Mathematical Model Of YFWXHD Branching Type With Hyphal Death

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Abstract— Mathematical modeling is used to describe the fungus growth process. This model depicts the growth-related behavior of Dichotomous branching, Lateral branching , Tip-tip anastomosis , Tip death due to Overcrowding, Tip-hypha anastomosis with haphal death , we are aware that fungi require money to flourish. Money and effort. Thus, we get a mathematical solution. Although the error ratio, to reduce the time, expense, and work needed to get the right conclusion. In this paper, we will use a system of partial differential equations to solve a mathematical model (PDEs), and for the numerical analysis, we applied several codes, (pplane8, pdepe).

Keywords— Lateral branching, Tip-tip anastomosis, Tip death due to overcrowdin, Tip-hypha anastomosis , Haphal death.

1 Introduction

There are several papers explaining mathematical models that have been put out by numerous researchers, such as:

- In (2022) Z. Hussein Khalil and A. Shuaa [7], In this study, researchers investigated the possibility of fungi growing when four different forms of them were combined. These types used all the energy.

- In (2022) A. Shuaa and A. Saleem Habeeb [8], In this paper, they explain The mathematical model illustrates energy consumption, tip-hypha anastomosis, lateral branching, dichotomous branching, and hyphal death.

- In (2022) N. Fawzi Khwedim and A. Hussein Shuaa [9], The mathematical model explains the dysplasia phenomenon. The model that shows the growth of filament tip anastomosis, limb anastomosis, bilateral branching, lateral branching, and limb death owing to crowding with thread death.

In this study, a new model for the growth of fungus is created.Table 1 lists many fungus species, their biological classifications, and versions of these biological phenomena in mathematical form. We combined many types of fungus in this model to explain how the parameters are described. The first person to transform biological events into mathematical form was Leah-Keshet[1].

Table 1:Biological type, symbol of this type, version and description of these parameters.

Biological type	Symbol	Version	Parameters description
Dichotomous branching	Y	$\delta = \alpha_1 n$	α_1 is the number of tips produced per tip per unit time
Lateral branching	F	$\delta = \alpha_2 \rho$	α_2 is the number of branches produced per unit length hypha per unit time.
Tip_tip anastomosis	W	$\delta = -\beta_1 n^2$	β_1 is the rate of tip reconnections per unit time.
Tip death due to over crowding	X	$\delta = -\beta_2 p^2$	β_3 is the rate at which overcrowding density limitation eliminates branch in
Tip_ hypha anastomosis	H	$\delta = -\beta_2 np$	β_2 is the rate of tip reconnections per unit length hypha per unit time.
Haphal death	D	$d = \gamma_1 p$	γ_1 is the loss rate of hyphal (constant for hyphal death).

2 Mathematical Model

We will study a new type of branching of fungal growth with hyphal death, the system listed below can be used to describe hyphal growth

$$\frac{\partial \mathrm{p}}{\partial \mathrm{t}} = nv - dp$$

$$\frac{\partial \mathrm{n}}{\partial \mathrm{t}} = \frac{-\partial nv}{\partial x} + \sigma(p,n) \qquad (1)$$

Where $\sigma(p,n) = \alpha_1 n + \alpha_2 p - \beta_1 n^2 - \beta_3 p^2 - \beta_2 np + \gamma_1 p$
Then the system (1) becomes

$$\frac{\partial p}{\partial t} = nv - dp$$

$$\frac{\partial n}{\partial t} = -\frac{dnv}{dx} + \alpha_1 n + \alpha_2 p + \beta_1 n^2 - \beta_3 p^2 - \beta_2 np + \gamma_1 p \qquad (2)$$

3 Non-dimensionlision and Stability

We must reduce the amount of parameters when solving any mathematical system with multiple variables; this process is known as non-dimensionalisation. Ali H. Shuaa Al-Taie (2011) clear up how can put these parameters as dimensionlisionless[2].

$$\frac{\partial \text{d}}{\partial} = n - p$$

$$\frac{\partial \text{n}}{\partial \text{t}} = \frac{-\partial n}{\partial x} + \alpha(n+p) - \beta(n^2 + p^2 + np) \qquad (3)$$

Where $\alpha = \frac{\alpha_2}{\bar{n}\gamma}$, $\beta = \frac{\beta_3 \bar{p}^2}{\bar{n}\gamma}$

To find steady states for system (3)

$n - \rho = 0 \rightarrow n = \rho$

And

$\alpha(n+p) - \beta(n^2 + p^2 + np) = 0$

$\rightarrow \rho = \frac{2\alpha}{3\beta}$

Then The steady state are (0,0) and

$(\frac{2\alpha}{3\beta}, \frac{2\alpha}{3\beta})$

Hence we use Jacobian for these equations

$$J(p,n)\begin{bmatrix} -1 & 1 \\ \alpha - 2\beta p - \beta n & \alpha - 2\beta n - \beta p \end{bmatrix}$$

The steady state are : (0,0) saddle point

And $\left(\frac{2\alpha}{3\beta}\right)$ stable spiral

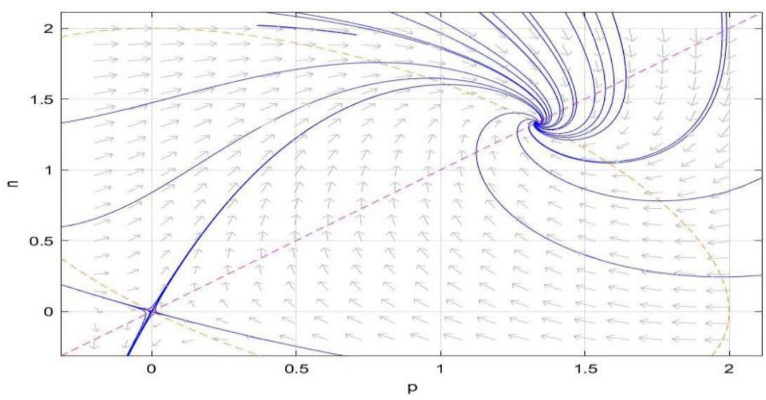

Figure l : The (p,n) plane note that The Irajectory Connects the Saddle point (0,0) and the stable spiral $(\frac{2\alpha}{3\beta}$, $\frac{2\alpha}{3\beta})$.

4 Traveling wave solution

We'll discuss the traveling wave solution. Assume that $P(x,t) = P(z)$, and $n(x,t) = N(z)$ where $z = x - ct, P(z)$ and $N(z)$ are density profile and c rate of propagation of colony edge. $P(z)$ and $N(z)$ non-negative function of z, the function $p(x,t), n(x,t)$ are traveling waves and are moves at constant speed c in positive x direction. Where $c > 0$, and =2, and =1. To look for traveling wave solution of equations in x and t in the form (3).

$$\frac{dp}{dt} = -c\frac{dP}{dz}, \frac{dn}{\partial t} = -c\frac{dN}{dz} \ and \ \frac{dn}{dx} = \frac{dy}{dz}$$

See [3], therefore the equation above becomes

$$\frac{dP}{dZ} = -\frac{1}{c}\ [N - P]$$

$$\frac{dN}{dZ} = \frac{1}{1-C}\left[\frac{-\partial n}{\partial x} + \alpha(n+p) - \beta(n^2 + p^2 + np)\right] , c \neq 1, -\infty < z < \infty \qquad (4)$$

Then the steady state of equation (4) is :

$$-\frac{1}{c}[N - P] = 0$$

$$\frac{1}{1-C}\left[\frac{-\partial n}{\partial x} + \alpha(n+p) - \beta(n^2 + p^2 + np)\right] = 0 \quad , c \neq 1, -\infty < z < \infty \qquad (5)$$

To eliminate the system's above steady state, we obtain $(N,P) = (0,0)$ Saddle point and $(N,P) = (\frac{2\alpha}{3\beta}, \frac{2\alpha}{3\beta})$ unstable spiral, for all $\alpha > \beta$, and $0 < c < 1$ See Figure (2) Using pplane8 in MATLAB.

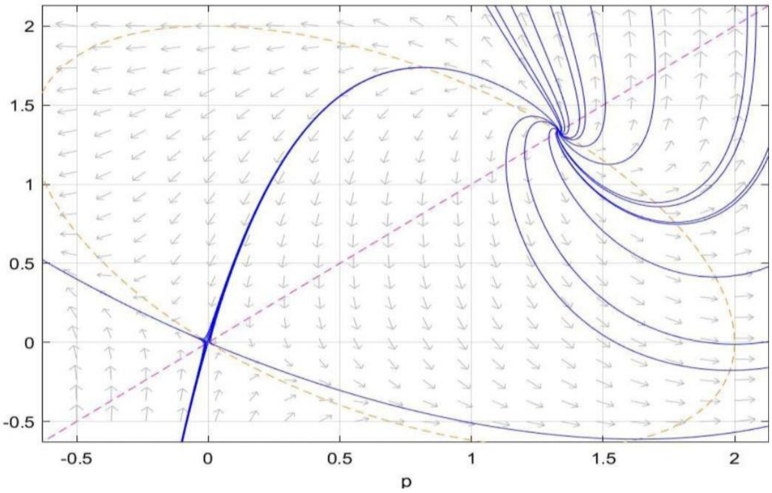

Figure 2 : The (N , P) plane , note that the trajectory Connects Saddle Point (0,0) to the unstable spiral ($\frac{2\alpha}{3\beta}$).

5 Numerical Solution

We use the pdepe code in MATLAP to solve the system (3) numerically because it cannot be solved directly.

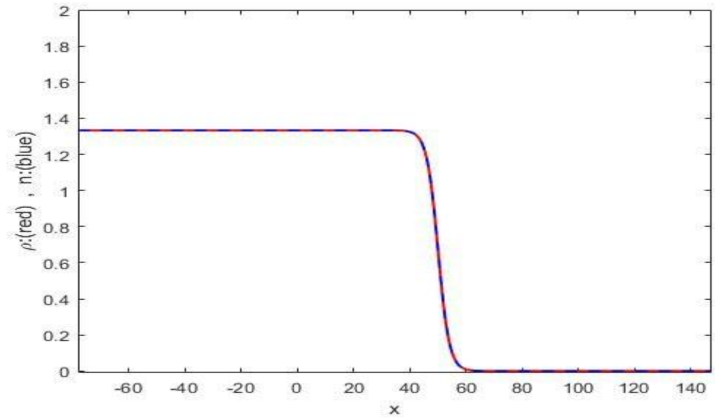

Figure 3: The initial condition of (n), and (p) with the parameters $\alpha = 2$ and $\beta = 1$.

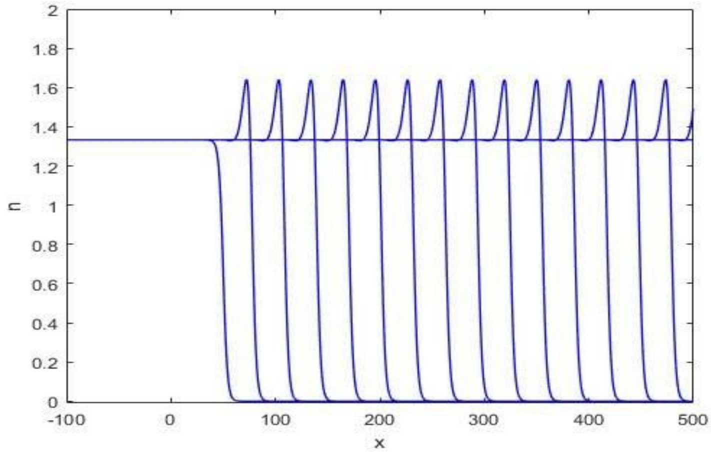

Figure 4: The blue line represented tips n.

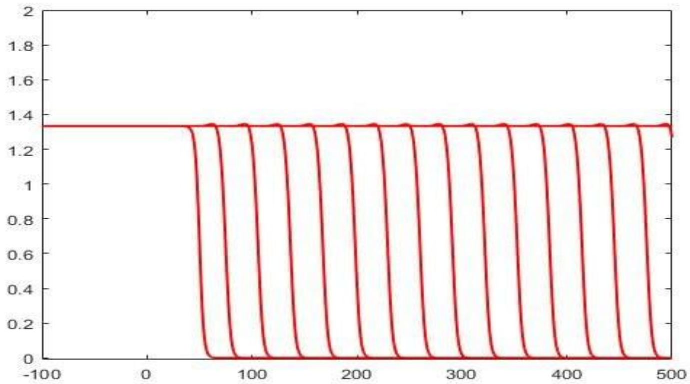

Figure 5: The red line represented branches p.

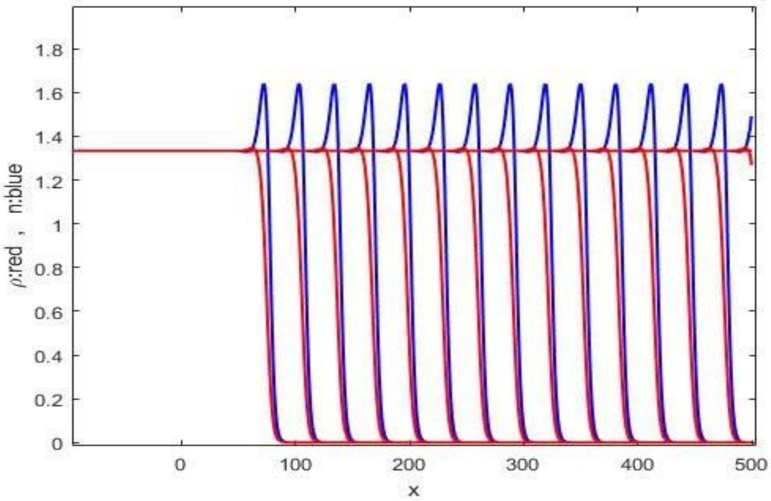

Figure 6: The blue line represented tips n ,with the red line represented branches p.

This paper came to the conclusion that traveling wave solution c and parameter had a relationship where traveling wave increased whenever the values of α increased. View Fig (7).

Table 2: The relation between values and waves speed c with taking v=β=d=1.

α	0.5	1	2	3	4	5	6	7	8	9	10
c	0.83	2.52	8.18	16.74	28.12	42.28	59.21	78.91	101.35	126.53	154.16

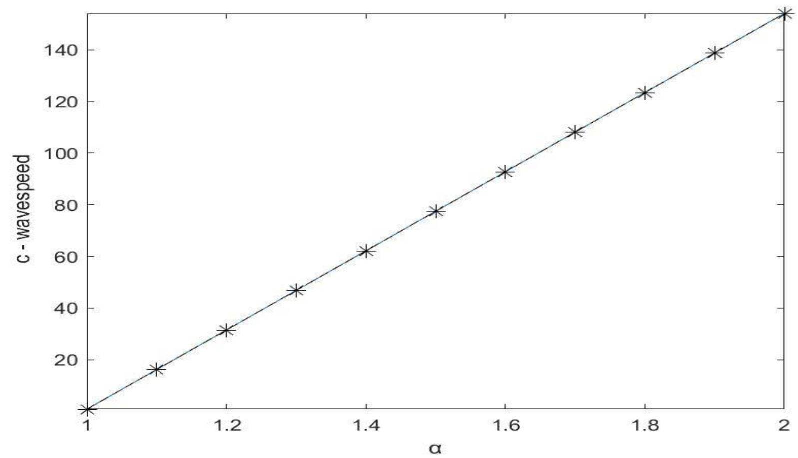

Figure 7: The relation between the wave speed c and parameter α .

Now, the relationship between the traveling wave solution c and the values β, such that whenever the values of β increase, we observe the traveling wave decreasing. View Fig (8)

Table 3: The relation between values β and waves speed c with taking $v = \beta = d = 1$.

β	0.5	1	2	3	4	5	6	7	8	9	10
c	5.04	2.5	2.36	2.1	1.98	0.72	0.61	0.42	0.35	0.29	0.15

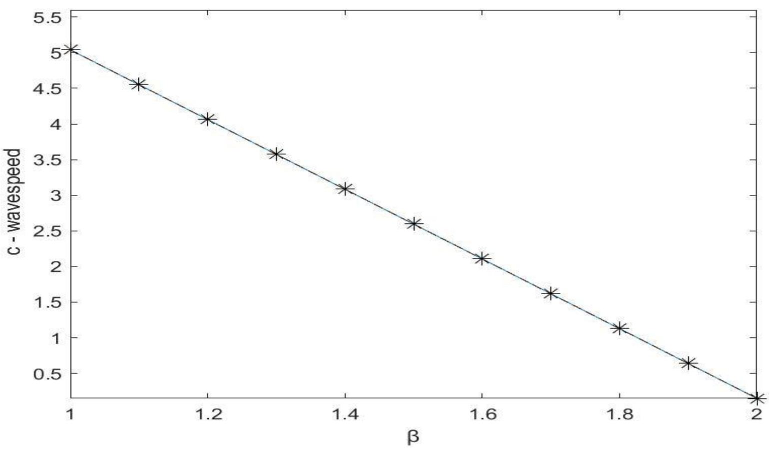

Figure 8: The relation between the wave speed c and parameter β .

So that we may observe the traveling wave growing as the values of v increase, we now consider the relationship between the wave speed c and the values v. View Fig (9).

Table 4: The relation between values v and waves speed c with taking α =β =d=1.

v	0.5	1	2	3	4	5	6	7	8	9	10
c	1.47	2.52	3.74	4.50	5.08	5.56	9.21	6.78	7.78	8.65	9.34

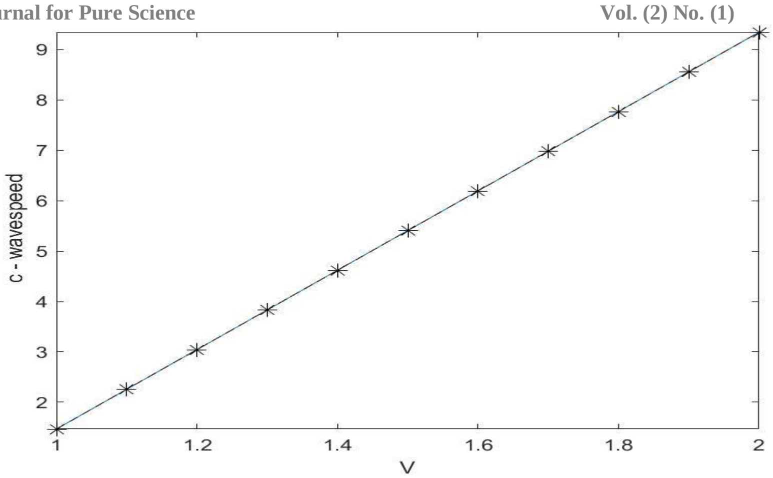

Figure 9: The relation between the wave speed c and values of v.

Consider the connection between wave speed c and d value ranges, and you'll see that the wave speed decreases as d value rises. View Fig (10).

Table 5: The relation between values d and waves speed c with taking α =β = v =1.

d	0.5	1	2	3	4	5	6	7	8	9	10
c	3.35	2.5	1.5	1.07	0.81	0.6	0.5	0.46	0.40	0.35	0.32

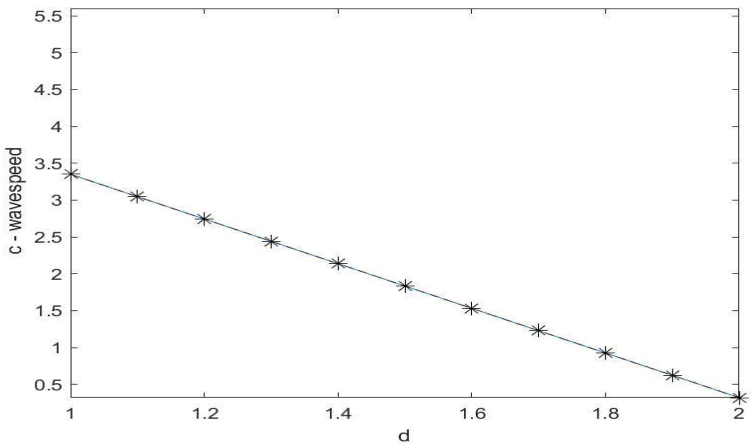

Figure 10: The relation between the wave speed c and the values of d.

6 Conclusion

The wave speed c is clearly increasing when α is an increase function when we plot the connection between c and α, as shown in Fig 7. Furthermore, we plot the connection between c and β (See Fig. 8), so that the wave speed c decreases as the function β increases. Furthermore, we plot the connection between c and v (See Fig. 9), which clearly shows that the wave speed c is increasing when v is an increase function. In fact, Fig. 10 shows that the wave speed is decreasing when the values of d are increasing. Since $\alpha = \frac{\alpha_2}{\bar{n}\gamma}$ and $\beta = \frac{\beta_3 \bar{p}^2}{\bar{n}\gamma}$ therefore the growth rate is increasing with α_2, while keeping $\bar{n}\gamma$ are fixed , and the growth rate decreasing with $\beta_3\bar{p}^2$, while keeping $\bar{n}\gamma$ fixed, [5,6]. In terms of biology, this means that when α grows, the growth also rises. And finally that the growth increases according to α_2 increasing , and everytime β_3 and $\bar{p}^2$ rise, the growth decreases.

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