many catastrophes and disease epidemics that changed the course of life and at the same time caused catastrophic tragedies, impeding everyday life in those countries, and the Coronavirus (COVID-19) is one of these epidemics as this epidemic spread rapidly and widely during the first months of 2020. Coronavirus Cases in the word in 13 July 2022 is 563,149,822 [1]. COVID-19 epidemic is a modern disease that is transmitted and spreads faster and more easily, more broadly than other diseases among people, some infected people don´t appear infected Until two or more symptoms of this epidemic appear in the infected person. The incubation period for the virus ranges from 7 to 12 days, and the maximum is up to two weeks. The proportion of people infected with the epidemic who did not show symptoms ranged from 40–45% [2, 3]. This virus affects the respiratory system, and one of the hypotheses claims that the reason for the emergence of this virus is the human eating of bats. We analyze the real data and information on the Coronavirus and suggest a model understand the patterns of the virus considering both public contact and participation, to help develop an effective strategy during a public emergency [1, 3]. Where data sites indicate that the numberof people infected with this virus continues to rise despite the preventive measures taken. No country in the world has been spared from coronavirus [4].

Many researchers interested in studying epidemiological systems especially in Covid 19 pandemic, SIR COVID -19 model spread in Iraq was studied with stochasticity to die out the epidemic [5], see [6],[7],[8],[9]. Other researchers have studied some numerical methods to solve these systems, such as [10],[11],[12],[6],[7],[8]. Some analytical methods have studied; SIR epidemic model was solved by Banach contraction method, Temimi-Ansari method, and Daftardar-Jafari method in [13], LTAM was applied the first time to solve the epidemic nonlinear model, this method is mixed Laplace transform with Tamimi and Ansari iterative method [14], see also [11],[15],[16],[17]. While there are those who are interested in studying epidemiological systems with random coefficients using appropriate methods to solve them called numerical simulation methods such as [18], [19], [20], [21], [9].

The proposed numerical method is Finite Difference (FD) that is used to solve the mathematical model under study, and it is considered an efficient approximation method for solving differential equations in general [22]. The discrete numerical values obtained from the FD method represent the exact approximate solution [22]. There is also a semi-analytical method which is the Variation Iteration Method, it is considered a reliable iterative method and gives approximate solutions to differential equations, as it was first introduced by Ji-Huan in 1997 [23, 24]. This method is used to solve linear and nonlinear equations, as well as homogenous and inhomogeneous ones, and it has applications in various engineering and applied fields [23]. The correction functional of a modification for the general Lagrange multiplier method represented an iteration method which is VIM [23, 25].

The two numerical simulation methods are used to solve the epidemic in this study, the first one is the Mean Monte Carlo Finite Difference method (MMC_FD) which is mixed between the Monte Carlo process (MC) [26] and Finite Difference Method (FD) [22].

-**In (2019)** Maha. A. M. and Ibrahim. S. et al. discussed the new approach that mixed two different methods which are the Mean Monte Carlo simulation technique and numerical iteration method which is finite difference method to sample randomly from a nonlinear epidemic model [18]. The second numerical simulation method is the Mean Latin Hybercub Finite Difference Method (MLH_FD) which is a merge of two different methods Latin Hyber Cube Sampling (LH) and Finite Difference Method (FD).

-**In (2018**) Mohammed, M. A, and Noor, N. F. M. et al. Studied A non-conventional hybrid numerical approach with multi-dimensional random sampling for cocaine abuse in Spain [19].

The nonlinear epidemic model in our research paper is solved by the method mentioned above.The importance of proposed methods gives more reliable numerical simulation solutions for such models that don't have the exact solution. More precisely, the numerical simulation methods are more suitable and give accurate results for such models whose coefficients are random variables. The study is divided into the following: in Section 2, the mathematical model details of COVID-19 are described; Section 3 has derived the numerical method FD and the analytical method VIM for the COVID-19 model under study. Section 4 explains numerical simulation methods MMC_FD and MLH_FD to solve the nonlinear system of the COVID-19 model in Iraq respectively. In Section 5, the finding results for the used methods are discussed and put in tables as well as represented graphically. Finally, Section 6 is a summary of research.

2 COVID-19 Mathematical Model

The epidemic model in our study includes the COVID-19 of people quarantined due to infection with the Coronavirus epidemic in Iraq [1] . The population consists of five types of individuals S, E, I, Q and R represent susceptible, exposed, the infected, the hospital quarantined and the recovery respectively. They are functions of time. The governing equations for the epidemic under study by the nonlinear ordinary differential equations of first order [27].

$$S'(t) = A - \left(\beta_1 - \beta_2 \frac{1}{m+1}\right) SI - dS,$$
$$E'(t) = (1-c)\left(\beta_1 - \beta_2 \frac{1}{m+1}\right) SI - k\left(\beta_1 - \beta_2 \frac{1}{m+1}\right) EI - dE,$$

$$I'(t) = c\left(\beta_1 - \beta_2 \frac{1}{m+1}\right) SI + k\left(\beta_1 - \beta_2 \frac{1}{m+1}\right) EI - (\epsilon + \gamma_1 + d + \mu)I, \quad (1)$$
$$Q'(t) = \epsilon I - (d + \gamma_2)Q,$$
$$R'(t) = \gamma_1 I + \gamma_2 Q - dR,$$

where Tables 1 and 2 represent variables S, E, I, Q, R and parameters $A, \beta_1, \beta_2, \epsilon, k, \gamma_1, \gamma_2, m, c, \mu, d$ sequentially. The system (1) can be solved according to the initial conditions that have been taken from the World Health Organization (WHO) website. WHO is the source of Iraq data and initial values for our system [3].

$S(0) = 500$, $E(0) = 50$, $I(0) = 25$, $Q(0) = 10$ and $R(0) = 15$, (2)

with the predicted parameters that are given in Table 2:

Table 1. Variables of COVID-19 model [27]

Variables	Description
$S(t)$	Susceptible category
$E(t)$	Exposed category
$I(t)$	Infected category
$Q(t)$	Hospital quarantined category
$R(t)$	Recovery category

Table 2. Parameters of COVID-19 model [27]

Parameter	Description	Value	Source
A	Birth rate	1541.8	[28]
β_1	Transmission contact rate between S and I	0.5	Estimated
β_2	Awareness rate	0.1	Estimated
μ	Death due to disease rate	0.38	[28]
ϵ	Quarantined rate	$\frac{1}{7}$	[29]
k	Fraction denoting the level of exogenous re-infection	0.05	Estimated
d	Natural death rate	3.8545×10^{-5}	[28], [30]
γ_1	Recovery rate from infected	0.033	Estimate
γ_2	Recovery rate from quarantine class	$\frac{1}{18}$	[29]
c	Fraction constsnt	[0,1]	Estimated
m	Half saturation of media constant	70	Estimated

3 Solving Covid-19 Model in Iraq

3.1 Numerical Method for Solving Covid-19 Model in Iraq

The nonlinear mathematical Epidemiological Covid-19 model with the estimated parameters the estimated parameters $A, \beta_1, \beta_2, \epsilon, k, \gamma_1, \gamma_2, m, c, \mu$ and d, that are explain in Table 2, the system (1) that represent quarantine model of Covid-19 in Iraq has been solved according to the initial conditions: $S_0(t)$, $E_0(t)$, $I_0(t)$, $Q_0(t)$ and $R_0(t)$, by the finite difference method (FD) with real step size $h = 0.02, 0.08$ where $h = \frac{Upper\ bound - Lower\ bound}{m}$. In this study, $m = 52, 12$ means the numbers of weaks and months through one year. The zero terms become as in (2). In order to find $S_1(t)$, $E_1(t)$, $I_1(t)$, $Q_1(t)$ and $R_1(t)$, Backward Finite Difference (BFD) can be used as follows:

$$S_1(t) = S_0(t) + h\left(A - \left(\beta_1 - \beta_2\left(\frac{1}{m+1}\right)\right) S_0(t) I_0(t) - dS_0(t)\right), \tag{3}$$

$$E_1(t) = E(t) + h\left((1-c)\left(\beta_1 - \beta_2\left(\frac{1}{m+1}\right)\right) S_0(t) I_0(t) - k\left(\beta_1 - \beta_2 \frac{1}{m+1}\right) E_0(t) I_0(t) - dE_0(t)\right),$$
(4)

$$I_1(t) = I_0(t) + h\left(c\left(\beta_1 - \beta_2 \frac{1}{m+1}\right) S_0(t) I_0(t) + k\left(\beta_1 - \beta_2 \frac{1}{m+1}\right) E_0(t) I_0(t) - (\epsilon + \gamma_1 + d + \mu) I_0(t)\right), \tag{5}$$

$$Q_1(t) = Q_0(t) + h\big(\epsilon I_0(t) - (d + \gamma_2) Q_0(t)\big), \tag{6}$$

$$R_1(t) = R_0(t) + h\big(\gamma_1 I_0(t) + \gamma_2 Q_0(t) - dR_0(t)\big). \tag{7}$$

The $S_1(t)$, $E_1(t)$, $I_1(t)$, $Q_1(t)$ and $R_1(t)$ are calculated from Eqs. (3-7) to obtain the following values: $S_1(t) = 487.4237$, $E_1(t) = 63.0707$, $I_1(t) = 25.0924$, $Q_1(t) = 10.0082$ and $R_1(t) = 15.0039$, respectively.

Now, the Central Finite Difference (CFD) to find the other terms can use as the follow:

$$S_{i+1}(t) = S_{i-1}(t) + 2h\left(A - \left(\beta_1 - \beta_2\left(\frac{1}{m+1}\right)\right) S_i(t) I_i(t) - dS_i(t)\right), \tag{8}$$

$$E_{i+1}(t) = E_{i-1}(t) + 2h\left((1-c)\left(\beta_1 - \beta_2\left(\frac{1}{m+1}\right)\right)S_i(t)I_i(t) - k\left(\beta_1 - \beta_2\frac{1}{m+1}\right)E_i(t)I_i(t) - dE_i(t)\right), \quad (9)$$

$$I_{i+1}(t) = I_{i-1}(t) + 2h\left(c\left(\beta_1 - \beta_2\frac{1}{m+1}\right)S_i(t)I_i(t) + k\left(\beta_1 - \beta_2\frac{1}{m+1}\right)E_i(t)I_i(t) - (\epsilon + \gamma_1 + d + \mu)I_i(t)\right), \quad (10)$$

$$Q_{i+1}(t) = Q_{i-1}(t) + 2h\left(\epsilon I_i(t) - (d+\gamma_2)Q_i(t)\right), \quad (11)$$

$$R_{i+1}(t) = R_{i-1}(t) + 2h\left(\gamma_1 I_0(t) + \gamma_2 Q_0(t) - dR_0(t)\right),$$

(12) For all $i = 1,2,\ldots,m$. To find $S_1, S_2, \ldots, S_m$, E_1, $E_2, \ldots, E_m$, $I_1, I_2, \ldots, I_m$, Q_1, $Q_2, \ldots, Q_m$ and $R_1, R_2, \ldots, R_m$ that consider as numerical solutions for Covid-19 Model.

3.2 Analytical Methods for Solving Covid-19 Model in Iraq

The nonlinear system (1) of COVID-19 model in Iraq can be solved by the Variation Iteration Method (VIM) with given initial conditions (2). The correction functional for the system (1) given by:

$$S_{n+1}(t) = S_n(t) + \int_0^t \delta\left(S'_n(t) - \left(A - \left(\beta_1 - \beta_2\left(\frac{1}{m+1}\right)\right)S_k(t)I_n(t) - dS_n(t)\right)\right), n \geq 0, \quad (13)$$

$$E_{n+1}(t) = E_n(t) + \int_0^t \delta\left(E'_n(t) - \left((1-c)\left(\beta_1 - \beta_2\left(\frac{1}{m+1}\right)\right)S_n(t)I_n(t) - k\left(\beta_1 - \beta_2\frac{1}{m+1}\right)E_n(t)I_n(t) - dE_n(t)\right)\right), n \geq 0, \quad (14)$$

$$I_{n+1}(t) = I_n + \int_0^t \delta\left(I'_n(t) - \left(c\left(\beta_1 - \beta_2\frac{1}{m+1}\right)S_n(t)I_n(t) + k\left(\beta_1 - \beta_2\frac{1}{m+1}\right)E_n(t)I_n(t) - (\epsilon + \gamma_1 + d + \mu)I_n(t)\right)\right), n \geq 0,$$

(15)

$$Q_{n+1}(t) = Q_n(t) + \int_0^t \delta\left(Q'_n(t) - \left(\epsilon I_n(t) - (d+\gamma_2)Q_n(t)\right)\right), n \geq 0. \quad (16)$$

$$R_{n+1}(t) = R_n(t) + \int_0^t \delta \left(R'_n(t) - \left(\gamma_1 I_n(t) + \gamma_2 Q_n(t) - dR_n(t)\right)\right), \quad (17)$$

where δ is a general Lagrange multiplier.
By choosing $\delta = -1$ and putting in Eqs (13-17) with initial conditions 2, using mathematica program to get the analytic solution of VIM. The first solutions are:
program to get the analytic solution of VIM. The first solutions are:
$S_1(t) = 500 - 3041.868\, t,$
$E_1(t) = 50 + 4548.727\, t,$
$I_1(t) = 25 + 25.003\, t,$
$Q_1(t) = 10 - 0.223\, t,$
$R_1(t) = 15 + 1.306\, t.$

Now, when (n =1), Eqs. (13-17) has been gotten the second solutions:

$S_2(t) = 500 - 3041.868\, t + 11451.527\, t^2 + 10104.615\, t^3$,
$E_2(t) = 50 + 4548.727\, t - 12471.249\, t^2 - 10829.810\, t^3$,
$I_2(t) = 25 + 25.003\, t + 1014.233\, t^2 + 725.195\, t^3,$
$Q_2(t) = 10 - 0.223\, t + 0.185\, t^2$,
$R_2(t) = 15 + 1.306\, t + 0.406\, t^2.$

For *n*=2: the results of the third iteration are found from Eqs.(18-22) as below:
$S_3(t) = 500 - 3041.868\, t + 11451.527\, t^2 - 92262.789\, t^3 + 219596.046\, t^4 - 770137.379\, t^5 - 1232450.922\, t^6 - 417236.721\, t^7,$ (18)
$E_3(t) = 50 + 4548.727\, t - 12471.249\, t^2 + 92826.068\, t^3 - 239293.879\, t^4 + 806172.822\, t^5 + 1295275.269\, t^6 + 438344.072\, t^7$, (19)
$I_3(t) = 25 + 25.003\, t + 1014.233\, t^2 - 707.734\, t^3 + 19620.366\, t^4 - 36035.443\, t^5 - 62824.347\, t^6 - 21107.351\, t^7$,
(20)
$Q_3(t) = 10 - 0.223\, t + 0.185\, t^2 + 4.826\, t^3 + 2.589\, t^4$, (21)
$R_3(t) = 15 + 1.306\, t + 0.406\, t^2 + 11.159\, t^3 + 5.983\, t^4.$ (22)

In order to obtain a better approximation, continue more iterations:
$S(t) = \lim_{n\to\infty} S_n(t), E(t) = \lim_{n\to\infty} E_n(t), I(t) = \lim_{n\to\infty} I_n(t), Q(t) = \lim_{n\to\infty} Q_n(t)$ and $R(t) = \lim_{n\to\infty} R_n(t).$

4 Numerical Simulation Methods for Solving Covid-19 Model

4.1 Mean Monte Carlo Finite Difference (MMCFD)

Mean Monte Carlo Finite Difference (MMC_FD) [18] is a combination of two different methods, one is a simulation technique which is Monte Carlo simulation (MC) and the other is a numerical method which is Finite Difference (FD). MMC_FD returns the simulation technique *n*-times for the coefficients random variables of the system that are generated by this technique. *m*-iterations if the numerical method happened in each simulation in order to solve the system. After that, the mean of the last iteration in each simulation is calculated. This average represents the estimated solution of the system, short name MMC_FD method. The MMC_FD method is programmed via MATLAB software, for more details, see [18].

4.2 Mean Latin Hyber Cube Finite Difference (MLHFD)

Mean Latin Hyber Cube Finite Difference (MLH_FD) [19] is a modified MLH_FD. It is a numerical simulation process that merges two different approaches which are Latin Hyber Cube strair simulation process (LH) to estimate the parameters of the system, with a numerical Finite Difference (FD) method to solve the system under study. The simulation processes LH can simulate the random coefficients for the model. With each repetition, a numerical method FD is used for solving the model numerically using simulated system parameters. The average of the last FD iteration results for all LH repetition is computed as the estimated approximate solution for the system under search. The short name of it is MLH_FD. The MLH_FD is considered one of the reliable methods to solve epidemic models, at the same time, it is more efficient and faster than MMC_FD. It has been programmed via MATLAB software, for more details, see [19].

5 Results and Discussion

The results of numerical and numerical simulation solutions for the nonlinear Coronavirus model in Iraq are discussed and analyzed in this section. The initial conditions of the system are taken from the Iraq data from the World Health Organization website [1, 3]. Table 5 contains the results for the interval (2020-2024) with numerical simulation solutions for variables $S(t), E(t), I(t), Q(t)$ and $R(t)$ using VIM, FD, MMC_FD and MLH_FD from 2020-2024 with real step size 0.02 in a week (52 weeks in a year, the data of the COVID-19 epidemic is taken from each week, therefore, in order to change the weeks to a year, the real step size is calculated as $h = \frac{1}{52} \approx 0.02$), and 0.003 in a day (365 days in a year, the data of the COVID-19 epidemic is taken from each day, therefore, in order to change the days to a year, the real step size is calculated as $h = \frac{1}{365} \approx 0.003$). But the results of step size $h = 0.003$ are more accurate than the results of step size h = 0.02, this is the principle of numerical methods.

Table 3 explains the numerical FD and approximate simulation results MMC_FD and MLH_FD for 100 repetition of COVID-19 model from 2020 to 2022, daily (h=0.003) andweekly (h=0.02) for two years. The comparison between the numerical simulation methods with the analytical method (VIM) via the absolute error is shown in **Table 4** and **Figure 1**, where it is found that the MLH_FD method is closer to the VIM method, which is considered a reliable method for the purpose of comparsion, so the MLH_FD method is more efficient than the MMC_FD method because it has the lowest absolute error.

Table 5 illustrates the approximate expected results for four years from 2020 to 2024 for (100 rep.) of COVID-19 model when h=0.003 daily. Note that prediction intervals (5th percentile, 95th percentile) for MMC_FD expected results have been accounted for in **Table 6** and MLH_FD expected results have been shown in **Table 7**. All these expected results for MMC_FD and MLH_FD falls within these estimated intervals in **Tables 6** and **7**. Notice that the more significant the number of iterations, the more accuracy and efficiency increase. It's clear that as the iterations increase, there is a slight increase in accuracy this means that these methods do not need many iterations to reach efficiency, which increases the importance of these methods. See **Tables 3** and **4**.

Table 3. Numerical simulation results for $P = 100$ repetition and $m = 730, 104$ number of iteration daily and monthly through two years

Model Variables	VIM (2 years)	Step Size, *h* (daily & weekly)	FD (2 years)	MMC_FD 100 rep. (2 years)	MLH_FD 100 rep. (2 years)
$S(t)$	66.15770111	0.003 (daily)	66.15581623	66.15265027	66.15293196
		0.02 (weekly)	66.15730122	66.15301346	66.15415839
$E(t)$	78.56821053	0.003 (daily)	78.56613062	78.56233134	78.56319038
		0.02 (weekly)	78.56804524	78.56590213	78.56631073
$I(t)$	93.82263177	0.003 (daily)	93.82131012	93.81774512	93.81955377
		0.02 (weekly)	93.82205899	93.82186389	93.82196025
$Q(t)$	62.87890174	0.003 (daily)	62.87904713	62.88953923	62.88512036
		0.02 (weekly)	62.88263471	62.88913605	62.88501503
$R(t)$	300.31674122	0.003 (daily)	300.31762273	300.32005461	300.31911364
		0.02 (weekly)	300.31951072	300.32292512	300.32102871

Table 4. Absolute error for MMC_FD and MLH_FD with VIM from 2020 to 2022

Model Variables	Step Size, *h* (daily & weekly)	Number of Iteration, *m*	MMC_FD 100 rep. (2 years)	MLH_FD 100 rep. (2 years)
$S(t)$	0.003 (daily)	740	0.005050841	0.004769151
	0.02 (weekly)	104	0.004687651	0.003542721
$E(t)$	0.003 (daily)	740	0.00587919	0.00502015
	0.02 (weekly)	104	0.00230840	0.00189980
$I(t)$	0.003 (daily)	740	0.00488665	0.00307801
	0.02 (weekly)	104	0.00076788	0.00067152
$Q(t)$	0.003 (daily)	740	0.01063749	0.00621862
	0.02 (weekly)	104	0.01023431	0.00611329
$R(t)$	0.003 (daily)	740	0.00331339	0.00237242
	0.02 (weekly)	104	0.00618392	0.00428749

Table 5. Expected numerical and numerical simulation results (100 rep.) of COVID-19 model in four years.

Model Variables	VIM (4 years)	Step Size, *h* (daily & weekly)	FD (4 years)	MLH_FD 1000 repitition (4 years)	MMC_FD 1000 repition (4 years)
$S(t)$	31.35372204	0.003 (daily)	31.37701463	31.39721722	31.38706092
		0.02 (weekly)	31.36632082	31.38302197	31.38163843
$E(t)$	63.35501193	0.003 (daily)	63.34089142	63.31730232	63.32099790
		0.02 (weekly)	63.33065503	63.29861225	63.30769692
$I(t)$	75.24431572	0.003 (daily)	75.38192078	75.42290144	75.40206733
		0.02 (weekly)	75.27425601	75.21026201	75.21948057

$\boldsymbol{Q(t)}$	42.00382274	0.003 (daily)	42.14514566	42.19138712	42.17018204
		0.02 (weekly)	42.20102135	42.24063026	42.23101830
$\boldsymbol{R(t)}$	388.33010244	0.003 (daily)	388.36403724	388.39154324	388.38420156
		0.02 (weekly)	388.39923505	388.44563927	388.42307823

Table 6. Prediction intervals (5th percentile, 95th percentile) for MMC_FD solutions

MMC_FD from 2020 to 2024 ($\boldsymbol{t \leq 4}$)		
Subpopulation	**(100 repetitions)**	**(1000 repetitions)**
$\boldsymbol{S(t)}$	(20.947223 , 141.168311)	(19.955301 , 140.447421)
$\boldsymbol{E(t)}$	(53.300054 , 165.085578)	(52.804973 , 167.798897)
$\boldsymbol{I(t)}$	(69.391802 , 172.355637)	(71.283209 , 176.165402)
$\boldsymbol{Q(t)}$	(37.611542 , 62.446360)	(38.634711 , 69.324810)
$\boldsymbol{R(t)}$	(375.256948 , 422.435681)	(373.253756 , 426.412655)

Table 7. Prediction intervals (5th percentile, 95th percentile) for MLH_FD solutions

MLH_FD from 2020 to 2024 ($\boldsymbol{t \leq 4}$)		
Subpopulation	**(100 repetitions)**	**(1000 repetitions)**
$\boldsymbol{S(t)}$	(19.805932 , 135.762404)	(21.807820 , 136.146641)
$\boldsymbol{E(t)}$	(51.786806 , 162.893688)	(50.657571 , 161.937494)
$\boldsymbol{I(t)}$	(65.060154 , 172.155659)	(64.048223 , 169.569295)
$\boldsymbol{Q(t)}$	(38.596893 , 64.356162)	(39.607302 , 67.261998)
$\boldsymbol{R(t)}$	(378.244381 , 428.404307)	(377.244872 , 430.398793)

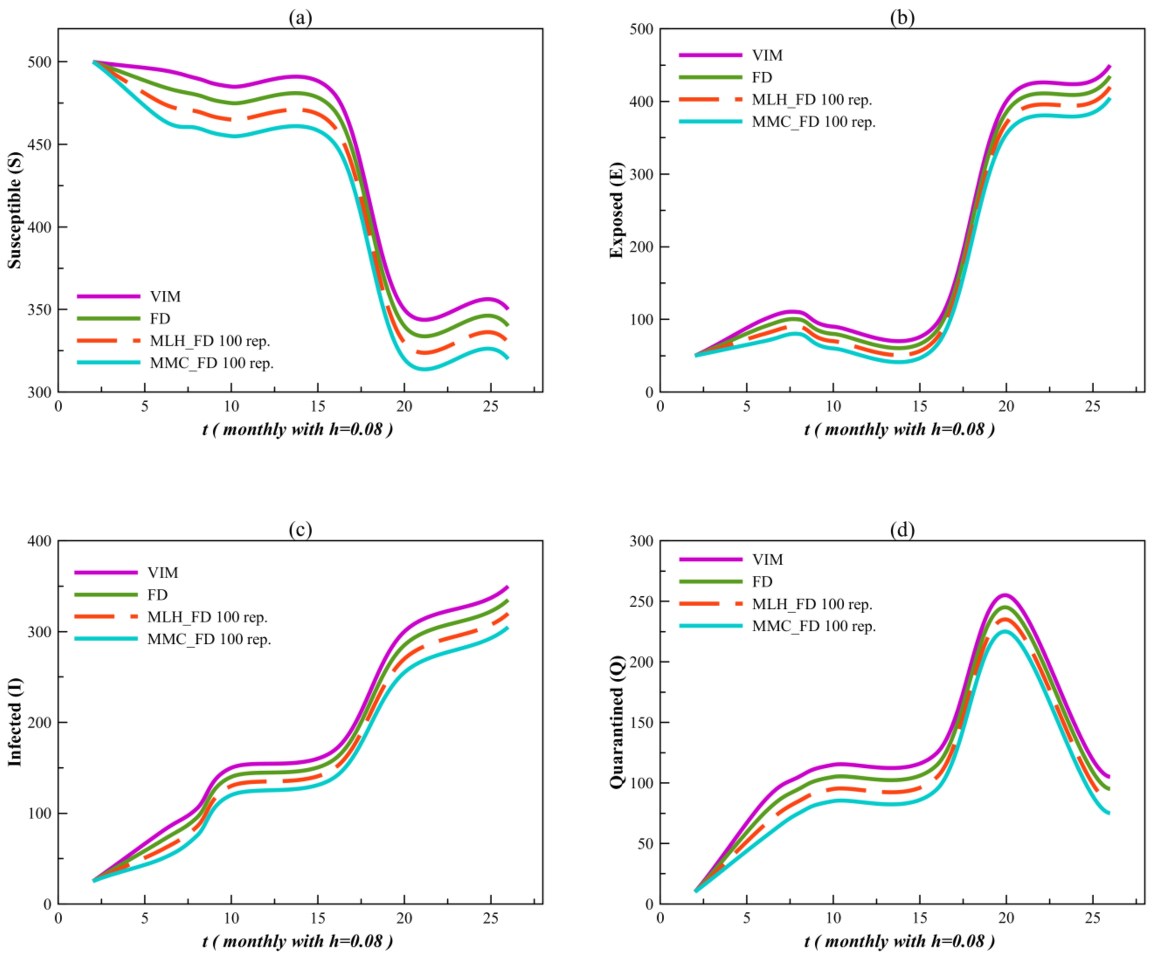
Figure 1 shows the curves of the methods used in our study for 24^{th} months, the interval 2021 to 2022, in which all groups of society are shown according to the impact of the virus on them. **Figure 1(a)**, represents a group of people who are not infected with an epidemic but They are susceptible to infection for this class $S(t)$ of all VIM, MLH_FD, MMC_FD, and FD methods are used with step size $h = \{0.003, 0.02\}$ daily and weekly through two years as follows, a sudden drop in the curves for all the methods used in the study after 17^{th} months. It is noticeable that the sudden descent in the curve as a result of a large number of infections of the study period to still down after the end of 2022.

Figure 1 (b), observe the curves of group $E(t)$ with people who are carriers of the virus without showing symptoms of infection. It is noticeable that the curves rise gradually until the 15^{th} month, after which they increase with greater upwards until the end of the study period.

As for **Figure 1(c)**, curves represents the infected people $I(t)$ with the epidemic, they have rise from 15th month dramatically to reach the highest level until the end of period study.

Figure 1(d), is associated with the group of people who are lying in quarantine places Q(t), notes that a gradual increase in the curves of this group in the 20^{th} month of society for all the methods used in the study, after that a sudden decrease until 2022.

Figure 1(e), represents the group of people $R(t)$ who have been cured of the disease, as they have been removed from the list of positive cases. There is a gradual rise of the curves especially from the 15^{th} month to the 20^{th} month, then it is continues to decline until the end of the study period.



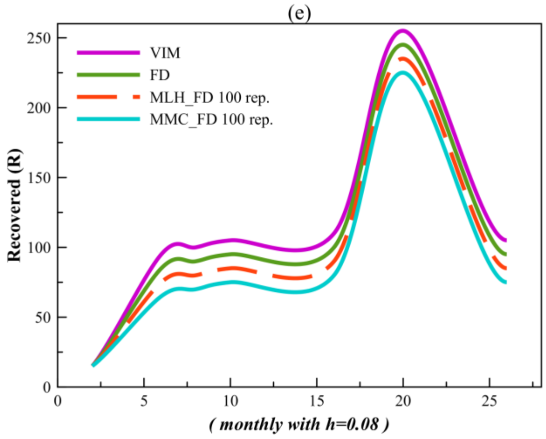


Figure 1. Comparstion of numerical and numerical simulation solutions for VIM, FDM, MMC_FD and MLH_FD of (a) $S(t)$, (b) $E(t)$, (c) $I(t)$, (d) $Q(t)$ and (e) $R(t)$ from 2020 to 2022.

Now, we discuss **Figure 2**, which shows the curves of the methods used in our study for the interval 2020 to 2024, in which all groups of society are shown according to the impact of the virus on them. **Figure 2 (a)**, represents a group of healthy people $S(t)$, but they are predisposed to infection. We note the gradual decline of this group of people for all the methods used in the study VIM, MLH_FD, MMC_FD, and FD in the early months as a result of mixing and non-compliance with health prevention methods, which led to a frightening descent that we observe during the first 2 years to settle after 2023 until the end of the study period.

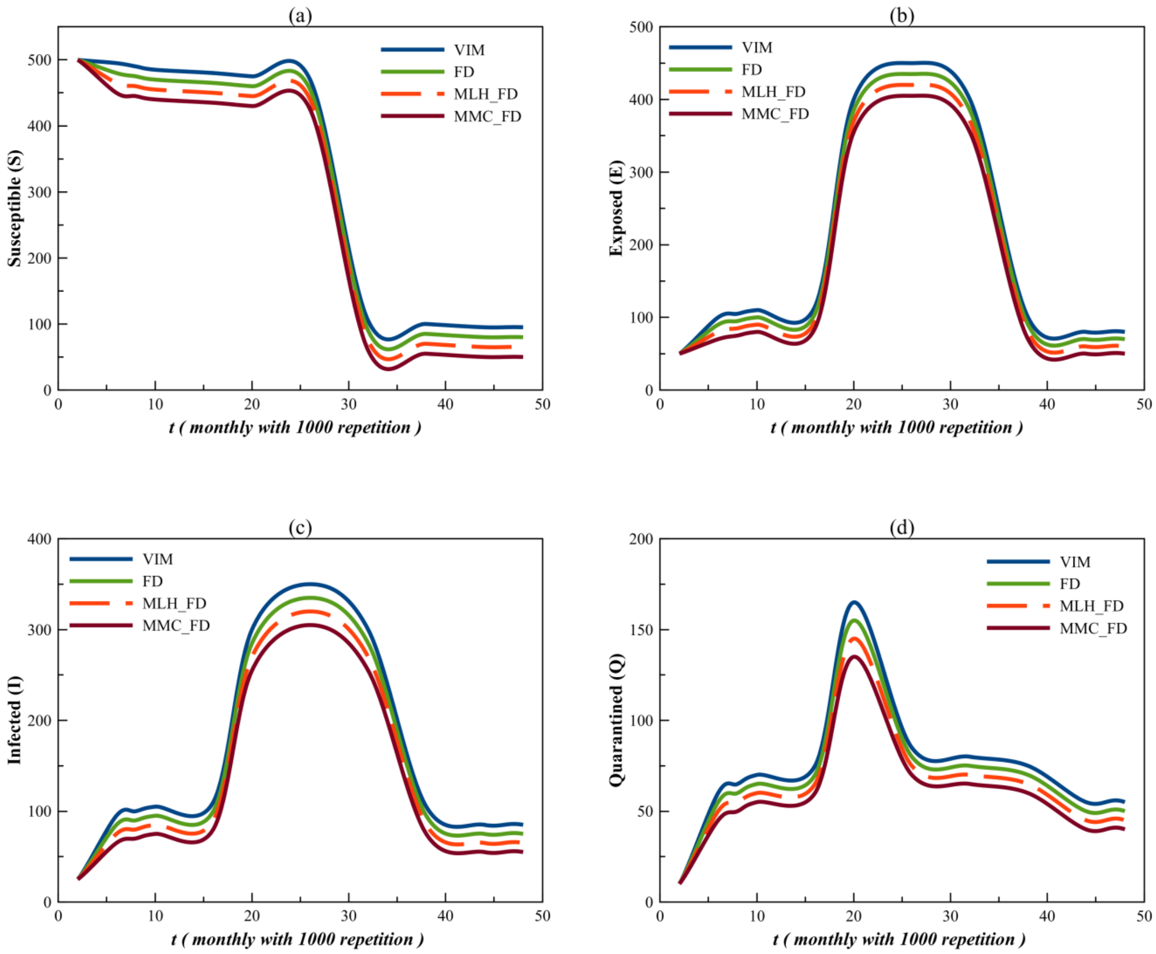
For **Figure 2 (b)**, is associated with people who are carriers of the virus without clearly showing signs of infection $E(t)$. It is clear that a rise in the curve of this group of society is apparent during the first months until reaching its peak in 2022 for all the methods used in the study, to stabilize after that until 2024 as a result of health awareness and following health instructions.

Figure 2 (c), represents the group of people who have been infected with the virus accompanying obvious symptoms $I(t)$. The curve of this group has rise from 15th month dramatically to reach the highest level in 25th month, and after that, the curve gradually rises until 2021, then A sudden rise from 2021 to 2022, all the methods used in the study have their highest level in the year 2023. After that, the number of infections will decline until the end of the study period in 2024.

Figure 2 (d), is associated with the group of people who are lying in quarantine places Q(t), It notices that a gradual increase in the curve of this group of society for all the methods used in the study happens, then a sudden rise in the 21th to 27th month , then reach the stable level after 2023.

Figure 2 (e), represents the group $R(t)$ of people who have been cured of the disease. It is noted that there is a slight increase in recovery cases from 2020 to 2022, the situation is almost stable in these years, then the number of recovery cases increases after 2022 to 2024, as a result of reaching some solutions by following the instructions of the World Health Organization and adhering to health prevention methods and taking Appropriate treatments.

Also **Figure 2**, explains there is a convergence of numerical simulation results for MMC_FD and MLH_FD methods (100 repetition) with each of subpoplutions (a) $S(t)$, (b) $E(t)$, (c) $I(t)$, (d) $Q(t)$ and (e) $R(t)$ for COVID-19 model in four years from 2020 to 2024.



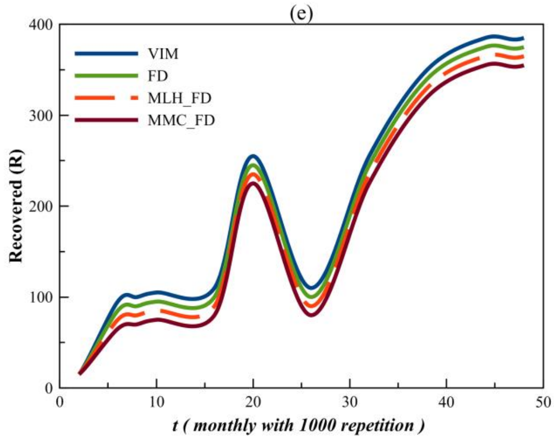


Figure 2. Comparstion of numerical and numerical simulation solutions for VIM, FDM, MMC_FD and MLH_FD of (a) $S(t)$, (b) $E(t)$, (c) $I(t)$, (d) $Q(t)$ and (e) $R(t)$ from 2020 to 2024.

6 Conclusion

In our research paper, the epidemiological model used talks about COVID-19 in Iraq, this mathematical model was designed in the form of a system of nonlinear first-order differential equations. Some reliable methods are used to solve this model for 4 years from 2020 to 2024, including the numerical FD method, an efficient semi-analytical VIM method is also used for comparison. On the other hand, two suitable numerical simulation methods which are MMC_FD and MLH_FD in this study. By the absolute error as a comparison tool between the numerical simulation methods which are used in our study, it was found that the MLH_FD method is more efficient than the MMC_FD because it has the lowest absolute error during the study period. The efficiency and accuracy increase with the increasing number of iterations, but this accuracy is slight with the increasing iterations. This shows that the numerical simulation methods used do not need many iterations to reach accuracy and efficiency, and this is what distinguishes them. Studying this epidemic model gives us an idea and impression of the impact of this virus on the population. The results show us that the group of healthy people $S(t)$ started decreasing in 2024, while the group of people infected with the epidemic without symptoms $E(t)$, we notice an increase in this group gradually until the end of the study period. The group of people infected with the disease $I(t)$ this group of people there is an increase in the number of infections clearly during the study period from 2020 to 2024, there is also an increase in the group of people quarantined $Q(t)$as a result of the high number of injuries. Finally, in the group of people who have fully recovered from infection with the virus due to infection $R(t)$, we also note that there is an increase in this group of people and it persists until the end of the study period. The behavior of the Coronavirus in Iraq has been expected for 4 years from 2020 to 2024 using the proposed numerical simulation methods.

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