Construction of q-ary(n,M,d)-codes in PG(2,16)

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Abstract The goal of this paper was to study the applications of the projective plane PG (2, q) over a Galois field of order q in the projective q-ary (n, M, d) -code such that the parameters length of code n, the maximum value size code M, and the minimum distance d with the error-correcting e according to an incidence matrix have been calculated. Also, this research provides examples and theorems of links between the combinatorial structures and coding theory. The method of the research depends on the classification of the points and lines in PG (2, q).

Keywords: Algebraic geometry, coding theory, incidence matrix.

1.Introduction

In coding theory, many mathematicians have studied the application of a projective plane over a Galois field Al-Seraji [6],[13] introduced the links between the projective plane of order 17 and error-correcting codes and Al-Zangana [5] described the link between the projective plane of order 19 and error-correcting codes. and also provided some other important results.. For example, Hirschfeld [1], [2], [8], [9] demonstrated more than a few theorems and definitions about the relations between finite projective geometry and coding theory. Hill [3] classified the concepts and tools of coding theory. Applications of the projective plane over a Galois field of order q, PG (2, q) for q=2,3,4 in projective linear codes have also been studied [1], [14], [15]. In [6],[7] Yahya and Al-Zangana studied linear codes, [10],[11] Khalaf. & Yahya studied perfect linear codes. In the current paper, we are looking at the projective plane over a finite field of order $q=4^2$ Background

Research in this area would benefit from the following results.

Theorem (1.1) [1].

Aq-ary (n, M,d)- code C

Satisfies:

$$M\left\{\binom{n}{0}+\binom{n}{1}(q-1)+\cdots+\binom{n}{e}(q-1)e\right\}\leq q^n\Big]$$

Where [Theorem 1.1,1] gives a necessary condition for the existence of [n,k,d]q code, with d= 2e+1 or 2e+2

Theorem (1.2) [1].

W is a subspace of a vector space V over a Finite (F_{16}+,0) if and only if :

Theorem (1.2)[1].

- W is a nonempty subset of V (i.e. ø ≠C ⊆ X).
- W is closed under the binary operation + defined on V (i.e. x+ c ∈ C)
- W is closed under the scalar multiplication defined on $F \times V$ (i.e. k– $v \in W \; \forall \; k \in F$ and $v \in W$).

Definition (2.1) [2]:

Let $F(x) = X^n - a_{n-1} X^{n-1} -\ldots.- a_0$

In this case, the polynomial must have a degree of n ≥ 1 over *Fq*. An nn matrix defines *C(f)*; it is a companion matrix to the adjoint matrix *C(f)*

$$C(f) = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 \\ . & . & . & \dots & . & . \\ . & . & . & \dots & . & . \\ 0 & 0 & 0 & \dots & . & 1 \\ a_0 & a_1 & a_2 & \dots & a_{n-2} & a_{n-1} \end{bmatrix}$$

Infinite fields with cubic curves of order 16 are classified

Let the polynomial $g_1(x) = x^3 + x^2 + x + \lambda$ and $F_{16} = \frac{F3[x]}{(g1(x))}$

Which has 16 elements namely 0,1, λ, λ^2, λ^3, λ^4, λ^6 , λ^7, λ^8, λ^9, λ^{10}, λ^{11}, λ^{12}, λ^{13}, λ^{14} where λ be x plus the ideal <g1(x)> generated by polynomial of degree 4 with coefficients in $F_3 = \{0,1,2,3\}$.

The companion matrix of $g_2(x) = x^3 + x^2 + x + \lambda$ Based on PG (2,16), the following points and lines were generated::

$$Pi = [\,1,0,0]\; C(g)^i = [\,1,\,0,\,0] \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \lambda & 1 & 1 \end{pmatrix}, i = 0, 1, 2, \dots\dots\dots\dots\dots,273$$

TABLE 1. The points of PG (2,16)

I	PI	I	PI	I	PI
0	(2, 1, 1)	41	(2 ,9 ,14)	82	(2, 10,11)
1	(1, 2, 1)	42	(2 ,15, 6)	83	(2, 14, 4)
2	(1, 1, 2)	43	(2 ,13, 8)	84	(2, 7 , 6)
3	(2,16,16)	44	(2 ,8 ,11)	85	(2, 13, 5)
4	(2, 5 , 1)	45	(2,14,12)	86	(2, 12 , 3)
5	(1 , 2, 5)	46	(2, 11, 9)	87	(2, 4 , 8)
6	(2,16,12)	47	(2 , 3 , 9)	88	(2, 8 ,13)
7	(2, 11, 2)	48	(2, 3 , 8)	89	(2, 2 , 6)
8	(2, 1, 8)	49	(2, 8, 6)	90	(2, 13, 13)
9	(2, 8, 16)	50	(2, 13, 9)	91	(2, 2 ,1)
10	(2, 5, 10)	51	(2 ,3, 2)	92	(1, 2, 2)
11	(2,10, 6)	52	(2, 1, 5)	93	(2, 16 , 1)
12	(2, 13, 2)	53	(2,12,16)	94	(1 , 2 ,16)
13	(2,1, 13)	54	(2, 5 ,13)	95	(2, 16 , 5)
14	(2,2, 16)	55	(2, 2 ,10)	96	(2, 12, 13)
15	(2, 5, 5)	56	(2,10,10)	97	(2, 2, 4)
16	(2,12, 1)	57	(2,10 , 1)	98	(2, 7 , 7)
17	(1, 2,12)	58	(1,2 ,10)	99	(2, 6, 1)
18	(2,16,11)	59	(2,16,10)	100	(1, 2, 6)
19	(2,14,11)	60	(2,10 ,4)	101	(2, 16, 13)
20	(2,14,15)	61	(2,15,13)	102	(2, 2, 15)
21	(2, 9, 4)	62	(2, 2 , 9)	103	(2, 9, 9)
22	(2,7, 11)	63	(2,3 , 3)	104	(2, 3, 1)
23	(2,14,13)	64	(2, 4 ,1)	105	(1, 2, 3)
24	(2, 2 , 5)	65	(1, 2, 4)	106	(2, 16, 4)

25	**(2,12,12)**	**66**	**(2,16, 7)**	**107**	**(2, 7, 12)**
26	**(2, 11, 1)**	**67**	**(2, 6 , 8)**	**108**	**(2, 11, 6)**
27	**(1, 2, 11)**	**68**	**(2, 8 ,7)**	**109**	**(2, 13 ,11)**
28	**(2,16,14)**	**69**	**(2 , 6 , 5)**	**110**	**(2, 14, 9)**
29	**(2, 15, 9)**	**70**	**(2, 12, 5)**	**111**	**(2, 3, 11)**
30	**(2,3,14)**	**71**	**(2, 12, 10)**	**112**	**(2, 14, 10)**
31	**(2,15, 2)**	**72**	**(2, 10, 9)**	**113**	**(2, 10, 2)**
32	**(2, 1 ,7)**	**73**	**(2, 3, 5)**	**114**	**(2, 1 , 3)**
33	**(2, 6,16)**	**74**	**(2, 12, 7)**	**115**	**(2, 4, 16)**
34	**(2, 5, 11)**	**75**	**(2, 6, 11)**	**116**	**(2 , 5, 15)**
35	**(2,14, 8)**	**76**	**(2, 14 , 6)**	**117**	**(2, 9 ,11)**
36	**(2, 8 ,14)**	**77**	**(2, 13, 3)**	**118**	**(2, 14, 7)**
37	**(2,15 , 8)**	**78**	**(2, 4, 6)**	**119**	**(2, 6, 10)**
38	**(2, 8,10)**	**79**	**(2, 13 , 7)**	**120**	**(2, 10, 13)**
39	**(2, 10, 7)**	**80**	**(2 , 6, 14)**	**121**	**(2, 2, 12)**
40	**(2 ,6, 15)**	**81**	**(2,5 ,10)**	**122**	**(2, 11, 11)**
123	**(2, 14 , 1)**	**155**	**(2, 10, 4)**	**199**	**(2,10 ,16)**
124	**(1 , 2, 14)**	**156**	**(2, 7 ,14)**	**200**	**(2 , 5 , 8)**
125	**(2, 16, 15)**	**157**	**(2, 15, 3)**	**201**	**(2 , 8, 12)**
126	**(2, 9 , 5)**	**158**	**(2, 4, 12)**	**202**	**(2, 11 ,13)**
127	**(2, 12 , 2)**	**159**	**(2, 11 ,10)**	**203**	**(2, 2, 7)**
128	**(2 , 1 , 6)**	**160**	**(2 ,10, 5)**	**204**	**(2 , 6 , 6)**
129	**(2, 13, 16)**	**161**	**(2 ,12, 11)**	**245**	**(2, 7 , 2)**
130	**(2 , 5, 12)**	**162**	**(2 ,14 , 5)**	**246**	**(2, 1 ,11)**
131	**(2 ,11, 3)**	**163**	**(2, 12, 8)**	**247**	**(2, 14 ,16)**
132	**(2, 4, 3)**	**164**	**(2 , 8 , 2)**	**248**	**(2, 5, 7)**
133	**(2, 4, 5)**	**165**	**(2 , 1, 14)**	**249**	**(2, 6 , 7)**
134	**(2, 12, 4)**	**166**	**(2 ,15 ,16)**	**250**	**(2 , 6 , 4)**
135	**(2, 7, 3)**	**167**	**(2, 5 , 4)**	**251**	**(2, 7, 9)**
136	**(2, 4, 2)**	**168**	**(2 ,7 ,5)**	**252**	**(2 , 3 , 7)**
137	**(2 , 1 , 9)**	**169**	**(2, 12, 9)**	**253**	**(2, 6 ,13)**
138	**(2, 3 ,16)**	**170**	**(2 , 3, 15)**	**254**	**(2 , 2 , 3)**
139	**(2 , 5 , 9)**	**171**	**(2 , 9 ,15)**	**255**	**(2, 4 ,, 4)**
140	**(2, 3, 13)**	**172**	**(2, 9 , 8)**	**256**	**(2, 7 , 1)**
141	**(2, 2, 11)**	**173**	**(2 , 8 , 5)**	**257**	**(1, 2, 7)**
142	**(2, 14, 14)**	**174**	**(2 12, 15)**	**258**	**(2 ,16 , 6)**
143	**(2, 15 , 1)**	**175**	**(2, 9 ,12)**	**259**	**(2 ,13, 6)**
144	**(1 , 2, 15)**	**176**	**(2 ,11, 12)**	**260**	**(2, 13, 10)**
145	**(2, 16, 9)**	**177**	**(2, 11, 4)**	**261**	**(2 ,10 ,15)**
146	**(2, 3 ,10)**	**178**	**(2 ,7, 10)**	**262**	**(2 , 9, 6)**
147	**(2, 10 ,3)**	**179**	**(2, 10, 12)**	**263**	**(2 ,13, 15)**
148	**(2, 4, 10)**	**180**	**(2 ,11 , 7)**	**264**	**(2 , 9 , 7)**
149	**(2, 10, 8)**	**181**	**(2 , 6, 2)**	**265**	**(2 , 6 , 9)**
150	**(2, 8 , 9)**	**182**	**(2 , 1, 2)**	**266**	**(2, 3, 12)**

151	**(2, 3 , 4)**	**183**	**(2 , 1 ,,16)**	**267**	**(2, 11, 14)**
152	**(2, 7 , 4)**	**184**	**(2 , 5,16)**	**268**	**(2 ,15 ,12)**
153	**(2, 7 ,15)**	**185**	**(2, 5 ,2)**	**269**	**(2 ,11, 15)**
154	**(2, 9 ,10)**	**186**	**(2 , 1 ,15)**	**270**	**(2 , 9, 13)**
155	**(2, 10, 4)**	**187**	**(2, 9, 16)**	**271**	**(2 , 2 ,13)**
156	**(2, 7 ,14)**	**188**	**(2 , 5, 3)**	**272**	**(2 , 2 , 2)**
157	**(2, 15, 3)**	**189**	**(2 , 4, 9)**		
158	**(2, 4, 12)**	**190**	**(2, 3 , 6)**		
159	**(2, 11 ,10)**	**191**	**(2, 13 ,12)**		
160	**(2 ,10, 5)**	**192**	**(2, 11, 5)**		
149	**(2, 10, 8)**	**193**	**(2 ,12, 14)**		
150	**(2, 8 , 9)**	**194**	**(2, 15 , 7)**		
151	**(2, 3 , 4)**	**195**	**(2, 6, 3)**		
152	**(2, 7 , 4)**	**196**	**(2 , 4 ,15)**		
153	**(2, 7 ,15)**	**197**	**(2 , 9 , 2)**		
154	**(2, 9 ,10)**	**198**	**(2 , 1 ,10)**		

With select the points in PG (2,16) such that ,In the third coordinate, zero means that it belongs to $\ell\ 0 = v(z)$ such that $v(z) = tz = z$ for all t in F16/{0} therefore, and with Pi = i, i = 0,1,…….272 , we obtain $\ell\ 0 =$ { 0, 1, 4, 16, 26, 57, 64, 91, 93, 99,104,123,143,205,219,228,256 } remove this,

$$\ell\ I = \ell\ 0\ c(g)I = \ell\ 0 \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \lambda & 1 & 1 \end{pmatrix}$$

TABLE 2. the lines and equation of PG (2,16)

LINES	Points	The Equations
Line1	0 1 4 16 26 57 64 91 93 99 104 123 143 205 219 228 256	**$z = 0$**
Line2	1 2 5 17 27 58 65 92 94 100 105 124 144 206 220 229 257	**$x = 0$**
Line3	2 3 6 18 28 59 66 93 95 101 106 125 145 207 221 230 258	**$x + \lambda y = 0$**
Line4	3 4 7 19 29 60 67 94 96 102 107 126 146 208 222 231 259	**$x + \lambda^{12} y + \lambda^{13} z = 0$**
Line5	4 5 8 20 30 61 68 95 97 103 108 127 147 209 223 232 260	**$x + \lambda^{12} y + \lambda^{9} z = 0$**
Line6	5 6 9 21 31 62 69 96 98 104 109 128 148 210 224 233 261	**$x + \lambda^{14} y + \lambda^{11} z = 0$**
Line7	6 7 10 22 32 63 70 97 99 105 110 129 149 211 225 234 262	**$x + \lambda^{11} y + \lambda^{10} z = 0$**
Line8	7 8 11 23 33 64 71 98 100 106 111 130 150 212 226 235 263	**$x + \lambda^{13} y + \lambda^{9} z = 0$**
Line9	8 9 12 24 34 65 72 99 101 107 112 131 151 213 227 236 264	**$x + \lambda^{11} y + \lambda^{9} z = 0$**
Line10	9 10 13 25 35 66 73 100 102 108 113 132 152 214 228 237 265	**$x + \lambda^{8} y + \lambda^{4} z = 0$**
Line11	10 11 14 26 36 67 74 101 103 109 114 133 153 215 229 238 266	**$x + \lambda^{6} y + \lambda^{14} z = 0$**
Line12	11 12 15 27 37 68 75 102 104 110 115 134 154 216 230 239 267	**$x + \lambda^{4} y + \lambda^{5} z = 0$**
Line13	12 13 16 28 38 69 76 103 105 111 116 135 155 217 231 240 268	**$x + \lambda^{5} y + \lambda^{4} z = 0$**
Line14	13 14 17 29 39 70 77 104 106 112 117 136 156 218 232 241 269	**$x + \lambda^{14} y + \lambda^{4} z = 0$**
Line15	14 15 18 30 40 71 78 105 107 113 118 137 157 219 233 242 270	**$x + \lambda^{9} y + \lambda^{8} z = 0$**
Line16	15 16 19 31 41 72 79 106 108 114 119 138 158 220 234 243 271	**$x + \lambda^{5} y + \lambda^{14} z = 0$**
Line17	16 17 20 32 42 73 80 107 109 115 120 139 159 221 235 244 272	**$x + \lambda^{5} y + \lambda^{10} z = 0$**
Line18	17 18 21 33 43 74 81 108 110 116 121 140 160 222 236 245 0	**$y + \lambda^{5} z = 0$**

Line19	18 19 22 34 44 75 82 109 111 117 122 141 161 223 237 246 1	$x+\lambda^6z=0$
Line20	19 20 23 35 45 76 83 110 112 118 123 142 162 224 238 247 2	$x+\lambda^3y=0$
Line21	20 21 24 36 46 77 84 111 113 119 124 143 163 225 239 248 3	$x+\lambda^2y+\lambda^5z=0$
Line22	21 22 25 37 47 78 85 112 114 120 125 144 164 226 240 249 4	$x+\lambda^{12}y+\lambda^{14}z=0$
Line23	22 23 26 38 48 79 86 113 115 121 126 145 165 227 241 250 5	$x+\lambda^6y+\lambda^3z=0$
Line24	23 24 27 39 49 80 87 114 116 122 127 146 166 228 242 251 6	$x+\lambda^8y+\lambda^{14}z=0$
Line25	24 25 28 40 50 81 88 115 117 123 128 147 167 229 243 252 7	$x+\lambda^3y+\lambda^{11}z=0$
Line26	25 26 29 41 51 82 89 116 118 124 129 148 168 230 244 253 8	$x+\lambda^6y+\lambda^9z=0$
Line27	26 27 30 42 52 83 90 117 119 125 130 149 169 231 245 254 9	$x+\lambda^6y+\lambda^{12}z=0$
Line28	27 28 31 43 53 84 91 118 120 126 131 150 170 232 246 255 10	$x+y+\lambda^6z=0$
Line29	28 29 32 44 54 85 92 119 121 127 132 151 171 233 247 256 11	$x+\lambda^{10}y+\lambda^{10}z=0$
Line30	29 30 33 45 55 86 93 120 122 128 133 152 172 234 248 257 12	$x+\lambda y+\lambda^{11}z=0$
Line31	30 31 34 46 56 87 94 121 123 129 134 153 173 235 249 258 13	$x+\lambda^3y+\lambda^4z=0$
Line32	31 32 35 47 57 88 95 122 124 130 135 154 174 236 250 259 14	$x+\lambda^7y+\lambda^{10}z=0$
Line33	32 33 36 48 58 89 96 123 125 131 136 155 175 237 251 260 15	$x+\lambda^3y+\lambda^{10}z=0$
Line34	33 34 37 49 59 90 97 124 126 132 137 156 176 238 252 261 16	$x+\lambda^5y+\lambda^8z=0$
Line35	34 35 38 50 60 91 98 125 127 133 138 157 177 239 253 262 17	$x+y+\lambda^5z=0$
Line36	35 36 39 51 61 92 99 126 128 134 139 158 178 240 254 263 18	$x+\lambda^{11}y+\lambda^{11}z=0$
Line37	36 37 40 52 62 93 100 127 129 135 140 159 179 241 255 264 19	$x+\lambda y+\lambda^{12}z=0$
Line38	37 38 41 53 63 94 101 128 130 136 141 160 180 242 256 265 20	$x+\lambda^{10}y+\lambda^{11}z=0$
Line39	38 39 42 54 64 95 102 129 131 137 142 161 181 243 257 266 21	$x+\lambda^{13}y+\lambda^8z=0$
Line40	39 40 43 55 65 96 103 130 132 138 143 162 182 244 258 267 22	$x+\lambda^2y+z=0$
Line41	40 41 44 56 66 97 104 131 133 139 144 163 183 245 259 268 23	$x+\lambda^{14}y+\lambda z=0$
Line42	41 42 45 57 67 98 105 132 134 140 145 164 184 246 260 269 24	$x+\lambda^7y+\lambda^6z=0$
Line43	42 43 46 58 68 99 106 133 135 141 146 165 185 247 261 270 25	$x+\lambda^{11}y+\lambda^3z=0$
Line44	43 44 47 59 69 100 107 134 136 142 147 166 186 248 262 271 26	$x+\lambda^6y+\lambda^2z=0$
Line45	44 45 48 60 70 101 108 135 137 143 148 167 187 249 263 272 27	$x+\lambda^2y+\lambda^8z=0$
Line46	45 46 49 61 71 102 109 136 138 144 149 168 188 250 264 0 28	$y+\lambda^2z=0$
Line47	46 47 50 62 72 103 110 137 139 145 150 169 189 251 265 1 29	$x+\lambda^8z=0$
Line48	47 48 51 63 73 104 111 138 140 146 151 170 190 252 266 2 30	$x+\lambda^{14}y=0$
Line49	48 49 52 64 74 105 112 139 141 147 152 171 191 253 267 3 31	$x+\lambda^{13}y+\lambda^{12}z=0$
Line50	49 50 53 65 75 106 113 140 142 148 153 172 192 254 268 4 32	$x+\lambda^{12}y+\lambda^{10}z=0$
Line51	50 51 54 66 76 107 114 141 143 149 154 173 193 255 269 5 33	$x+\lambda^2y+\lambda^{14}z=0$
Line52	51 52 55 67 77 108 115 142 144 150 155 174 194 256 270 6 34	$x+\lambda^{10}y+\lambda^{12}z=0$
Line53	52 53 56 68 78 109 116 143 145 151 156 175 195 257 271 7 35	$x+\lambda^2y+\lambda^{12}z=0$
Line54	53 54 57 69 79 110 117 144 146 152 157 176 196 258 272 8 36	$x+\lambda^7y+\lambda^9z=0$
Line55	54 55 58 70 80 111 118 145 147 153 158 177 197 259 0 9 37	$y+\lambda^7z=0$
Line56	55 56 59 71 81 112 119 146 148 154 159 178 198 260 1 10 38	$x+\lambda^8z=0$
Line57	56 57 60 72 82 113 120 147 149 155 160 179 199 261 2 11 39	$x+\lambda^7y=0$
Line58	57 58 61 73 83 114 121 148 150 156 161 180 200 262 3 12 40	$x+\lambda^7y+\lambda^{14}z=0$
Line59	58 59 62 74 84 115 122 149 151 157 162 181 201 263 4 13 41	$x+\lambda^{12}y+\lambda^4z=0$
Line60	59 60 63 75 85 116 123 150 152 158 163 182 202 264 5 14 42	$x+\lambda^3y+z=0$
Line61	60 61 64 76 86 117 124 151 153 159 164 183 203 265 6 15 43	$x+\lambda^{13}y+\lambda z=0$

Line62	61 62 65 77 87 118 125 152 154 160 165 184 204 266 7 16 44	$x+\lambda^5y+\lambda^3z=0$
Line63	62 63 66 78 88 119 126 153 155 161 166 185 205 267 8 17 45	$x+\lambda^4y+\lambda^9z=0$
Line64	63 64 67 79 89 120 127 154 156 162 167 186 206 268 9 18 46	$x+\lambda^{13}y+\lambda^2z=0$
Line65	64 65 68 80 90 121 128 155 157 163 168 187 207 269 10 19 47	$x+\lambda^{13}y+\lambda^{11}z=0$
Line66	65 66 69 81 91 122 129 156 158 164 169 188 208 270 11 20 48	$x+y+\lambda^{13}z=0$
Line67	66 67 70 82 92 123 130 157 159 165 170 189 209 271 12 21 49	$x+\lambda^3y+\lambda^3z=0$
Line68	67 68 71 83 93 124 131 158 160 166 171 190 210 272 13 22 50	$x+\lambda y+\lambda^4z=0$
Line69	68 69 72 84 94 125 132 159 161 167 172 191 211 0 14 23 51	$y+\lambda z=0$
Line70	69 70 73 85 95 126 133 160 162 168 173 192 212 1 15 24 52	$x+\lambda^{12}z=0$
Line71	70 71 74 86 96 127 134 161 163 169 174 193 213 2 16 25 53	$x+\lambda^5y=0$
Line72	71 72 75 87 97 128 135 162 164 170 175 194 214 3 17 26 54	$x+\lambda^6y+\lambda^{11}z=0$
Line73	72 73 76 88 98 129 136 163 165 171 176 195 215 4 18 27 55	$x+\lambda^{12}y+\lambda^3z=0$
Line74	73 74 77 89 99 130 137 164 166 172 177 196 216 5 19 28 56	$x+\lambda^{11}y+\lambda^8z=0$
Line75	74 75 78 90 100 131 138 165 167 173 178 197 217 6 20 29 57	$x+\lambda^7y+\lambda^3z=0$
Line76	75 76 79 91 101 132 139 166 168 174 179 198 218 7 21 30 58	$x+y+\lambda^7z=0$
Line77	76 77 80 92 102 133 140 167 169 175 180 199 219 8 22 31 59	$x+\lambda^9y+\lambda^9z=0$
Line78	77 78 81 93 103 134 141 168 170 176 181 200 220 9 23 32 60	$x+\lambda y+\lambda^{10}z=0$
Line79	78 79 82 94 104 135 142 169 171 177 182 201 221 10 24 33 61	$x+\lambda^{14}y+z=0$
Line80	79 80 83 95 105 136 143 170 172 178 183 202 222 11 25 34 62	$x+\lambda^2y+\lambda z=0$
Line81	80 81 84 96 106 137 144 171 173 179 184 203 223 12 26 35 63	$x+\lambda^{13}y+\lambda^{11}z=0$
Line82	81 82 85 97 107 138 145 172 174 180 185 204 224 13 27 36 64	$x+\lambda^{14}y+z=0$
Line83	82 83 86 98 108 139 146 173 175 181 186 205 225 14 28 37 65	$x+\lambda^4y+\lambda^2z=0$
Line84	83 84 87 99 109 140 147 174 176 182 187 206 226 15 29 38 66	$x+\lambda^{11}y+z=0$
Line85	84 85 88 100 110 141 148 175 177 183 188 207 227 16 30 39 67	$x+\lambda^5y+\lambda z=0$
Line86	85 86 89 101 111 142 149 176 178 184 189 208 228 17 31 40 68	$x+\lambda^8y+\lambda^{13}z=0$
Line87	86 87 90 102 112 143 150 177 179 185 190 209 229 18 32 41 69	$x+\lambda^2y+\lambda^{10}z=0$
Line88	87 88 91 103 113 144 151 178 180 186 191 210 230 19 33 42 70	$x+y+\lambda^2z=0$
Line89	88 89 92 104 114 145 152 179 181 187 192 211 231 20 34 43 71	$x+\lambda^{14}y+\lambda^{14}z=0$
Line90	89 90 93 105 115 146 153 180 182 188 193 212 232 21 35 44 72	$x+\lambda y+z=0$
Line91	90 91 94 106 116 147 154 181 183 189 194 213 233 22 36 45 73	$x+y+\lambda z=0$
Line92	91 92 95 107 117 148 155 182 184 190 195 214 234 23 37 46 74	$x+y+z=0$
Line93	92 93 96 108 118 149 156 183 185 191 196 215 235 24 38 47 75	$x+\lambda y+\lambda z=0$
Line94	93 94 97 109 119 150 157 184 186 192 197 216 236 25 39 48 76	$x+\lambda y+\lambda^2 z=0$
Line95	94 95 98 110 120 151 158 185 187 193 198 217 237 26 40 49 77	$x+\lambda^6y+\lambda^7z=0$
Line96	95 96 99 111 121 152 159 186 188 194 199 218 238 27 41 50 78	$x+\lambda^{11}y+\lambda^2 z=0$
Line97	96 97 100 112 122 153 160 187 189 195 200 219 239 28 42 51 79	$x+\lambda^9y+\lambda^5z=0$
Line98	97 98 101 113 123 154 161 188 190 196 201 220 240 29 43 52 80	$x+\lambda^3y+\lambda^{12}z=0$
Line99	98 99 102 114 124 155 162 189 191 197 202 221 241 30 44 53 81	$x+\lambda^{11}y+\lambda^{14}z=0$
Line100	99 100 103 115 125 156 163 190 192 198 203 222 242 31 45 54 82	$x+\lambda^{11}y+\lambda^7z=0$
Line101	100 101 104 116 126 157 164 191 193 199 204 223 243 32 46 55 83	$x+\lambda^{14}y+\lambda^{10}z=0$
Line102	101 102 105 117 127 158 165 192 194 200 205 224 244 33 47 56 84	$x+\lambda^{11}y+\lambda^9z=0$
Line103	102 103 106 118 128 159 166 193 195 201 206 225 245 34 48 57 85	$x+\lambda^{13}y+\lambda^3z=0$
Line104	103 104 107 119 129 160 167 194 196 202 207 226 246 35 49 58 86	$x+\lambda^{14}y+\lambda^6z=0$

Line105	104 105 108 120 130 161 168 195 197 203 208 227 247 36 50 59 87	$x+\lambda^{14}y+\lambda^{13}z=0$
Line106	105 106 109 121 131 162 169 196 198 204 209 228 248 37 51 60 88	$x+\lambda^{8}y+\lambda^{7}z=0$
Line107	106 107 110 122 132 163 170 197 199 205 210 229 249 38 52 61 89	$x+\lambda^{4}y+\lambda^{12}z=0$
Line108	107 108 111 123 133 164 171 198 200 206 211 230 250 39 53 62 90	$x+\lambda^{3}y+\lambda^{7}z=0$
Line109	108 109 112 124 134 165 172 199 201 207 212 231 251 40 54 63 91	$x+y+\lambda^{3}z=0$
Line110	109 110 113 125 135 166 173 200 202 208 213 232 252 41 55 64 92	$x+\lambda^{13}y+\lambda^{13}z=0$
Line111	110 111 114 126 136 167 174 201 203 209 214 233 253 42 56 65 93	$x+\lambda y+\lambda^{4}z=0$
Line112	111 112 115 127 137 168 175 202 204 210 215 234 254 43 57 66 94	$x+\lambda^{7}y+\lambda^{8}z=0$
Line113	112 113 116 128 138 169 176 203 205 211 216 235 255 44 58 67 95	$x+\lambda^{4}y+\lambda^{11}z=0$
Line114	113 114 117 129 139 170 177 204 206 212 217 236 256 45 59 68 96	$x+\lambda^{10}y+\lambda^{14}z=0$
Line115	114 115 118 130 140 171 178 205 207 213 218 237 257 46 60 69 97	$x+\lambda^{4}y+\lambda^{14}z=0$
Line116	115 116 119 131 141 172 179 206 208 214 219 238 258 47 61 70 98	$x+\lambda^{9}y+\lambda^{13}z=0$
Line117	116 117 120 132 142 173 180 207 209 215 220 239 259 48 62 71 99	$x+\lambda^{11}y+\lambda^{5}z=0$
Line118	117 118 121 133 143 174 181 208 210 216 221 240 260 49 63 72 100	$x+\lambda^{2}y+\lambda^{13}z=0$
Line119	118 119 122 134 144 175 182 209 211 217 222 241 261 50 64 73 101	$x+\lambda^{13}y+z=0$
Line120	119 120 123 135 145 176 183 210 212 218 223 242 262 51 65 74 102	$x+\lambda^{3}y+\lambda z=0$
Line121	120 121 124 136 146 177 184 211 213 219 224 243 263 52 66 75 103	$x+\lambda^{9}y+\lambda^{12}z=0$
Line122	121 122 125 137 147 178 185 212 214 220 225 244 264 53 67 76 104	$x+\lambda^{14}y+\lambda^{8}z=0$
Line123	122 123 126 138 148 179 186 213 215 221 226 245 265 54 68 77 105	$x+\lambda^{13}y+\lambda^{2}z=0$
Line124	123 124 127 139 149 180 187 214 216 222 227 246 266 55 69 78 106	$x+\lambda^{3}y+\lambda^{6}z=0$
Line125	124 125 128 140 150 181 188 215 217 223 228 247 267 56 70 79 107	$x+\lambda^{8}y+\lambda^{11}z=0$
Line126	125 126 129 141 151 182 189 216 218 224 229 248 268 57 71 80 108	$x+\lambda^{7}y+z=0$
Line127	126 127 130 142 152 183 190 217 219 225 230 249 269 58 72 81 109	$x+\lambda^{9}y+\lambda z=0$
Line128	127 128 131 143 153 184 191 218 220 226 231 250 270 59 73 82 110	$x+\lambda^{2}y+\lambda^{11}z=0$
Line129	128 129 132 144 154 185 192 219 221 227 232 251 271 60 74 83 111	$x+\lambda^{9}y+\lambda^{11}z=0$
Line130	129 130 133 145 155 186 193 220 222 228 233 252 272 61 75 84 112	$x+\lambda^{7}y+\lambda^{2}z=0$
Line131	130 131 134 146 156 187 194 221 223 229 234 253 0 62 76 85 113	$y+\lambda^{8}z=0$
Line132	131 132 135 147 157 188 195 222 224 230 235 254 1 63 77 86 114	$x+\lambda^{14}z=0$
Line133	132 133 136 148 158 189 196 223 225 231 236 255 2 64 78 87 115	$x+\lambda^{13}y=0$
Line134	133 134 137 149 159 190 197 224 226 232 237 256 3 65 79 88 116	$x+\lambda^{10}y+\lambda^{8}z=0$
Line135	134 135 138 150 160 191 198 225 227 233 238 257 4 66 80 89 117	$x+\lambda^{12}y+\lambda^{7}z=0$
Line136	135 136 139 151 161 192 199 226 228 234 239 258 5 67 81 90 118	$x+\lambda^{8}y+\lambda^{5}z=0$
Line137	136 137 140 152 162 193 200 227 229 235 240 259 6 68 82 91 119	$x+y+\lambda^{8}z=0$
Line138	137 138 141 153 163 194 201 228 230 236 241 260 7 69 83 92 120	$x+\lambda^{8}y+\lambda^{8}z=0$
Line139	138 139 142 154 164 195 202 229 231 237 242 261 8 70 84 93 121	$x+\lambda y+\lambda^{9}z=0$
Line140	139 140 143 155 165 196 203 230 232 238 243 262 9 71 85 94 122	$x+\lambda^{2}y+\lambda^{3}z=0$
Line141	140 141 144 156 166 197 204 231 233 239 244 263 10 72 86 95 123	$x+\lambda^{3}y+\lambda^{5}z=0$
Line142	141 142 145 157 167 198 205 232 234 240 245 264 11 73 87 96 124	$x+\lambda^{4}y+\lambda^{7}z=0$
Line143	142 143 146 158 168 199 206 233 235 241 246 265 12 74 88 97 125	$x+\lambda^{2}y+\lambda^{6}z=0$
Line144	143 144 147 159 169 200 207 234 236 242 247 266 13 75 89 98 126	$x+\lambda^{2}y+\lambda^{4}z=0$
Line145	144 145 148 160 170 201 208 235 237 243 248 267 14 76 90 99 127	$x+\lambda^{11}y+\lambda^{13}z=0$
Line146	145 146 149 161 171 202 209 236 238 244 249 268 15 77 91 100 128	$x+y+\lambda^{11}z=0$
Line147	146 147 150 162 172 203 210 237 239 245 250 269 16 78 92 101 129	$x+\lambda^{5}y+\lambda^{5}z=0$

Line148	147 148 151 163 173 204 211 238 240 246 251 270 17 79 93 102 130	$x+\lambda\ y+\lambda^{6}z=0$
Line149	148 149 152 164 174 205 212 239 241 247 252 271 18 80 94 103 131	$x+\lambda^{4}y+\lambda^{5}z=0$
Line150	149 150 153 165 175 206 213 240 242 248 253 272 19 81 95 104 132	$x+\lambda^{14}y+\lambda^{3}z=0$
Line151	150 151 154 166 176 207 214 241 243 249 254 0 20 82 96 105 133	$y+\lambda^{14}z=0$
Line152	151 152 155 167 177 208 215 242 244 250 255 1 21 83 97 106 134	$x+\lambda^{13}z=0$
Line153	152 153 156 168 178 209 216 243 245 251 256 2 22 84 98 107 135	$x+\lambda^{10}y=0$
Line154	153 154 157 169 179 210 217 244 246 252 257 3 23 85 99 108 136	$x+\lambda^{11}y+\lambda^{6}z=0$
Line155	154 155 158 170 180 211 218 245 247 253 258 4 24 86 100 109 137	$x+\lambda^{12}y+\lambda^{8}z=0$
Line156	155 156 159 171 181 212 219 246 248 254 259 5 25 87 101 110 138	$x+\lambda^{9}y+\lambda^{6}z=0$
Line157	156 157 160 172 182 213 220 247 249 255 260 6 26 88 102 111 139	$x+\lambda^{6}y+z=0$
Line158	157 158 161 173 183 214 221 248 250 256 261 7 27 89 103 112 140	$x+\lambda^{10}y+\lambda\ z=0$
Line159	158 159 162 174 184 215 222 249 251 257 262 8 28 90 104 113 141	$x+\lambda^{14}y+\lambda^{9}z=0$
Line160	159 160 163 175 185 216 223 250 252 258 263 9 29 91 105 114 142	$x+y+\lambda^{14}z=0$
Line161	160 161 164 176 186 217 224 251 253 259 264 10 30 92 106 115 143	$x+\lambda^{2}y+\lambda^{2}z=0$
Line162	161 162 165 177 187 218 225 252 254 260 265 11 31 93 107 116 144	$x+\lambda\ y+\lambda^{3}z=0$
Line163	162 163 166 178 188 219 226 253 255 261 266 12 32 94 108 117 145	$x+\lambda^{9}y+\lambda^{10}z=0$
Line164	163 164 167 179 189 220 227 254 256 262 267 13 33 95 109 118 146	$x+\lambda^{10}y+\lambda^{4}z=0$
Line165	164 165 168 180 190 221 228 255 257 263 268 14 34 96 110 119 147	$x+\lambda^{8}y+\lambda^{3}z=0$
Line166	165 166 169 181 191 222 229 256 258 264 269 15 35 97 111 120 148	$x+\lambda^{10}y+\lambda^{3}z=0$
Line167	166 167 170 182 192 223 230 257 259 265 270 16 36 98 112 121 149	$x+\lambda^{5}y+z=0$
Line168	167 168 171 183 193 224 231 258 260 266 271 17 37 99 113 122 150	$x+\lambda^{11}y+\lambda^{11}z=0$
Line169	168 169 172 184 194 225 232 259 261 267 272 18 38 100 114 123 151	$x+\lambda^{3}y+\lambda^{14}z=0$
Line170	169 170 173 185 195 226 233 260 262 268 0 19 39 101 115 124 152	$y+\lambda^{3}z=0$
Line171	170 171 174 186 196 227 234 261 263 269 1 20 40 102 116 125 153	$x+\lambda^{2}z=0$
Line172	171 172 175 187 197 228 235 262 264 270 2 21 41 103 117 126 154	$x+\lambda^{8}y=0$
Line173	172 173 176 188 198 229 236 263 265 271 3 22 42 104 118 127 155	$x+\lambda^{14}y+\lambda^{7}z=0$
Line174	173 174 177 189 199 230 237 264 266 272 4 23 43 105 119 128 156	$x+\lambda^{12}y+\lambda^{11}z=0$
Line175	174 175 178 190 200 231 238 265 267 0 5 24 44 106 120 129 157	$y+\lambda^{12}z=0$
Line176	175 176 179 191 201 232 239 266 268 1 6 25 45 107 121 130 158	$x+\lambda^{5}z=0$
Line177	176 177 180 192 202 233 240 267 269 2 7 26 46 108 122 131 159	$x+\lambda^{6}y=0$
Line178	177 178 181 193 203 234 241 268 270 3 8 27 47 109 123 132 160	$x+\lambda^{3}y+\lambda^{9}z=0$
Line179	178 179 182 194 204 235 242 269 271 4 9 28 48 110 124 133 161	$x+\lambda^{12}y+z=0$
Line180	179 180 183 195 205 236 243 270 272 5 10 29 49 111 125 134 162	$x+\lambda^{4}y+\lambda z=0$
Line181	180 181 184 196 206 237 244 271 0 6 11 30 50 112 126 135 163	$y+\lambda^{4}z=0$
Line182	181 182 185 197 207 238 245 272 1 7 12 31 51 113 127 136 164	$x+z=0$
Line183	182 183 186 198 208 239 246 0 2 8 13 32 52 114 128 137 165	$y=0$
Line184	183 184 187 199 209 240 247 1 3 9 14 33 53 115 129 138 166	$x+\lambda\ z=0$
Line185	184 185 188 200 210 241 248 2 4 10 15 34 54 116 130 139 167	$x+\lambda^{12}y=0$
Line186	185 186 189 201 211 242 249 3 5 11 16 35 55 117 131 140 168	$x+\lambda^{5}y+\lambda^{2}z=0$
Line187	186 187 190 202 212 243 250 4 6 12 17 36 56 118 132 141 169	$x+\lambda^{12}y+\lambda^{2}z=0$
Line188	187 188 191 203 213 244 251 5 7 13 18 37 57 119 133 142 170	$x+\lambda^{7}y+\lambda^{4}z=0$
Line189	188 189 192 204 214 245 252 6 8 14 19 38 58 120 134 143 171	$x+\lambda^{2}y+\lambda^{9}z=0$
Line190	189 190 193 205 215 246 253 7 9 15 20 39 59 121 135 144 172	$x+\lambda^{4}y+\lambda^{6}z=0$

Line191	190 191 194 206 216 247 254 8 10 16 21 40 60 122 136 145 173	$x+\lambda^5y+\lambda^9z=0$
Line192	191 192 195 207 217 248 255 9 11 17 22 41 61 123 137 146 174	$x+\lambda^3y+\lambda^8z=0$
Line193	192 193 196 208 218 249 256 10 12 18 23 42 62 124 138 147 175	$x+\lambda^{10}y+\lambda^{13}z=0$
Line194	193 194 197 209 219 250 257 11 13 19 24 43 63 125 139 148 176	$x+\lambda^9y+\lambda^4z=0$
Line195	194 195 198 210 220 251 258 12 14 20 25 44 64 126 140 149 177	$x+\lambda^{13}y+\lambda^7z=0$
Line196	195 196 199 211 221 252 259 13 15 21 26 45 65 127 141 150 178	$x+\lambda^6y+\lambda^4z=0$
Line197	196 197 200 212 222 253 260 14 16 22 27 46 66 128 142 151 179	$x+\lambda^5y+\lambda^{11}z=0$
Line198	197 198 201 213 223 254 261 15 17 23 28 47 67 129 143 152 180	$x+\lambda^2y+\lambda^7z=0$
Line199	198 199 202 214 224 255 262 16 18 24 29 48 68 130 144 153 181	$x+\lambda^5y+\lambda^7z=0$
Line200	199 200 203 215 225 256 263 17 19 25 30 49 69 131 145 154 182	$x+\lambda^{10}y+z=0$
Line201	200 201 204 216 226 257 264 18 20 26 31 50 70 132 146 155 183	$x+\lambda^6y+\lambda z=0$
Line202	201 202 205 217 227 258 265 19 21 27 32 51 71 133 147 156 184	$x+\lambda^4y+\lambda^{10}z=0$
Line203	202 203 206 218 228 259 266 20 22 28 33 52 72 134 148 157 185	$x+\lambda^8y+\lambda^{12}z=0$
Line204	203 204 207 219 229 260 267 21 23 29 34 53 73 135 149 158 186	$x+\lambda^9y+\lambda^2z=0$
Line205	204 205 208 220 230 261 268 22 24 30 35 54 74 136 150 159 187	$x+\lambda^4y+\lambda^{13}z=0$
Line206	205 206 209 221 231 262 269 23 25 31 36 55 75 137 151 160 188	$x+\lambda^4y+\lambda^8z=0$
Line207	206 207 210 222 232 263 270 24 26 32 37 56 76 138 152 161 189	$x+\lambda^6y+\lambda^{10}z=0$
Line208	207 208 211 223 233 264 271 25 27 33 38 57 77 139 153 162 190	$x+\lambda^7y+\lambda^{13}z=0$
Line209	208 209 212 224 234 265 272 26 28 34 39 58 78 140 154 163 191	$x+\lambda^6y+\lambda^{13}z=0$
Line210	209 210 213 225 235 266 0 27 29 35 40 59 79 141 155 164 192	$y+\lambda^6z=0$
Line211	210 211 214 226 236 267 1 28 30 36 41 60 80 142 156 165 193	$x+\lambda^3z=0$
Line212	211 212 215 227 237 268 2 29 31 37 42 61 81 143 157 166 194	$x+\lambda^2y=0$
Line213	212 213 216 228 238 269 3 30 32 38 43 62 82 144 158 167 195	$x+\lambda^8y+\lambda^{10}z=0$
Line214	213 214 217 229 239 270 4 31 33 39 44 63 83 145 159 168 196	$x+\lambda^{12}y+\lambda^5z=0$
Line215	214 215 218 230 240 271 5 32 34 40 45 64 84 146 160 169 197	$x+\lambda^{13}y+\lambda^{10}z=0$
Line216	215 216 219 231 241 272 6 33 35 41 46 65 85 147 161 170 198	$x+\lambda^9y+\lambda^7z=0$
Line217	216 217 220 232 242 0 7 34 36 42 47 66 86 148 162 171 199	$y+\lambda^9z=0$
Line218	217 218 221 233 243 1 8 35 37 43 48 67 87 149 163 172 200	$x+\lambda^9z=0$
Line219	218 219 222 234 244 2 9 36 38 44 49 68 88 150 164 173 201	$x+\lambda^9y=0$
Line220	219 220 223 235 245 3 10 37 39 45 50 69 89 151 165 174 202	$x+\lambda^9y+\lambda^3z=0$
Line221	220 221 224 236 246 4 11 38 40 46 51 70 90 152 166 175 203	$x+\lambda^{12}y+\lambda^6z=0$
Line222	221 222 225 237 247 5 12 39 41 47 52 71 91 153 167 176 204	$x+y+\lambda^{12}z=0$
Line223	222 223 226 238 248 6 13 40 42 48 53 72 92 154 168 177 205	$x+\lambda^4y+\lambda^4z=0$
Line224	223 224 227 239 249 7 14 41 43 49 54 73 93 155 169 178 206	$x+\lambda y+\lambda^5z=0$
Line225	224 225 228 240 250 8 15 42 44 50 55 74 94 156 170 179 207	$x+\lambda^8y+\lambda^9z=0$
Line226	225 226 229 241 251 9 16 43 45 51 56 75 95 157 171 180 208	$x+\lambda^5y+\lambda^{13}z=0$
Line227	226 227 230 242 252 10 17 44 46 52 57 76 96 158 172 181 209	$x+\lambda^7y+\lambda^{12}z=0$
Line228	227 228 231 243 253 11 18 45 47 53 58 77 97 159 173 182 210	$x+\lambda^8y+z=0$
Line229	228 229 232 244 254 12 19 46 48 54 59 78 98 160 174 183 211	$x+\lambda^8y+\lambda z=0$
Line230	229 230 233 245 255 13 20 47 49 55 60 79 99 161 175 184 212	$x+\lambda^{11}y+\lambda^4z=0$
Line231	230 231 234 246 256 14 21 48 50 56 61 80 100 162 176 185 213	$x+\lambda^{10}y+\lambda^6z=0$
Line232	231 232 235 247 257 15 22 49 51 57 62 81 101 163 177 186 214	$x+\lambda^7y+\lambda^2z=0$
Line233	232 233 236 248 258 16 23 50 52 58 63 82 102 164 178 187 215	$x+\lambda^5y+\lambda^{12}z=0$

Line234	233 234 237 249 259 17 24 51 53 59 64 83 103 165 179 188 216	$x+\lambda^{13} y+\lambda^{3} z=0$
Line235	234 235 238 250 260 18 25 52 54 60 65 84 104 166 180 189 217	$x+\lambda^{14}y+ \lambda^{12}z=0$
Line236	235 236 239 251 261 19 26 53 55 61 66 85 105 167 181 190 218	$x+\lambda^{6}y+ \lambda^{5}z=0$
Line237	236 237 240 252 262 20 27 54 56 62 67 86 106 168 182 191 219	$x+\lambda^{9}y+ z=0$
Line238	237 238 241 253 263 21 28 55 57 63 68 87 107 169 183 192 220	$x+\lambda^{7}y+ \lambda z=0$
Line239	238 239 242 254 264 22 29 56 58 64 69 88 108 170 184 193 221	$x+\lambda^{13}y+ \lambda^{10}z=0$
Line240	239 240 243 255 265 23 30 57 59 65 70 89 109 171 185 194 222	$x+\lambda^{7}y+\lambda^{5}z=0$
Line241	240 241 244 256 266 24 31 58 60 66 71 90 110 172 186 195 223	$x+\lambda^{10}y+\lambda^{2}z =0$
Line242	241 242 245 257 267 25 32 59 61 67 72 91 111 173 187 196 224	$x+ y+\lambda^{10} z=0$
Line243	242 243 246 258 268 26 33 60 62 68 73 92 112 174 188 197 225	$x+\lambda^{6} y+ \lambda^{6}z=0$
Line244	243 244 247 259 269 27 34 61 63 69 74 93 113 175 189 198 226	$x+\lambda y+ \lambda^{7}z=0$
Line245	244 245 248 260 270 28 35 62 64 70 75 94 114 176 190 199 227	$x+\lambda^{13} y+\lambda^{14} z=0$
Line246	245 246 249 261 271 29 36 63 65 71 76 95 115 177 191 200 228	$x+\lambda^{8}y+ \lambda^{6}z=0$
Line247	246 247 250 262 272 30 37 64 66 72 77 96 116 178 192 201 229	$x+\lambda^{13}y+ \lambda^{6}z=0$
Line248	247 248 251 263 0 31 38 65 67 73 78 97 117 179 193 202 230	$y+ \lambda^{13}z=0$
Line249	248 249 252 264 1 32 39 66 68 74 79 98 118 180 194 203 231	$X+662+ \lambda^{10}z=0$
Line250	249 250 253 265 2 33 40 67 69 75 80 99 119 181 195 204 232	$x+\lambda^{11}y =0$
Line251	250 251 254 266 3 34 41 68 70 76 81 100 120 182 196 205 233	$x+\lambda^{4} y+ z=0$
Line252	251 252 255 267 4 35 42 69 71 77 82 101 121 183 197 206 234	$x+\lambda^{12}y+ \lambda z=0$
Line253	252 253 256 268 5 36 43 70 72 78 83 102 122 184 198 207 235	$x+\lambda^{10} y+\lambda^{7} z=0$
Line254	253 254 257 269 6 37 44 71 73 79 84 103 123 185 199 208 236	$x+\lambda^{3}y+ \lambda^{13}z=0$
Line255	254 255 258 270 7 38 45 72 74 80 85 104 124 186 200 209 237	$x+\lambda^{14}y+ \lambda^{2}z=0$
Line256	255 256 259 271 8 39 46 73 75 81 86 105 125 187 201 210 238	$x+\lambda^{10}y+ \lambda^{9}z=0$
Line257	256 257 260 272 9 40 47 74 76 82 87 106 126 188 202 211 239	$x+\lambda^{10} y+\lambda^{5}z=0$
Line258	257 258 261 0 10 41 48 75 77 83 88 107 127 189 203 212 240	$y+ \lambda^{10}z=0$
Line259	258 259 262 1 11 42 49 76 78 84 89 108 128 190 204 213 241	$x +\lambda^{11}z=0$
Line260	259 260 263 2 12 43 50 77 79 85 90 109 129 191 205 214 242	$x+\lambda^{4}y =0$
Line261	260 261 264 3 13 44 51 78 80 86 91 110 130 192 206 215 243	$x+ y+ \lambda^{4}z =0$
Line262	261 262 265 4 14 45 52 79 81 87 92 111 131 193 207 216 244	$x+\lambda^{12}y+ \lambda^{12}z=0$
Line263	262 263 266 5 15 46 53 80 82 88 93 112 132 194 208 217 245	$x+\lambda^{7}y+\lambda^{13} z=0$
Line264	263 264 267 6 16 47 54 81 83 89 94 113 133 195 209 218 246	$x+\lambda^{5}y+ \lambda^{6}z=0$
Line265	264 265 268 7 17 48 55 82 84 90 95 114 134 196 210 219 247	$x+\lambda^{5}y+ \lambda^{10}z=0$
Line266	265 266 269 8 18 49 56 83 85 91 96 115 135 197 211 220 248	$x+ y+\lambda^{9}z=0$
Line267	266 267 270 9 19 50 57 84 86 92 97 116 136 198 212 221 249	$x+\lambda^{7}y+\lambda^{7}z =0$
Line268	267 268 271 10 20 51 58 85 87 93 98 117 137 199 213 222 250	$x+\lambda y+\lambda^{8}z=0$
Line269	268 269 272 11 21 52 59 86 88 94 99 118 138 200 214 223 251	$x+\lambda^{11} y+ \lambda^{12}z=0$
Line270	269 270 0 12 22 53 60 87 89 95 100 119 139 201 215 224 252	$y+ \lambda^{11}z=0$
Line271	270 271 1 13 23 54 61 88 90 96 101 120 140 202 216 225 253	$x+\lambda^{4} z=0$
Line272	271 272 2 14 24 55 62 89 91 97 102 121 141 203 217 226 254	$x+ y=0$
Line273	272 0 3 15 25 56 63 90 92 98 103 122 142 204 218 227 255	$y+ z =0$

The parameters n,M,d are constructed in the following theorem.

Theorem (1,3) Code *C* with order 16 is the projective plane of order 16
[n=273, $M \leq 16^{270}$,d=17]

proof:

Suppose that the plane πq has an incidence matrix A=(aij), where

$$aij = \begin{cases} 1 & if \quad P_j \in \ell \text{ i} \\ 0 & if \quad P_j \notin \ell \text{ i} \end{cases}$$

TABLE 3. The points of a_{ij}

	P0	P1	P2	P3	P4	.	.	.	P268	P269	P270	P271	P272
ℓ0	1	1	0	0	0	.	.	.	0	0	0	1	0
ℓ1	0	1	1	0	0	.	.	.	0	0	0	0	1
ℓ2	1	0	1	1	0	.	.	.	0	0	0	0	0
ℓ3	0	1	0	1	1	.	.	.	1	0	0	0	0
ℓ4	0	0	1	0	1	.	.	.	0	1	0	0	0
.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.
ℓ268	0	0	0	0	0	.	.	.	1	1	0	0	0
ℓ269	0	0	0	0	0	.	.	.	0	1	1	0	0
ℓ270	0	0	0	0	0	.	.	.	1	0	1	1	0
ℓ271	0	0	0	0	0	.	.	.	0	1	0	1	1
ℓ272	1	0	0	0	0	.	.	.	0	0	1	0	1

Let

aa= [0,0,0,…,0]
ab = [1,1,1,…,1]
ac = [λ, λ, λ,…, λ]
ad = [λ^2,λ^2 ,λ^2,…,λ^2]
ae = [λ^3,λ^3,λ^3,…,λ^3]
af = [λ^4,λ^4 ,λ^4,…,λ^4]
ag = [λ^5,λ^5,λ^5,…,λ^5]
ah = [λ^6,λ^6 ,λ^6,…,λ^6]
ak = [λ^7,λ^7,λ^7,…,λ^7]
am = [λ^8,λ^8 ,λ^8,…,λ^8]
an = [λ^9,λ^9,λ^9,…,λ^9]
ao = [λ^{10},λ^{10},λ^{10},…,λ^{10}]
ap = [λ^{11},λ^{11} ,λ^{11},…,λ^{11}]
aq = [λ^{12},λ^{12},λ^{12},…,λ^{12}]
ar = [λ^{13},λ^{13} ,λ^{13},…,λ^{13}]
as = [λ^{14},λ^{14},λ^{14},…,λ^{14}]

ab= {1,1,1……..,1}

TABLE 4. table of a_i = ab + ℓ_i

	P0	P1	P2	P3	P4	.	.		.	P268	P269	P270	P271	P272
a0	0	0	1	1	1	.	.		.	1	1	1	0	1
a1	1	0	0	1	1	.	.		.	1	1	1	1	0
a2	0	1	0	0	1	.	.		.	1	1	1	1	1
a3	1	0	1	0	0	.	.		.	0	1	1	1	1
a4	1	1	0	1	0	.	.		.	1	0	1	1	1
.	.	.	.	.	.	.	.		.	.	.	.	.	.
.	.	.	.	.	.	.	.		.	.	.	.	.	.
.	.	.	.	.	.	.	.		.	.	.	.	.	.
a268	1	1	1	1	1	.	.		.	0	0	1	1	1

a269	1	1	1	1	1	.	.		.	1	0	0	1	1
a270	1	1	1	1	1	.	.		.	0	1	0	0	1
a271	1	1	1	1	1	.	.		.	1	0	1	0	0
a272	0	1	1	1	1	.	.		.	1	1	0	1	0

Let bi = ac + ℓ_i

ac = [λ, λ, λ,…, λ]

TABLE 5. table of bi = ac + ℓ_i

	P0	P1	P2	P3	P4	.	.	.	P268	P269	P270	P271	P272
b0	0	0	λ	λ	λ	.	.	.	λ	λ	λ	0	λ
b1	λ	0	0	λ	λ	.	.	.	λ	λ	λ	λ	0
b2	0	λ	0	0	λ	.	.	.	λ	λ	λ	λ	-1
b3	λ	0	λ	0	0	.	.	.	0	λ	λ	λ	λ
b4	λ	λ	0	λ	0	.	.	.	λ	0	λ	λ	λ
.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.
b268	λ	λ	λ	λ	λ	.	.	.	0	0	λ	λ	λ
b269	λ	λ	λ	λ	λ	.	.	.	λ	0	0	λ	λ
b270	λ	λ	λ	λ	λ	.	.	.	0	λ	0	0	λ
b271	λ	λ	λ	λ	λ	.	.	.	λ	0	λ	0	0
b272	0	λ	λ	λ	λ	.	.	.	λ	λ	0	λ	0

Let c_i = ad + ℓ_i

ad = [λ^2,λ^2 ,λ^2,…,λ^2]

TABLE 6. table of c_i = ad + ℓ_i

	P0	P1	P2	P3	P4	.	.	.	P268	P269	P270	P271	P272
c0	0	0	λ^2	λ^2	λ^2	.	.	.	λ^2	λ^2	λ^2	0	λ^2
c1	λ^2	0	0	λ^2	λ^2	.	.	.	λ^2	λ^2	λ^2	λ	0
c2	0	λ^2	0	0	λ^2	.	.	.	λ^2	λ^2	λ^2	λ	λ
c3	λ^2	0	λ^2	0	0	.	.	.	0	λ^2	λ^2	λ^2	λ^2
c4	λ^2	λ	0	λ^2	0	.	.	.	λ^2	0	λ^2	λ^2	λ^2
.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.
c268	λ^2	λ^2	λ^2	λ^2	λ^2	.	.	.	0	0	λ^2	λ^2	λ^2
c269	λ^2	λ^2	λ^2	λ^2	λ^2	.	.	.	λ^2	0	0	λ	λ^2
c270	λ^2	λ^2	λ^2	λ^2	λ^2	.	.	.	0	λ^2	0	0	λ^2
c271	λ^2	λ^2	λ^2	λ^2	λ^2	.	.	.	λ^2	0	λ^2	0	0
c272	0	λ^2	λ^2	λ^2	λ^2	.	.	.	λ^2	λ^2	0	λ^2	0

ae = [λ^3,λ^3,λ^3,…,λ^3]

TABLE 7. table of d_i = ae + ℓ_i

	P0	P1	P2	P3	P4	.	.	.	P268	P269	P270	P271	P272
d 0	0	0	λ^3	λ^3	λ^3	.	.	.	λ^3	λ^3	λ^3	0	λ^2
d1	λ^3	0	0	λ^3	λ^3	.	.	.	λ^3	λ^3	λ^3	λ	0
d2	0	λ^3	0	0	λ^3	.	.	.	λ^3	λ^3	λ^3	λ	λ
d3	λ^3	0	λ^3	0	0	.	.	.	0	λ^3	λ^3	λ^3	λ^3
d4	λ^3	λ	0	λ^3	0	.	.	.	λ^3	0	λ^3	λ^3	λ^3

.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.
d268	λ^3	λ^3	λ^3	λ^3	λ^3	.	.	.	0	0	λ^3	λ^3	λ^3
d269	λ^3	λ^3	λ^3	λ^3	λ^3	.	.	.	λ^3	0	0	λ	λ^3
d270	λ^3	λ^3	λ^3	λ^3	λ^3	.	.	.	0	λ^3	0	0	λ^3
d271	λ^3	λ^3	λ^3	λ^3	λ^3	.	.	.	λ^3	0	λ^3	0	0
d272	0	λ^3	λ^3	λ^3	λ^3	.	.	.	λ^3	λ^2	0	λ^3	0

af = [λ^4,λ^4 ,λ^4,…,λ^4]

TABLE 8. table of e_i = af + ℓ i

	P0	P1	P2	P3	P4	.	.	.	P268	P269	P270	P271	P272
e 0	0	0	λ^4	λ^4	λ^4	.	.	.	λ^4	λ^4	λ^4	0	λ^4
e1	λ^4	0	0	λ^4	λ^4	.	.	.	λ^4	λ^4	λ^4	λ	0
e2	0	λ^4	0	0	λ^4	.	.	.	λ^4	λ^4	λ^4	λ	λ
e3	λ^4	0	λ^4	0	0	.	.	.	0	λ^4	λ^4	λ^4	λ^4
e4	λ^4	λ	0	λ^4	0	.	.	.	λ^4	0	λ^4	λ^4	λ^4
.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.
e268	λ^4	λ^4	λ^4	λ^4	λ^4	.	.	.	0	0	λ^4	λ^4	λ^4
e269	λ^4	λ^4	λ^4	λ^4	λ^4	.	.	.	λ^4	0	0	λ^4	λ^4
e270	λ^4	λ^4	λ^4	λ^4	λ^4	.	.	.	0	λ^4	0	0	λ^4
e271	λ^4	λ^4	λ^4	λ^4	λ^4	.	.	.	λ^4	0	λ^4	0	0
e272	0	λ^4	λ^4	λ^4	λ^4	.	.	.	λ^4	λ^2	0	λ^4	0

ag = [λ^5,λ^5,λ^5,…,λ^5]

TABLE 9. table of f_i = ag + ℓ_i

	P0	P1	P2	P3	P4	.	.	.	P268	P269	P270	P271	P272
f 0	0	0	λ^5	λ^5	λ^5	.	.	.	λ^5	λ^5	λ^5	0	λ^5
f1	λ^5	0	0	λ^5	λ^5	.	.	.	λ^5	λ^5	λ^5	λ	0
f2	0	λ^5	0	0	λ^5	.	.	.	λ^5	λ^5	λ^5	λ	λ
f3	λ^5	0	λ^5	0	0	.	.	.	0	λ^5	λ^5	λ^5	λ^5
f4	λ^5	λ	0	λ^5	0	.	.	.	λ^5	0	λ^5	λ^5	λ^5
.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.
f268	λ^5	λ^5	λ^5	λ^5	λ^5	.	.	.	0	0	λ^5	λ^5	λ^5
f269	λ^5	λ^5	λ^5	λ^5	λ^5	.	.	.	λ^5	0	0	λ^5	λ^5
f270	λ^5	λ^5	λ^5	λ^5	λ^5	.	.	.	0	λ^5	0	0	λ^5
f271	λ^5	λ^5	λ^5	λ^5	λ^5	.	.	.	λ^5	0	λ^5	0	0
f272	0	λ^5	λ^5	λ^5	λ^5	.	.	.	λ^5	λ^5	0	λ^5	0

ah = [λ^6,λ^6 ,λ^6,…,λ^6]

TABLE 10. table of g_i = ah + ℓ_i

	P0	P1	P2	P3	P4	.	.	.	P268	P269	P270	P271	P272
g 0	0	0	λ^6	λ^6	λ^6	.	.	.	λ^6	λ^6	λ^6	0	λ^6
g1	λ^6	0	0	λ^6	λ^6	.	.	.	λ^6	λ^6	λ^6	λ^6	0
g2	0	λ^6	0	0	λ^6	.	.	.	λ^6	λ^6	λ^6	λ^6	λ

g3	λ^6	0	λ^6	0	0	.	.	.	0	λ^6	λ^6	λ^6	λ^6
g4	λ^6	λ	0	λ^6	0	.	.	.	λ^6	0	λ^6	λ^6	λ^6
.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.
g268	λ^6	λ^6	λ^6	λ^6	λ^6	.	.	.	0	0	λ^6	λ^6	λ^6
g269	λ^6	λ^6	λ^6	λ^6	λ^6	.	.	.	λ^6	0	0	λ^6	λ^6
g270	λ^6	λ^6	λ^6	λ^6	λ^6	.	.	.	0	λ^6	0	0	λ^6
g271	λ^6	λ^6	λ^6	λ^6	λ^6	.	.	.	λ^6	0	λ^6	0	0
g272	0	λ^6	λ^6	λ^6	λ^6	.	.	.	λ^6	λ^6	0	λ^6	0

ak = $[\lambda^7,\lambda^7,\lambda^7,\ldots,\lambda^7]$

TABLE 11. table of h_i = ak + ℓ_i

	P0	P1	P2	P3	P4	.	.	.	P268	P269	P270	P271	P272
h 0	0	0	λ^7	λ^7	λ^7	.	.	.	λ^7	λ^7	λ^7	0	λ^7
h1	λ^7	0	0	λ^7	λ^7	.	.	.	λ^7	λ^7	λ^7	λ^7	0
h2	0	λ^7	0	0	λ^7	.	.	.	λ^7	λ^7	λ^7	λ^7	λ^7
h3	λ^7	0	λ^7	0	0	.	.	.	0	λ^7	λ^7	λ^7	λ^7
h4	λ^7	λ	0	λ^7	0	.	.	.	λ^7	0	λ^7	λ^7	λ^7
.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.
h268	λ^7	λ^7	λ^7	λ^7	λ^7	.	.	.	0	0	λ^7	λ^7	λ^7
h269	λ^7	λ^7	λ^7	λ^7	λ^7	.	.	.	λ^7	0	0	λ^7	λ^7
h270	λ^7	λ^7	λ^7	λ^7	λ^7	.	.	.	0	λ^7	0	0	λ^7
h271	λ^7	λ^7	λ^7	λ^7	λ^7	.	.	.	λ^7	0	λ^7	0	0
h272	0	λ^7	λ^7	λ^7	λ^7	.	.	.	λ^7	λ^7	0	λ^7	0

am = $[\lambda^8,\lambda^8,\lambda^8,\ldots,\lambda^8]$

TABLE 12. table of j_i = am + ℓ_i

	P0	P1	P2	P3	P4	.	.	.	P268	P269	P270	P271	P272
j 0	0	0	λ^8	λ^8	λ^8	.	.	.	λ^8	λ^8	λ^8	0	λ^8
j1	λ^8	0	0	λ^8	λ^8	.	.	.	λ^8	λ^8	λ^8	λ^8	0
j2	0	λ^8	0	0	λ^8	.	.	.	λ^8	λ^8	λ^8	λ^8	λ^8
j3	λ^8	0	λ^8	0	0	.	.	.	0	λ^8	λ^8	λ^8	λ^8
j4	λ^8	λ	0	λ^8	0	.	.	.	λ^8	0	λ^8	λ^8	λ^8
.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.
j268	λ^8	λ^8	λ^8	λ^8	λ^8	.	.	.	0	0	λ^8	λ^8	λ^8
j269	λ^8	λ^8	λ^8	λ^8	λ^8	.	.	.	λ^8	0	0	λ^8	λ^8
j270	λ^8	λ^8	λ^8	λ^8	λ^8	.	.	.	0	λ^8	0	0	λ^8
j271	λ^8	λ^8	λ^8	λ^8	λ^8	.	.	.	λ^8	0	λ^8	0	0
j272	0	λ^8	λ^8	λ^8	λ^8	.	.	.	λ^8	λ^8	0	λ^8	0

an = [λ^9,λ^9,λ^9,…,λ^9]

TABLE 13. table of l_i = an + ℓ_i

	P0	P1	P2	P3	P4	.	.	.	P268	P269	P270	P271	P272
l 0	0	0	λ^9	λ^9	λ^9	.	.	.	λ^9	λ^9	λ^9	0	λ^9
l1	λ^9	0	0	λ^9	λ^9	.	.	.	λ^9	λ^9	λ^9	λ^9	0
l2	0	λ^9	0	0	λ^9	.	.	.	λ^9	λ^9	λ^9	λ^9	λ^9
l3	λ^9	0	λ^9	0	0	.	.	.	0	λ^9	λ^9	λ^9	λ^9
l4	λ^9	λ	0	λ^9	0	.	.	.	λ^9	0	λ^9	λ^9	λ^9
.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.
l268	λ^9	λ^9	λ^9	λ^9	λ^9	.	.	.	0	0	λ^9	λ^9	λ^9
l269	λ^9	λ^9	λ^9	λ^9	λ^9	.	.	.	λ^9	0	0	λ^9	λ^9
l270	λ^9	λ^9	λ^9	λ^9	λ^9	.	.	.	0	λ^9	0	0	λ^9
l271	λ^9	λ^9	λ^9	λ^9	λ^9	.	.	.	λ^9	0	λ^9	0	0
l272	0	λ^9	λ^9	λ^9	λ^9	.	.	.	λ^9	λ^9	0	λ^9	0

ao = [λ^{10},λ^{10},λ^{10},…,λ^{10}]

TABLE 14. table of m_i = ao + ℓ_i

	P0	P1	P2	P3	P4	.	.	.	P268	P269	P270	P271	P272
m 0	0	0	λ^{10}	λ^{10}	λ^{10}	.	.	.	λ^{10}	λ^{10}	λ^{10}	0	λ^{10}
m1	λ^{10}	0	0	λ^{10}	λ^{10}	.	.	.	λ^{10}	λ^{10}	λ^{10}	λ^{10}	0
m2	0	λ^{10}	0	0	λ^{10}	.	.	.	λ^{10}	λ^{10}	λ^{10}	λ^{10}	λ^{10}
m3	λ^{10}	0	λ^{10}	0	0	.	.	.	0	λ^{10}	λ^{10}	λ^{10}	λ^{10}
m4	λ^{10}	λ	0	λ^{10}	0	.	.	.	λ^{10}	0	λ^{10}	λ^{10}	λ^{10}
.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.
m268	λ^{10}	λ^{10}	λ^{10}	λ^{10}	λ^{10}	.	.	.	0	0	λ^{10}	λ^{10}	λ^{10}
m269	λ^{10}	λ^{10}	λ^{10}	λ^{10}	λ^{10}	.	.	.	λ^{10}	0	0	λ^{10}	λ^{10}
m270	λ^{10}	λ^{10}	λ^{10}	λ^{10}	λ^{10}	.	.	.	0	λ^{10}	*0*	*0*	λ^{10}
m271	λ^{10}	λ^{10}	λ^{10}	λ^{10}	λ^{10}	.	.	.	λ^{10}	0	λ^{10}	0	0
m272	0	λ^{10}	λ^{10}	λ^{10}	λ^{10}	.	.	.	λ^{10}	λ^{10}	0	λ^{10}	0

ap = [λ^{11},λ^{11} ,λ^{11},…,λ^{11}]

TABLE 15. table of n_i = ap + ℓ_i

	P0	P1	P2	P3	P4	.	.	.	P268	P269	P270	P271	P272
n 0	0	0	λ^{11}	λ^{11}	λ^{11}	.	.	.	λ^{11}	λ^{11}	λ^{11}	0	λ^{11}
n1	λ^{11}	0	0	λ^{11}	λ^{11}	.	.	.	λ^{11}	λ^{11}	λ^{11}	λ^{11}	0
n2	0	λ^{11}	0	0	λ^{11}	.	.	.	λ^{11}	λ^{11}	λ^{11}	λ^{11}	λ^{11}
n3	λ^{11}	0	λ^{11}	0	0	.	.	.	0	λ^{11}	λ^{11}	λ^{11}	λ^{11}
n4	λ^{11}	λ	0	λ^{11}	0	.	.	.	λ^{11}	0	λ^{11}	λ^{11}	λ^{11}
.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.
n268	λ^{11}	λ^{11}	λ^{11}	λ^{11}	λ^{11}	.	.	.	0	0	λ^{11}	λ^{11}	λ^{11}
n269	λ^{11}	λ^{11}	λ^{11}	λ^{11}	λ^{11}	.	.	.	λ^{11}	0	0	λ^{11}	λ^{11}
n270	λ^{11}	λ^{11}	λ^{11}	λ^{11}	λ^{11}	.	.	.	0	λ^{11}	*0*	*0*	λ^{11}

n271	λ^{11}	λ^{11}	λ^{11}	λ^{11}	λ^{11}	.	.	.	λ^{11}	0	λ^{11}	0	0
n272	0	λ^{11}	λ^{11}	λ^{11}	λ^{11}	.	.	.	λ^{11}	λ^{11}	0	λ^{11}	0

aq = [λ^{12},λ^{12},λ^{12},…,λ^{12}]

TABLE 16. table of o_i = aq + ℓ_i

	P0	P1	P2	P3	P4	.	.	.	P268	P269	P270	P271	P272
o 0	0	0	λ^{12}	λ^{12}	λ^{12}	.	.	.	λ^{12}	λ^{12}	λ^{12}	0	λ^{12}
o1	λ^{12}	0	0	λ^{12}	λ^{12}	.	.	.	λ^{12}	λ^{12}	λ^{12}	λ^{12}	0
o2	0	λ^{12}	0	0	λ^{12}	.	.	.	λ^{12}	λ^{12}	λ^{12}	λ^{12}	λ^{12}
o3	λ^{12}	0	λ^{12}	0	0	.	.	.	0	λ^{12}	λ^{12}	λ^{12}	λ^{12}
o4	λ^{12}	λ	0	λ^{12}	0	.	.	.	λ^{12}	0	λ^{12}	λ^{12}	λ^{12}
.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.
o268	λ^{12}	λ^{12}	λ^{12}	λ^{12}	λ^{12}	.	.	.	0	0	λ^{12}	λ^{12}	λ^{12}
o269	λ^{12}	λ^{12}	λ^{12}	λ^{12}	λ^{12}	.	.	.	λ^{12}	0	0	λ^{12}	λ^{12}
o270	λ^{12}	λ^{12}	λ^{12}	λ^{12}	λ^{12}	.	.	.	0	λ^{12}	0	0	λ^{12}
o271	λ^{12}	λ^{12}	λ^{12}	λ^{12}	λ^{12}	.	.	.	λ^{12}	0	λ^{12}	0	0
o272	0	λ^{12}	λ^{12}	λ^{12}	λ^{12}	.	.	.	λ^{12}	λ^{12}	0	λ^{12}	0

ar = [λ^{13},λ^{13} ,λ^{13},…,λ^{13}]

TABLE 17. table of p_i = ar + ℓ_i

	P0	P1	P2	P3	P4	.	.	.	P268	P269	P270	P271	P272
p 0	0	0	λ^{13}	λ^{13}	λ^{13}	.	.	.	λ^{13}	λ^{13}	λ^{13}	0	λ^{13}
p1	λ^{13}	0	0	λ^{13}	λ^{13}	.	.	.	λ^{13}	λ^{13}	λ^{13}	λ^{13}	0
p2	0	λ^{13}	0	0	λ^{13}	.	.	.	λ^{13}	λ^{13}	λ^{13}	λ^{13}	λ^{12}
p3	λ^{13}	0	λ^{13}	0	0	.	.	.	0	λ^{13}	λ^{13}	λ^{13}	λ^{13}
p4	λ^{13}	λ^{13}	0	λ^{13}	0	.	.	.	λ^{13}	0	λ^{13}	λ^{13}	λ^{13}
.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.
p268	λ^{13}	λ^{13}	λ^{13}	λ^{13}	λ^{13}	.	.	.	0	0	λ^{13}	λ^{13}	λ^{13}
p269	λ^{13}	λ^{13}	λ^{13}	λ^{13}	λ^{13}	.	.	.	λ^{13}	0	0	λ^{13}	λ^{13}
p270	λ^{13}	λ^{13}	λ^{13}	λ^{13}	λ^{13}	.	.	.	0	λ^{13}	0	0	λ^{13}
p271	λ^{13}	λ^{13}	λ^{13}	λ^{13}	λ^{13}	.	.	.	λ^{13}	0	λ^{13}	0	0
p272	0	λ^{13}	λ^{13}	λ^{13}	λ^{13}	.	.	.	λ^{13}	λ^{13}	0	λ^{13}	0

as = [λ^{14},λ^{14},λ^{14},…,λ^{14}]

TABLE 18. table of q_i = as + ℓ_i

	P0	P1	P2	P3	P4	.	.	.	P268	P269	P270	P271	P272
q 0	0	0	λ^{14}	λ^{14}	λ^{14}	.	.	.	λ^{14}	λ^{14}	λ^{14}	0	λ^{14}
q1	λ^{14}	0	0	λ^{14}	λ^{14}	.	.	.	λ^{14}	λ^{14}	λ^{14}	λ^{14}	0
q2	0	λ^{14}	0	0	λ^{14}	.	.	.	λ^{14}	λ^{14}	λ^{14}	λ^{14}	λ^{14}
q3	λ^{14}	0	λ^{14}	0	0	.	.	.	0	λ^{14}	λ^{14}	λ^{14}	λ^{14}
q4	λ^{14}	λ^{14}	0	λ^{14}	0	.	.	.	λ^{14}	0	λ^{13}	λ^{14}	λ^{14}
.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.
q268	λ^{14}	λ^{14}	λ^{14}	λ^{14}	λ^{14}	.	.	.	0	0	λ^{14}	λ^{14}	λ^{14}

q269	λ^{14}	λ^{14}	λ^{14}	λ^{14}	λ^{14}	.	.	.	λ^{14}	0	0	λ^{14}	λ^{14}
q270	λ^{14}	λ^{14}	λ^{14}	λ^{14}	λ^{14}	.	.	.	0	λ^{14}	0	0	λ^{14}
q271	λ^{14}	λ^{14}	λ^{14}	λ^{14}	λ^{14}	.	.	.	λ^{14}	0	λ^{14}	0	0
q272	0	λ^{14}	λ^{14}	λ^{14}	λ^{14}	.	.	.	λ^{14}	λ^{14}	0	λ^{14}	0

d(aa,ab) = 273	d(aa,ab) = 273	d(ac,aa) = 273	d(ad,aa) = 273
d(aa, ac) = 273	d(ab, ac) = 273	d(ac, ab) = 273	d(ad, ab) = 273
d(aa, ad) = 273	d(ab, ad) = 273	d(ac, ad) = 273	d(ad, ac) = 273
d(aa,ae) = 273	d(ab,ae) = 273	d(ac,ae) = 273	d(ad,ae) = 273
d(aa,af) = 273	d(ab,af) = 273	d(ac,af) = 273	d(ad,af) = 273
d(aa,ag) = 273	d(ab,ag) = 273	d(ac,ag) = 273	d(ad,ag) = 273
d(aa,ah) = 273	d(ab,ah) = 273	d(ac,ah) = 273	d(ad,ah) = 273
d(aa, ak) = 273	d(ab, ak) = 273	d(ac, ak) = 273	d(ad, ak) = 273
d(aa,am) = 273	d(ab,am) = 273	d(ac,am) = 273	d(ad,am) = 273
d(aa,an) = 273	d(ab,an) = 273	d(ac,an) = 273	d(ad,an) = 273
d(aa,ao) = 273	d(ab,ao) = 273	d(ac,ao) = 273	d(ad,ao) = 273
d(aa, ap) = 273	d(ab, ap) = 273	d(ac, ap) = 273	d(ad, ap) = 273
d(aa,aq) = 273	d(ab,aq) = 273	d(ac,aq) = 273	d(ad,aq) = 273
d(aa, ar) = 273	d(ab, ar) = 273	d(ac, ar) = 273	d(ad, ar) = 273
d(aa,as) = 273	d(ab,as) = 273	d(ac,as) = 273	d(ad,as) = 273

d(ae,aa) = 273	d(af,aa) = 273	d(ag,aa) = 273	d(ah,aa) = 273
d(ae, ab) = 273	d(af, ab) = 273	d(ag, ab) = 273	d(ah, ab) = 273
d(ae, ac) = 273	d(af, ac) = 273	d(ag, ac) = 273	d(ah, ac) = 273
d(ae,ae) = 273	d(af,af) = 273	d(ag,ag) = 273	d(ah,ah) = 273
d(ae,af) = 273	d(af,af) = 273	d(ag,ag) = 273	d(ah,ah) = 273
d(ae,ag) = 273	d(af,ag) = 273	d(ag,ag) = 273	d(ah,ah) = 273
d(ae,ah) = 273	d(af,ah) = 273	d(ag,ah) = 273	d(ah,ah) = 273
d(ae, ak) = 273	d(af, ak) = 273	d(ag, ak) = 273	d(ah, ak) = 273
d(ae,am) = 273	d(af,am) = 273	d(ag,am) = 273	d(ah,am) = 273
d(ae,an) = 273	d(af,an) = 273	d(ag,an) = 273	d(ah,an) = 273
d(ae,ao) = 273	d(af,ao) = 273	d(ag,ao) = 273	d(ah,ao) = 273
d(ae, ap) = 273	d(af, ap) = 273	d(ag, ap) = 273	d(ah, ap) = 273
d(ae,aq) = 273	d(af,aq) = 273	d(ag,aq) = 273	d(ah,aq) = 273
d(ae, ar) = 273	d(af, ar) = 273	d(ag, ar) = 273	d(ah, ar) = 273
d(ae,as) = 273	d(af,as) = 273	d(ag,as) = 273	d(ah,as) = 273

d(ak,aa) = 273	d(am,aa) = 273	d(an,aa) = 273	d(ao,aa) = 273
d(ak, ab) = 273	d(am, ab) = 273	d(an, ab) = 273	d(ao, ab) = 273
d(ak, ac) = 273	d(am, ac) = 273	d(an, ac) = 273	d(ao, ac) = 273
d(ak,ak) = 273	d(am,am) = 273	d(an,an) = 273	d(ao,ao) = 273
d(ak,ak) = 273	d(am,am) = 273	d(an,an) = 273	d(ao,ao) = 273
d(ak,ak) = 273	d(am,am) = 273	d(an,an) = 273	d(ao,ao) = 273
d(ak,ak) = 273	d(am,am) = 273	d(an,an) = 273	d(ao,ao) = 273
d(ak, ak) = 273	d(am, am) = 273	d(an, an) = 273	d(ao, ao) = 273
d(ak,am) = 273	d(am,am) = 273	d(an,an) = 273	d(ao,ao) = 273
d(ak,an) = 273	d(am,an) = 273	d(an,an) = 273	d(ao,ao) = 273
d(ak,ao) = 273	d(am,ao) = 273	d(an,ao) = 273	d(ao,ao) = 273
d(ak, ap) = 273	d(am, ap) = 273	d(an, ap) = 273	d(ao, ap) = 273
d(ak,aq) = 273	d(am,aq) = 273	d(an,aq) = 273	d(ao,aq) = 273

d(ap,ab) = 273	d(aq,ab) = 273	d(ar,ab) = 273	d(as,ab) = 273
d(ap, ac) = 273	d(aq, ac) = 273	d(ar, ac) = 273	d(as, ac) = 273
d(ap, ad) = 273	d(aq, ad) = 273	d(ar, ad) = 273	d(as, ad) = 273
d(ap,ae) = 273	d(aq,ae) = 273	d(ar,ae) = 273	d(as,ae) = 273
d(ap,af) = 273	d(aq,af) = 273	d(ar,af) = 273	d(as,af) = 273
d(ap,ag) = 273	d(aq,ag) = 273	d(ar,ag) = 273	d(as,ag) = 273
d(ap,ah) = 273	d(aq,ah) = 273	d(ar,ah) = 273	d(as,ah) = 273
d(ap, ak) = 273	d(aq, ak) = 273	d(ar, ak) = 273	d(as, ak) = 273
d(ap,am) = 273	d(aq,am) = 273	d(ar,am) = 273	d(as,am) = 273
d(ap,an) = 273	d(aq,an) = 273	d(ar,an) = 273	d(as,an) = 273
d(ap,ao) = 273	d(aq,ao) = 273	d(ar,ao) = 273	d(as,ao) = 273
d(ap, ap) = 273	d(aq, ap) = 273	d(ar, ap) = 273	d(as, ap) = 273
d(ap,aq) = 273	d(aq,aq) = 273	d(ar,aq) = 273	d(as,aq) = 273
d(ap, ar) = 273	d(aq, ar) = 273	d(ar, ar) = 273	d(as, ar) = 273
d(ap,as) = 273	d(aq,as) = 273	d(ar,as) = 273	d(as,as) = 273

$d(a_i, a_j) = 272 \quad i \neq j$
$d(a_j, b_i) = 272 \quad i \neq j$
$d(a_j, Li) = 272 \quad i \neq j$

If we substitute the value of n =273, d = 17, e = 8

In inequality 1.1 we get M = 16^{270} hence C is a (273, 16^{270}, 17)-code

$16^{270}\{\binom{273}{0} + \binom{273}{1}(16-1) + \binom{273}{2}(16-1)^2$

$+\binom{273}{3}(16-1)^3 + \binom{273}{4}(16-1)^4 > 16^{273}$

By corollary therefore *C* is not perfect.

Such that 0 = λ In code *C*, the remaining vectors are formed by combining aa,ab,ac,ad,ae,af,ag,ah,ak,am,an,ao,ap,aq,ar,as,ai,bi,ci,di,ei,fi,gi,hi,mi,ni,oi,pi,qi,ri,si

Note that d (ℓ_i, ℓ_j) = There are exactly one point on each of ℓ_i or ℓ_j then if we substitute the value of n = 273 , d = 17 , e = 8 in equality of theorem (1.1) we get M ≤ 16^{270} hence *C* is a (273, *M* , 17) – code .

As a result of the addition modulo9 operation, the following theorem leads to the conclusion that the code *C* is closed:

Theorem

The code c = [n =q^2 +q+1, M <= q^2+q+l-k, d = the number of points located in the line which is derived from the projective Plane of order q= p^h. of any two elements of C is in C

proof:

Here is the geometry with Pi=i, where i=0,1…….q^2+q+1 such that.

ar = 1 Pr Lies on precisely one of li, lj

ar = 0 Pr Lies on the third Line through Herer

Definition 1.36 (companion matrix). [18] Let

f(x) be a monic polynomial in F[x]:

$f(x) = x^n + a_{n-1}x^{n-1} + \ldots\ldots\ldots + a_1x + q0$

The companion matrix c(f) is the n * n matrix given by

$$C(f) = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ ta0 & ta1 & ta2 & \cdots & tan-2 & tan-1 \end{pmatrix}$$

Galo's field: GF(q)

Let f(x) is irreducible polynomial of degree h
Over Gf (P) (≃ Zp). p is prime then
Gf (p^h) = Gf (P) [x] / < f(x) >
Gf (p^h) = {$\sum_{i=0}^{h-1}${ aiji : ai € Gf (P), f(λ) =0}
is called Galo's field of order q
$q=p^h$, $\lambda^{q-1}=1$, λ is a root of f(x)
G F(p) ≃Zp

2.Conclusion

We have constructed the projective linear codes with the parameters n,M,d depending on the order of Galois Field *F*q, and have also studied the relationship between the finite projective plane and coding theory, such that the columns of the generator matrix of any linear code are considered the points in the projective plane .Finally we construction of q-ary(n,M,d)-codes in PG(2,9)theorem 1.3.

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