Continuum Model on (HYTXW) of Fungal Growth

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Abstract-: In general the growth of fungi needs to be resolved until its goal becomes a correction and thus, we minimize cost, effort, time and money. Therefore, we arrived at a mathematical solution using some techniques. In this paper, we studied its growth behavior and the effect of each branch on the fungus, then we combined a number of branches represented as mathematical model as partial differential equations (PDEs), approximate numerical solutions, and some mathematical steps, such as non-dimensionalisation, finding stability or steady state and representing it on phase plane, we found approximate results for these types using **MATLAB's** codes such as **Pplane8** and **Pdepe** [1,7].

Keywords- Continuum Model, Lateral tip-hypha, Anastomosis and Hyphal, Fungal Growth, Mathematical Model, Tip-hypha Anostomosis.

1 Introduction:

There are many papers of mathematical models that have been proposed by many researchers to explain the mathematical model , for example:

-In (2011) Shuaa [1], Studied to develop a model for the growth of fungi which can be used to create a source term in a single root model to account for nutrient uptake by the fungi. Therefore ,focus on the hyphal loss or death.

-In(2013) Mudhafar [2] , Proposed the different modelling procedures, with a special emphasis on their ability to reproduce the biological system and to predict measured quantities which describe the overall processes.

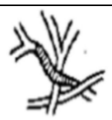
-In (2021) Zainab Jaafar, Shuaa Al-Taie [4], studied a mathematical model of branching using the solution of a system of partial equations (PDEs). The results of this solution was be describe a success or failure of the growth of the fungus species studied.

-In(2022) Nabaa Fauzi, Shuaa[5], thesis presents developed types of mathematical models using partial differential equations for fungi plants, which helped in diagnosing the best types for cultivation.

-In(2022) Ayat , Shuaa [6],they talked about types of fungal, where four branches were combined together, then discussed the results of these types.

Mathematical modeling is best understood as an active process, rather than a static object of study. In practice, modeling entails a systematic approach to problem solving that brings the techniques and structures of mathematics to bear in an effort to describe, understand, and make predictions about a wide range of empirical phenomena, here we have transformed the phenomenon of fungi growth into a formula and mathematical equations[7]. Where the table[1] represents the biological type of branches, clarifying the parameters, and version of each branch.

Table 1 : Illustrate branching, Biological type, symbol of this type and version.

Branching	Biological type	Sym-bol	Version	Parameters description
	Dichotomous branching	Y	$\delta = \alpha_1 n$	α_1 is the number of tips produced per tip per unit time
	Lateral branching	F	$\delta = \alpha_2 \rho$	α_2 is the number of branches produced per unit length hypha per unit time.
	Tip-hypha anastomosis	H	$\delta = -\beta_2 n\rho$	β_2 is the rate of tip reconnections per unit length hypha per unit time
	Tip-tip anastomosis	W	$\delta = -\beta_1 n^2$	β_1 is the rate of tip reconnections per unit time.
	Tip death	T	$\delta = -\alpha_3 n$	α_3 is the loss rate of tips (constant for tip death)
	Tip death due to overcrowding	X	$\delta = -\beta_3 \rho^2$	β_3 is the rate at which overcrowding density limitation eliminates branching.
	Hyphal death	D	$d = \gamma_1 \rho$	γ_1 is the loss rate of hyphal (constant for hyphal death).

2 Mathematical Model

Mathematicians saw that they transform the branches of the fungi into letters, and these letters depend on the behavior of the species in terms of $\rho = density$ of the hypha in unit filament length per unit area , $n = tip\ density$. We will study a new type of branching of fungal growth $H, Y, T, X\ and\ W$.
We can describe hyphal growth by the system below:

$$\frac{\partial \rho}{\partial t} = nv - d\rho$$
$$\frac{\partial n}{\partial t} = -\frac{\partial (nv)}{\partial x} + \delta(p, n) \qquad (1)$$

Where: $\delta(p, n) = \alpha_1 n - \beta_2 np - \beta_1 n^2 - \alpha_3 n - \beta_3 p^2$
Then this system becomes:

$$\frac{\partial \rho}{\partial t} = nv - \gamma\rho$$
$$\frac{\partial n}{\partial t} = -\frac{\partial (nv)}{\partial x} + \alpha_1 n - \beta_2 np - \beta_1 n^2 - \alpha_3 n - \beta_3 p^2 \qquad (2)$$

2.1 Non-dimensionlision and Stability

Leah-keshet (1982) and Ali H. Shuaa Al-Taie (2011)[1,7], clear up how can put these parameters as dimensionlision less

$$\frac{\partial \rho}{\partial t} = n - \rho$$
$$\frac{\partial n}{\partial t} = -\frac{\partial (nv)}{\partial x} + \alpha n(1 - p) - \beta(n(n + 1) + p^2) \qquad (3)$$

Where: $\alpha = \frac{\alpha_1 \beta_1}{\beta_2\, v}$, $\beta = \frac{\alpha 3 \gamma}{v \beta 3}$

Now, to find steady states when take from system (3)

$$\frac{\partial\, \rho}{\partial t} = n - \rho = 0 \rightarrow n = \rho$$

And on the other hand

$$\frac{\partial n}{\partial t} = \alpha n(1 - p) - \beta(n(n + 1) + p^2) = 0$$
$$\rightarrow \alpha p(1 - p) - \beta(p(p + 1) + p^2) = 0$$

The solution of equation, we'll find the values of (p,n), the steady state are :
(p,n) = (0,0) and, $\left(\frac{\alpha-\beta}{\alpha+2\beta}, \frac{\alpha-\beta}{\alpha+2\beta}\right)$ therefor, we take Jacobain of these equations. [5, 8, 6]

$$f = n - p$$
$$g = \alpha n(1 - n) - \beta p(n + p)$$

J(p, n) = $\begin{bmatrix} \frac{\partial f}{\partial p} & \frac{\partial f}{\partial n} \\ \frac{\partial g}{\partial p} & \frac{\partial g}{\partial n} \end{bmatrix}$

$$J(p,n) = \begin{bmatrix} -1 & 1 \\ (-\alpha n \; - 2\beta p) & (\alpha - \alpha p - 2\beta n - \beta) \end{bmatrix}$$

We can classify the cirtical points according to the eigenvalues of this matrix Jacobain at (0,0):

$$J(p,n) = \begin{bmatrix} -1 & 1 \\ 0 & \alpha - \beta \end{bmatrix}$$

Thus | A − λI | = 0 , we get two values of λ : $\lambda_1 = -1$, $\lambda_2 = \alpha - \beta$ .The stability of the steady state is saddle point for all α,β > 0, α > β , see Fig[1].

Then we take the Jacobain for $\left(\frac{\alpha-\beta}{\alpha+2\beta}, \frac{\alpha-\beta}{\alpha+2\beta}\right)$:

$$J\left(\frac{\alpha}{\alpha+2\beta}, \frac{\alpha}{\alpha+2\beta}\right) = \begin{bmatrix} -1 & 1 \\ -\frac{(-\alpha-2\beta)(\alpha-\beta)}{\alpha+2\beta} & \frac{(-\alpha-2\beta)(\alpha-\beta)}{\alpha+2\beta} \end{bmatrix}$$

The stability of the steady state is stable spiral for all α, β > 0 and β < α, see Fig[1].

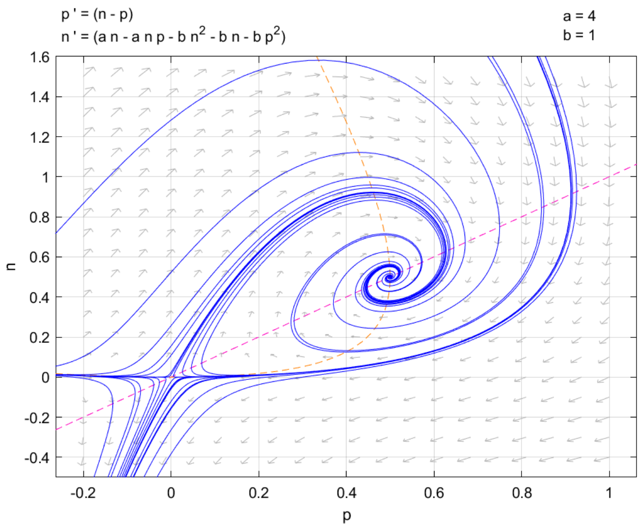


Figure [1]: The (p, n) plane for the ordinary differential equation (3)

Not that when α = 4, β = 1. The solid blue line corresponds to the model a trajectory connects the saddle point (0, 0) to the stable spiral $\left(\frac{1}{2}, \frac{1}{2}\right)$, the dashed lines corresponds the model to the null-cline. Solution is produced using **MATLAB pplane8**.

2.2 Traveling wave solution

We will now discuss the traveling wave solution, we assume that:

$\rho(x,t) = \rho(z)$ and, $n(x,t) = N(z)$ where $z = x - ct$, $P(z)$ profile density and propagation rate c of edge of the colony. $P(z)$ And, $N(z)$ is a non-negative function of z. The function, $p(x,t)$,

$n(x,t)$ are moving waves, moving with a constant speed c in a positive x direction, where $c > 0$, and $\alpha = 1$ to search for a traveling wave solution to the equations in x and t of the system (3).
$\frac{d\rho}{dt} = -c\frac{d\rho}{dx}$, $\frac{dn}{dt} = -\frac{dN}{dx}$, and ,$\frac{dn}{dt} = \frac{dN}{dx}$

Thus we can reduce the system (3) to a set of two ordinary differential equation:

$$-\frac{1}{c}[N - P] = 0 \qquad (1)$$

$$\frac{1}{1-c}[\alpha N(1 - P) - \beta(N(N + 1) + P^2)] = 0 \qquad (2)$$

The system (3) has two uniform steady state points (0,0) and $\left(\frac{\alpha-\beta}{\alpha+2\beta}, \frac{\alpha-\beta}{\alpha+2\beta}\right)$, we can take Jacobian and by using determinate eigenvalue of λ , | A − λI |= 0 to solve the system, we get two of the value of λ, where c≠0, c≠1 we get (N, P) = (0,0) saddle point and $\left(\frac{3}{2}, \frac{3}{2}\right)$ stable spiral constant for c=-0.5 , α =4 and β=1. This helps us to determine the initial conditions of p and n . See Fig [2].

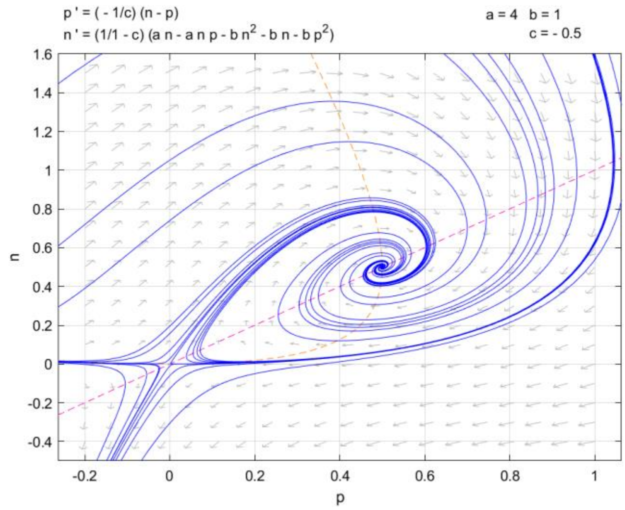


Figure [2]: The (n, p) plane, the solid blue line corresponds the model a trajectory connects the saddle point (0, 0) to the $\left(\frac{3}{2}, \frac{3}{2}\right)$ stable spiral.

2.3 Numerical Solution

Because the system (3) cannot be solved exactly so we resort to numerical solution, and here we using **pdepe** code in **MATLAB,** see Fig[3,4,5,6].

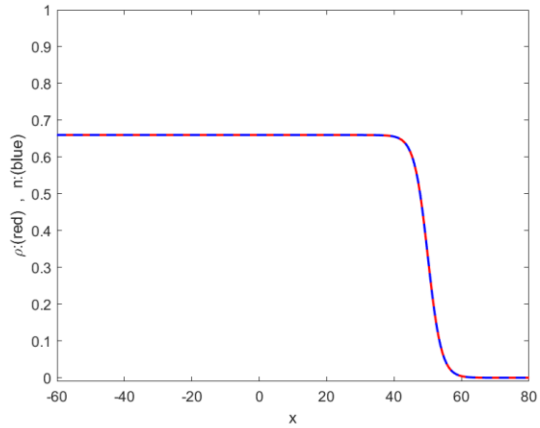


Figure [3]: The initial condition of solution to the system (3) with the parameters α = 4 and β = 1.

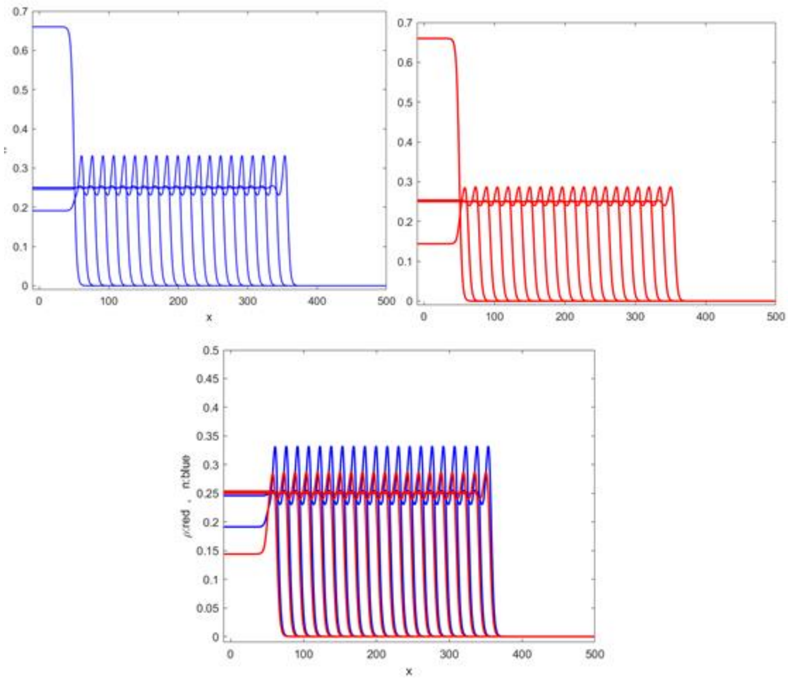


Figure [4,5,6]: Solution to the system (3)with parameters α = 2 and β = 1 and time t = 1, ..., 200, where blue line represented tips (n), and red line represented branches (p).

From this operations we get the relation between traveling waves solution c and α values with taking v, d and β=1 , we can show that the table[2]. Where α increasing then the traveling waves solution c is increasing, see Fig[7].

Table 2: the value of α and c for solution of the type HYTWX.

α	0.5	1	2	3	4	5	6	7	8	9	10
c	2.45	3.41	5.37	7.36	9.38	11.40	13.44	15.46	17.51	19.65	21.59

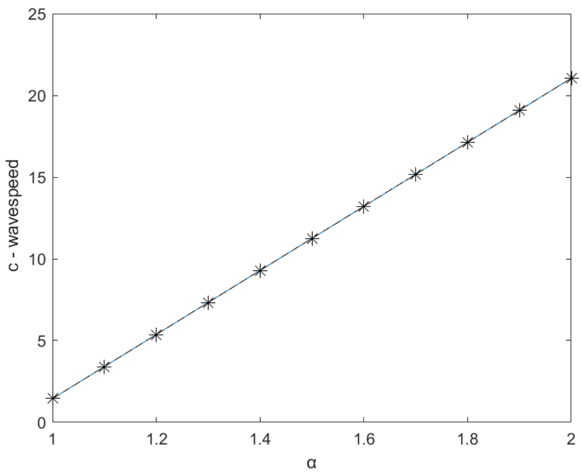


Figure [7]: The relation between wave speed c and α values

Now, we get the relation between traveling waves solution c and β values with taking α ,v = 1 , we can show that the table[3], where β increasing then the traveling waves solution c is increasing, see Fig[8]

Table 3: The value of β and c for solution of the type HYTXW.

β	0.5	1	2	3	4	5	6	7	8	9	10
c	3.06	3.41	4.29	5.26	6.25	7.26	8.25	9.26	10.26	11.24	12.25

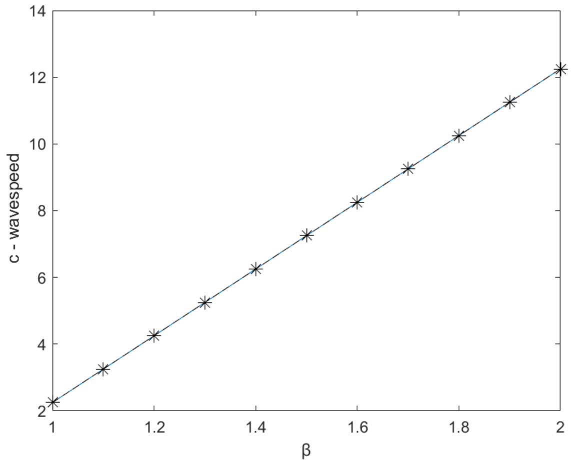


Figure [8]: The relation between wave speed c and β values.

Now, we take relation between traveling waves solution c and v values with taking $\alpha = \beta = 1$, we can show that the table[4], where v increasing then the traveling waves solution c is increasing, see Fig[9]

Table 4: the value of v and c for solution of the type HYTXW.

v	0.5	1	2	3	4	5	6	7	8	9	10
c	2.92	3.41	4.39	5.32	6.25	7.18	8.11	9.04	9.97	10.9	11.83

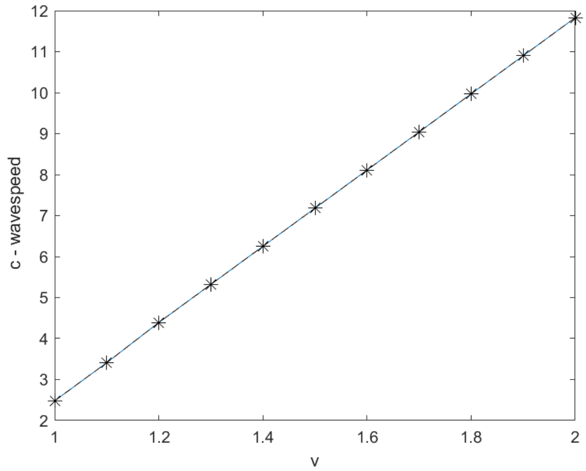


Figure [9]: The relation between wave speed c and v values.

Also, we take relation between traveling waves solution c and d values with taking $\alpha = \beta = v = 1$, we can show that the table[5], where v increasing then the traveling waves solution c is decreasing, see Fig[10]

Table 5: the value of d and c for solution of the type HYTXW.

d	0.5	1	2	3	4	5	6	7	8	9	10
c	0.32	0.28	0.117	0.102	0.08	0.072	0.06	0.055	0.049	0.044	0.04

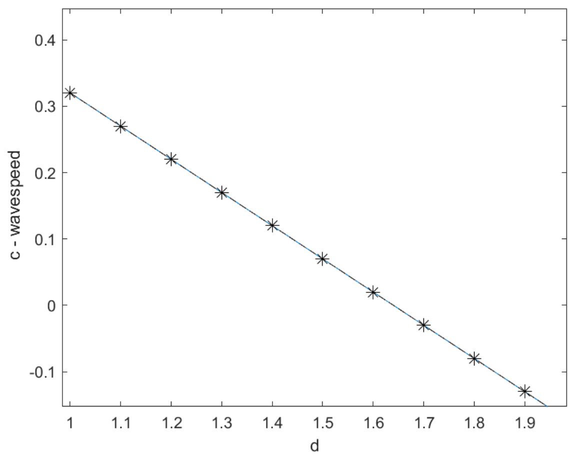


Figure 10: The relation between wave speed c and d values

3 The Conclusions

After we made a comparison in this paper it was concluded that there is a relationship between traveling wave solution (c) and parameter $\alpha = \frac{\alpha_1}{\gamma}$ where traveling wave (c) increase whenever the values of α increase. Through the following relation, we see that the value of (α) is directly proportional to the value of (α_1)and inversely to (γ), which means that the growth increases by increasing the value of (α), and we noted that the relation between traveling waves solution (c) and (β) values, where (β) increasing then the traveling waves solution (c) is increasing, also the relation between traveling waves solution (c) and (v) values, we've seen that (v) increasing then the traveling waves solution (c) is increasing too, finally, we take relation between traveling waves solution (c) and (d) values, (v) increasing then the traveling waves solution (c) is decreasing [3,8].

4 References

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