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Study on Fuzzy $\boldsymbol{\beta}$- Supercontinuous Function in Fuzzy Topological Spaces

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Abstract

The main aim of this research is introducing and characterizing new concepts of fuzzy continuity using the definition of fuzzy β-sets and fuzzy βs -sets including fuzzy β- supercontinuous. Some of their basic properties are examined. After that some theorem by using these properties have been proved.

Keywords. Fuzzy topological space, fuzzy b – open set, fuzzy β-sets, fuzzy βs -sets, fuzzy β- supercontinuous functions.

1. Introduction

The concept of fuzzy sets was introduced by Zadeh in 1965 [4] and it has an important role in the study of fuzzy topology which was explained by Chang in 1968 [1]. Chang also investigated fuzzy continuity that was proved to be of fundamental importance in the field of fuzzy topology. Mapping is a significant tool for the research of topological notions and for building a new topological space, Chang (1968) extended the concepts of continuous, open and closed mappings in fuzzy topological spaces. Therefore, various concepts in general topology have been extended to fuzzy topological spaces. The class of b -open sets in general topology was defined by Andrijevic in 1996 [2]. In fuzzy topological space the notion of fuzzy semi open sets and fuzzy semi closed sets were first explained in 1981 by Azad [5] and also he defined the concept of semi continuous and semi open functions in fuzzy topological spaces. In this study, we introduce definition of fuzzy $\boldsymbol{\beta}$-supercontinuous function and illustrate a deep study on their properties and its related with other types of fuzzy continuous functions.

2. Preliminaries

In this section, we illustrate some basic definition and results.

Definition 2.1.[1]A family $\mathfrak{T}$ of fuzzy sets in μ is called fuzzy topology if it satisfies the following axioms:

(i) $0,1 \in \mathfrak{T}$

(ii) $\mu_1, \mu_2 \in \mathfrak{T}$, then $\mu_1 \wedge \mu_2 \in \mathfrak{T}$;

(iii) If $\{\mu_j : j \in J\} \subset \mathfrak{T}$, where J denotes an index set, then $\vee \mu_j \in \mathfrak{T}$.

The pair $(\mu,\mathfrak{T})$ is named as a fuzzy topological space or shortly f.t.s. The members of $\mathfrak{T}$ are definedas $\mathfrak{T}$-open fuzzy set. If the complement of $\wp$ which denoted by 1- $\wp$ is $\mathfrak{T}$- open then an element $\wp \in [0,1]^{\mu}$ is said to be fuzzy closed set.

Example 2.2. Let $\mu = \{m_1, m_2, m_3\}$, $\lambda = (0.8, 0.7, 0.6)$, $\gamma = (0.5, 0.4, 0.3)$, $\Omega = (0.2, 0.2, 0.1)$, $\mathfrak{T} = \{0, \lambda, \gamma, \Omega, 1\}$, then

$\lambda \wedge \gamma = (0.8, 0.7, 0.6) \wedge (0.5, 0.4, 0.3) = (0.5, 0.4, 0.3) = \gamma \in \mathfrak{T}$

$\lambda \wedge \Omega = (0.8, 0.7, 0.6) \wedge (0.2, 0.2, 0.1) = (0.2, 0.2, 0.1) = \Omega \in \mathfrak{T}$

$\gamma \wedge \Omega = (0.5, 0.4, 0.3) \wedge (0.2, 0.2, 0.1) = (0.2, 0.2, 0.1) = \Omega \in \mathfrak{T}$

And,

$\lambda \vee \gamma = (0.8, 0.7, 0.6) \vee (0.5, 0.4, 0.3) = (0.8, 0.7, 0.6) = \lambda \in \mathfrak{T}$

$\lambda \vee \Omega = (0.8, 0.7, 0.6) \vee (0.2, 0.2, 0.1) = (0.8, 0.7, 0.6) = \lambda \in \mathfrak{T}$

$\gamma \vee \Omega = (0.5, 0.4, 0.3) \vee (0.2, 0.2, 0.1) = (0.5, 0.4, 0.3) = \gamma \in \mathfrak{T}$

Also,

$\lambda \vee \gamma \vee \Omega = (0.8, 0.7, 0.6) \vee (0.5, 0.4, 0.3) \vee (0.2, 0.2, 0.1) = (0.8, 0.7, 0.6) = \lambda \in \mathfrak{T}$

It is obvious that o, $1 \in \mathfrak{T}$

Hence, $(\mu, \mathfrak{T})$ is a fuzzy topological space.

Definition 2.3.[10]

1) A fuzzy set F in a fuzzy topological space $(\mu,\mathfrak{T})$ is said to be fuzzy b-open set if $F \leq (int(cl(F)) \vee (cl(int(F))$ and we can denote by fb- open.
2) A fuzzy set F in a fuzzy topological space $(\mu,\mathfrak{T})$ is said to be fuzzy b-closed set if $F \geq (int(cl(F)) \wedge (cl(int(F)))$

Example 2.4. Let $\mu = \{m_1, m_2, m_3\}$, $\lambda = (0.5, 0.5, 0.5)$, $\gamma = (0.2, 0.6, 0.2)$, $\Omega = (0.5, 0.2, 0.6)$, and $\psi = (0.3, 0.4, 0.3)$ such that $\lambda(m_1) = 0.5$, $\lambda(m_2) = 0.5$, $\lambda(m_3) = 0.5$ and $\gamma(m_1) = 0.2$, $\gamma(m_2) = 0.6$, $\gamma(m_3) = 0.2$, $\Omega(m_1) = 0.5$, $\Omega(m_2) = 0.2$, $\Omega(m_3) = 0.6$ and $\psi(m_1) = 0.3$, $\psi(m_2) = 0.4$, $\psi(m_3) = 0.3$, $\mathfrak{T} = \{0, \Omega, \psi, \Omega \vee \psi, \Omega \wedge \psi, 1\}$. It is clear that λ is fuzzy b – open set, but γ is not fuzzy b - open set.

Remarks 2.5.[10]

1) For a fuzzy set F in a fuzzy topological space$(\mu,\mathfrak{T})$ if F is fuzzy b-open set if and only if 1-F Is a fuzzy b-closed set.
2) For fuzzy set F in a fuzzy topological space $(\mu,\mathfrak{T})$, F is fuzzy b-closed set if and only if 1-F is a fuzzy b-open set.

Definitions 2.6.[6]

1) Let F be fuzzy set in a fuzzy topological space $(\mu,\mathfrak{T})$. Then its fuzzy b-closure is defined by $Fbcl(F) = \wedge \{L \geq F$: L is a fb-closed set of $(\mu,\mathfrak{T})$.
2) Let F be fuzzy set in fuzzy topological space $(\mu,\mathfrak{T})$. Then its fuzzy b-interior ifdefined by $Fbint(F)=\vee\{G\leq F$: G is afb-open set of $(\mu,\mathfrak{T})\}$.

Theorem 2.7.[6] let F be any fuzzy set in a fuzzy topological space$(\mu,\mathfrak{T})$ Then:

1) F bcl(1-F)=1-fb int(F)
2) F bint(1-F)=1-fbcl(F)

Remarks 2.8.[6]

1) If F is a fuzzy subset of fuzzy topological space$(\mu,\mathfrak{T})$. Then Fbcl(F) is the smallest fb-closed set containing F. Thus, $F\,bcl(F)=F\vee(int(cl(F)))\wedge(cl(int(F)))$
2) If F is a fuzzy subset of fuzzy topological space$(L,\mathfrak{T})$. Then Fbint(F) is the largest fb-open set containing F. Thus, $Fbint(F) = F\wedge ((int (cl(F))) \vee(cl(int(F)))$

Theorem 2.9.[10] In a fuzzy topological space $(\mu,\mathfrak{T})$

1) An arbitrary union of f b-open set is a f b-open set.
2) An arbitrary intersection of f b –closed set is a f b-closed set.

Definition 2.10.[8] A fuzzy subset A of a fuzzy topological space $(\mu,\mathfrak{T})$ is said to be fuzzy Locally closed set if $A=U\wedge V$, where $U\in \mathfrak{T}$and V is a fuzzy closed. The family of all fuzzy locally closed set is denoted by $FLC(\mu)$.

Remark 2.11.[8] The fuzzy topology $\mathcal{T} = \{0_\mu,1_\mu,\mathcal{M},N\}$ on μ. Then $\mathcal{W} = \mathcal{M} \wedge \mathcal{N}$ is a fuzzy locally closed set, but it can't be fuzzy open.

Proposition 2.12.[8] An intersection of a fuzzy locally closed set with any fuzzy open set is fuzzy locally closed.

Proposition 2.13.[8] If $\mathcal{W}$ *and* $\mathcal{V}$ are fuzzy locally closed subsets of f.t.s.$(\mu,\mathfrak{T})$, therefore $\mathcal{W} \wedge \mathcal{V}$ is fuzzy locally closed.

Proposition2.14.[3] A fuzzy set $\mathcal{W}$ in any f.t.s.$(\mu,\mathfrak{T})$ is fuzzy locally closed if and only if $\mathcal{W} = \mathcal{M} \wedge \mathcal{W}$ for some fuzzy open set $\mathcal{M}$.

Definition 2.15 [10]. A space μ is called fuzzy b – closed space if every f b – open subsets of f.t.s. $(\mu,\mathfrak{T})$ is fuzzy open sets.

Definition 2.16. [9] A f.t.s. $(\mu,\mathfrak{T})$ is said to be fuzzy extremally disconnected if the fuzzy closure of every fuzzy open subset of μ is fuzzy open.

Now we will recall some details of fuzzy β-sets and fuzzy βs -sets.

Definition 2.17 [7] A subset A of a fuzzy topological space $(\mu,\mathfrak{T})$ is said to be fuzzy β-set if A is fuzzy b-open and fuzzy locally closed set. The family of all fuzzy β-sets is denoted by $F\beta_s(\mu)$. It is clear that $F\beta_s(\mu)=FLC(\mu,\mathfrak{T})\wedge FBO(\mu,\mathfrak{T})$.

Proposition 2.18 [7] Every open subset of topological space $(\mu,\mathfrak{T})$is Fβ-set.

Proposition 2.19. [7] If μ is a sub maximal space, then the union of any family of fuzzy β-sets is fuzzy β-set.

Theorem 2.20 [7] Let $(\mu,\mathfrak{T})$ be externally disconnected fuzzy topological space. Then $A\leq\mu$ is fuzzy open if and only if A is fuzzy β-set.

Definition 2.21 [7] A subset A of a f.t.s. $(\mu,\mathfrak{T})$ is called a fuzzy β_s-set if $A \wedge B \in F\beta_s(\mu)$ for all $B\in F\beta_s(\mu)$. The class of all $F\beta_s$-set in$(\mu,\mathfrak{T})$will bedenoted by $\mathfrak{T}_{F\beta s}$. We will denote to the complement of F β_s-set by 1- F β_s–set.

Theorems 2.23 [7].

1) If μ is a sub maximal space, then $\mathfrak{T}_{F\beta s} \subseteq F\beta_s(\mu)$.
2) If topological space μ is a sub maximal externally disconnected, then $\mathfrak{T}_{F\beta s} = F\beta_s(\mu) = \mathfrak{T}$.

3) Let (μ,$\mathfrak{T}$) is a submaximal Fb-space, then $\mathfrak{T}_{F\beta s}$ =$\mathfrak{T}$.

Definition 2.26 [9] Let $\mu\ and\ \sigma$ be any non-empty sets and $\varphi\colon\mu\to\sigma$ is a function, then for Let λ be a fuzzy set in μ. any fuzzy set λ in μ then the inverse image of λ under φ, we can write as $\varphi^{-1}(\lambda)$ and it is fuzzy set in μ and defined by $\varphi^{-1}(\lambda)(\mu_1)=\lambda(f(\mu_1)$ for all $\mu_1\in\mu$. Additionally, the image of λ under f written as $\varphi(\lambda)$ is the fuzzy set defined by

$$\varphi(\lambda)(\mu_2)=\begin{cases}\sup\lambda(\mu_1)\ ,if\ \varphi^{-1}(\mu_2)\neq\varphi\\ 0 \qquad otherwise\end{cases}$$

Theorem 2.27 [9] Let φ be a function from $\mu\ to\ \sigma$ **.**Then

1) $\varphi^{-1}(1-\lambda)=1-\varphi^{-1}(\lambda)$ for any fuzzy set λ in σ
2) $1-\varphi(\lambda)\subset\varphi(1-\lambda)$ for any fuzzy set λ in μ.
3) $\mu_1\subset\mu_2\ \varphi(\mu_1)\subset\varphi^{-1}(\mu_2)$ where $\mu_1,\mu_2$are fuzzy set in σ .
4) $\lambda_1\subset\lambda_2\ \Longrightarrow\varphi(\lambda_1)\subset\varphi(\lambda_2)$ where $\lambda_1 and\lambda_2$are fuzzy set in μ.
5) $\varphi(\varphi^{-1}(\mu))\subset\mu$ for any fuzzy set μ in σ.
6) $\lambda\subset\varphi^{-1}\left(\varphi(\lambda)\right)$ for any fuzzy set λ in μ**.**
7) Let φ be a function from μ to σ and β be function from $\sigma\ to\ \Omega$. Then $(\beta o\varphi)^{-1}(\eta)=\varphi^{-1}(\beta^{-1}(\eta))$ for any fuzzy set η in Ω where (β o φ) is the composition of β and φ.

Definition 2.28 [9] A function φ from a fuzzy topological space $(\mu,\mathfrak{I})$ to $(\sigma,\mathcal{L})$ are said to be fuzzy continuous if $\varphi^{-1}(\lambda)$ is a fuzzy open set in $(\mu,\mathfrak{I})$ for each fuzzy open set $\lambda\ in\ (\sigma,\mathcal{L})$.

Definition 2.29. [9] Let $(\mu,\mathfrak{I})and(\sigma,\mathcal{L})$ be two fuzzy topological space. The mapping $\varphi\colon(\mu,\mathfrak{I})\longrightarrow(\sigma,\mathcal{L})$ is said to be a fuzzy homeomorphism if and only if φ directive, φ is fuzzy continuous and φ^{-1} is fuzzy continuous.

3. fuzzy β-supercontinuous

Definition 3.1. The function $\varphi\colon\mu\to\sigma$ is called fuzzy β-supercontinuous [briefly Fβ-SC] at a point $p\in\mu$ if for every fuzzy open subset H containing $\varphi(p)$ in σ, there exists a fuzzy β- set G containing p in μ such that $\varphi(G)\leq H$.

If φ is fuzzy β-supercontinuou at every point x of μ, then it is called fuzzy β-supercontinuous.

Proposition 3.2. Let $(\mu,\mathfrak{I})$ be submaximal space and lat B be a base for a topology on σ. Then, $\varphi\colon\mu\to\sigma$ is fuzzy β-supercontinuous if and only if $\varphi^{-1}(B)$ is fuzzy β-set subset of μ for each $B\in\mathbb{B}$.

Proof: Necessity, suppose that φ is fuzzy β-supercontinuous. Then, since each $B \in \mathbb{B}$ is fuzzy open subset of σ. So, $\varphi^{-1}(B)$ is fuzzy β-set subset of μ.

Sufficiency, let N be any fuzzy open subset of σ then, $N = \vee \{B_i : i \in I\}$ where every B_i is member of $\mathbb{B}$ and I is a suitable index set.

If follows that, $\varphi^{-1}(N) = \varphi^{-1}(\vee \{B_i : i \in I\})$

$$= \vee \{\varphi^{-1}(B_i) : i \in I\}$$

Since $\varphi^{-1}(B_i)$ is fuzzy β-set subset of μ for each $i \in I$. Hence, N is fuzzy β-set. Therefore, we have, φ is fuzzy β-supercontinuous.

Theorem 3.3. Let $(\mu, \mathfrak{J})$ be a submaximal space and let $\varphi: \mu \to \sigma$ then, the following statements are equivalent.

a) φ is fuzzy β-supercontinuous.
b) $\varphi([F]_{\beta_S} \leq cl(\varphi(F))$ for every $F \leq \mu$.
c) $[\varphi^{-1}(H)]_{\beta_s} \leq \varphi^{-1}(cl(H))$ for every $H \leq \sigma$.

Proof: $(a) \Longrightarrow (b)$

Let $F \leq \mu$ since $cl\big(\varphi(F)\big)$ is fuzzy closed in σ so $cl(\varphi(F))$ is fuzzy $\beta_s -$ set, since $F \leq \varphi^{-1}\left(cl\big(\varphi(F)\big)\right)$ is fuzzy $\beta_s -$ set in μ and $F \leq \varphi^{-1}\left(cl\big(\varphi(F)\big)\right)$, so $[F]_{\beta_s} \leq [\varphi^{-1}\left(cl\big(\varphi(F)\big)\right)]_{\beta_s} = \varphi^{-1}\left(cl\big(\varphi(F)\big)\right)$. Therefore, $\varphi\big([F]_{\beta_s}\big) \leq \varphi\left(\varphi^{-1}\left(cl\big(\varphi(F)\big)\right)\right) \leq cl\big(\varphi(F)\big)$.

$(b) \Rightarrow (c)$

Let $H \leq \sigma$. Then, $\varphi([\varphi^{-1}(H)]_{\beta_s} \leq cl\left(\varphi\big(\varphi^{-1}(H)\big)\right) \leq cl(H)$

And so it follows that $[\varphi^{-1}(H)]_{\beta_s} \leq \varphi^{-1}\big(cl(H)\big)$.

$(c) \Longrightarrow (a)$

Let H be any fuzzy open subset in σthen ,1- H is fuzzy closed in $\sigma$and so

$[\varphi^{-1}(1-H)]_{\beta_s} \leq \varphi^{-1}\big(cl(1-H)\big) = \varphi^{-1}(1-H)$.

Again from (b) $\varphi^{-1}(1-H) \leq cl\big(\varphi^{-1}(1-H)\big) \leq [\varphi^{-1}(1-H)]_{\beta_s}$

Therefore, $\varphi^{-1}(1-H) = [\varphi^{-1}(1-H)]_{\beta_s}$ which in turn implies that $\varphi^{-1}(1-H)$ is fuzzy closed in μ

Since $\varphi^{-1}(1-H) = 1-\varphi^{-1}(H)$and this leads to $\varphi^{-1}(H)$ is open set in μ, so from proposition (2.16)

We get, $\varphi^{-1}(H)$ is fuzzy β-set in μ.

Hence, from theorem (3.4), $\varphi$is fuzzy β −supercontinuous function.

Theorem 3.4. Let$(\mu,\Im)$ be submaximal space. then $\varphi: \mu \to \sigma$is fuzzy β-supercontinuous if and only if $\varphi^{-1}(H)$ is fuzzy β-set for each fuzzy open set H in σ.

Proof: To prove necessity, let H be a fuzzy open set in σ

Suppose that$p \in \varphi^{-1}(H)$, since φ is fuzzy β-supercontinuous, so φ is fuzzy β- continuous at $p \in \varphi^{-1}(H)$.

Therefore, for each fuzzy open set H containing $\varphi(H)$ in σ there exists a fuzzy β-set G_p containing p in μ such that, $\varphi(G_p) \leq H$,

Thus $\varphi^{-1}(H)$ =$\vee \{G_p|_p \in \varphi^{-1}(H)\}$and since G_x is fuzzy β-set in μ, therefore $\vee \{G_p|_p \in \varphi^{-1}(H)\}$ is a fuzzy β- set in μ. Hence, $\varphi^{-1}(H)$ is fuzzy β- set.

To prove sufficiency, let a point $p \in \mu$ and H is fuzzy open set containing $\varphi(H)$ in σ.

Since $\varphi^{-1}(H)$ is fuzzy β- set in μ such that, $\varphi(p) \in H$,therefore $p \in \varphi^{-1}(H)$.

Since $\varphi^{-1}(H)$ is fuzzy β- set containing x such that $\varphi(\varphi^{-1}(H)) \leq H$.

Thus, φ is a fuzzy β-supercontinuous at p. φ is a fuzzy β-supercontinuous.

Theorem 3.5. If$\varphi: (\mu,\Im) \to (\sigma,\Sigma)$ is fuzzy continuous, then φ is a fuzzy β- supercontinuous.

Proof: Let V be a fuzzy open subset of σ. Since φ is continuous, then $\varphi^{-1}(V)$is fuzzy open set in μ. So, from proposition (3.2) we get, $\varphi^{-1}(V)$is fuzzy β-set in μ.

Hence, φ is fuzzy β-supercontinuous.

Theorem 3.6 Let $(\mu,\Im)$, (σ,τ) be two topological space and let μ is a submaximal extremally disconnected space. Then, the function $\varphi: \mu \to \sigma$ is fuzzy β- supercontinuous if and only if $\varphi: (\mu,\Im_{\beta_s}) \to (\sigma,\tau)$ is fuzzy continuous.

Proof: Assume that φ is fuzzy β- supercontinuous such that μ is a submaximal extremally disconnected space. Then for any fuzzy open set V of σ by theorem (3.4), we have, $\varphi^{-1}(V)$ is fuzzy β- set in μ. Since μ is a submaximal extremally disconnected space so by theorem (3.5) $\varphi^{-1}(V)$ is fuzzy open set in μ. Hence, φ is fuzzy continuous.

Conversely, if φ is fuzzy continuous. Then, for any fuzzy open set V of φ, $\varphi(V)$ is fuzzy open set in μ. Since μ is a sub maximal externally disconnected space, so by theorem (3.5), we have $\Im_{\beta s} = \beta_s(\mathrm{ì})$. Hence, $\varphi^{-1}(V)$ is fuzzy β- set in μ. Therefore, φ is fuzzy β - supercontinuous.

Theorem 3.7. Every fuzzy supercontinuous is fuzzy β-supercontinuous function.

Proof: If follows directly from theorem (3.5).

Theorem 3.8. Every fuzzy β- supercontinuous is fuzzy b- continuous function.

Proof: let $\varphi: \mu \to \sigma$ is fuzzy β-supercontinuous and let V be a fuzzy open subset of σ. Since φ is fuzzy β-supercontinuous, so $\varphi^{-1}(V)$ is fuzzy β- set in μ. Hence,φ^{-1} is fuzzy b-open in μ. Therefore, φ is fuzzy b- continuous.

Theorem 3.9. Let μ is a fuzzy b-open space and $\varphi: \mu \to \sigma$ then φ is fuzzy b-continuous function if and only if $\varphi: \mu \to \sigma$ is fuzzy β - supercontinuous.

Proof: Necessity, suppose φ is fuzzy b-continuous and V be fuzzy open set in σ. Since φ is fuzzy b-continuous we get $\varphi^{-1}(V)$ is fuzzy b-open set in μ. Since μ is a fuzzy b-open space. then $\varphi^{-1}(V)$ is fuzzy open set in μ. Hence, from proposition (2.18) we get $\varphi^{-1}(V)$ is fuzzy β- set in μ. Therefore, φ is fuzzy β-supercontinuous.

Theorem 3.10. Every fuzzy β- supercontinuous function is fuzzy LC-continuous.

Proof: let $\varphi: \mu \to \sigma$ is fuzzy β-supercontinuous and let V be a fuzzy open subset of σ. Since φ is fuzzy β-supercontinuous, so $\varphi^{-1}(V)$ is β- set in μ. Hence, φ^{-1} is fuzzy locally closed in μ. then, φ is fuzzy LC-continuous.

Theorem 3.11. Let $\varphi: \mu \to \sigma$ is a function, then,φ is fuzzy b-continuous and fuzzy LC-continuous if and only if φ is fuzzy β- supercontinuous.

Proof: Necessity, let φ is fuzzy b-continuous and fuzzy LC-continuous and let V be fuzzy open set in σ. Since φ is fuzzy b-continuous, $\varphi^{-1}(V)$ is fuzzy b-open set in μ. Since φ is fuzzy LC-continuous, then we have, $\varphi^{-1}(V)$ is fuzzy locally closed set in μ. Hence, $\varphi^{-1}$is fuzzy b-open and fuzzy locally closed set in μ. Therefore, $\varphi^{-1}(V)$ is fuzzy β- set in μ. Hence, φ is fuzzy β-supercontinuous.

Sufficiency, the proof straight forward from theorem (3.8) and theorem (3.10)

Theorem 3.12. Let $\varphi: \mu \to \sigma$ is fuzzy b-continuous and $\psi: \sigma \to \mathcal{S}$ is fuzzy β-supercontinuous, then $\psi o\varphi: \mu \to \mathcal{S}$ is fuzzy b-continuous.

Proof: let V be fuzzy open set in $\mathcal{S}$. since ψ is fuzzy β-supercontinuous, then theorem (3.4) we get $\psi^{-1}(V)$ is fuzzy β-set in σ. So, $\psi^{-1}(V)$ is fuzzy b-open set in σ. Since φ is fuzzy b -continuous, so$\varphi^{-1}\left(\psi^{-1}(V)\right)$ is fuzzy b- open set in μ. But $\varphi^{-1}(\psi^{-1}(V) = (\psi o\varphi)^{-1}(V)$. Hence, $(\psi o\varphi)^{-1}(V)$ is fuzzy b- open set in μ. Therefore, $\psi o\varphi$ is fuzzy b- continuous.

Theorem 3.13. If $\varphi: \mu \to \sigma$ is fuzzy b-continuous $\psi: \sigma \to \mathcal{S}$ is fuzzy β-supercontinuous and $(\mu, \mathfrak{I})$ is submaximal extremally disconnected space then $\psi o\varphi: \mu \to \mathcal{S}$ is fuzzy β -supercontinuous.

Proof: let U be fuzzy open set in $\mathcal{S}$. Since ψ is fuzzy β-supercontinuous, then from Theorem (3.4) we get $\psi^{-1}(U)$ is fuzzy β- set in σ. So $\psi^{-1}(U)$ is fuzzy b-open set in σ. Since φ is fuzzy b -continuous. so $\varphi^{-1}(\psi^{-1}(U))$ is fuzzy b-open set in μ. But $\varphi^{-1}\left(\psi^{-1}(U)\right) = (\psi o\varphi)^{-1}(U)$. Hence, $(\psi o\varphi)^{-1}(U)$ is fuzzy b-open set in μ. Since μ is sub maximal externally disconnected space, then $(\psi o\varphi)^{-1}(U)$ is fuzzy locally closed set in μ. Therefore $(\psi o\varphi)^{-1}(U)$ is fuzzy β-set in μ. Hence, $\psi o\varphi$ is fuzzy β-supercontinuous.

Theorem 3.14 Let$(\mu, \mathfrak{I})$ be extremally disconnected topological space. Then, the function $\varphi: \mu \to \sigma$is fuzzy β-supercontinuous if and only if φ is fuzzy continuous.

Proof: To prove necessity, suppose that $\varphi$is fuzzy β-supercontinuous. Let Mbe a fuzzy open subset of σ. Then, from theorem (4.4) we have, φ^{-1}(M) is fuzzy β-set and so from theorem (3.5) φ^{-1}(M) is fuzzy open set. Hence φ is fuzzy continuous.

Definition 3.15. For topological space $(\mu, \mathfrak{I})$ and (σ, Σ) the function $\varphi: \mu \to \sigma$ is said to be fuzzy

β_s-continuous if for each fuzzy β- set subset H of σ, then $\varphi^{-1}(H)$ is fuzzy β-set

Theorem 3.16. Let $\varphi: \mu \to \sigma$be fuzzy β-supercontinuous function if $\sigma$is extremally disconnected space then φ is fuzzy β_s- continuous.

Proof: Let V be a fuzzy β-set in σ. Since is extremally disconnected space, then V is fuzzy open set.

Since φ is fuzzy β-supercontinuous function. Thus $\varphi^{-1}(V)$is fuzzy β-set in μ. Hence, φ is fuzzy β_s-continuous.

Proposition 3.17. Every fuzzy β_s-continuous function is fuzzy b-continuous.

The proof is obvious.

Conclusion. In this research, we continue the study of continuity, and we defined new concepts of fuzzy continuous that is fuzzy β- supercontinuous. Moreover, we examined the related properties and proved some theorems.

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