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Three Classes of Soft Functions Via Soft S_p-Open Sets and Soft S_p-Closed Sets

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ABSTRACT: This paper introduces novel concepts of soft functions known as soft S_p-irresolute, soft S_p-open, and soft S_p-closed function as well as some of their properties. The interrelationships of this newly defined soft functions with other types of soft functions are investigated, and the behaviors of soft S_p-irresolute (respectively, soft S_p-open, and soft S_p-closed) functions under soft composition are studied. Finally, using soft S_p-open sets, the concepts of soft S_p-Hausdorff space are introduced and investigated.

Keywords: soft S_p-open set, soft S_p-closed set, soft S_p-irresolute function, soft S_p-Hausdorff space, soft S_p-open function, soft S_p-closed function.

1. INTRODUCTION AND PRELIMINARIES

To deal with ambiguous items, Molodtsov provided the following definition of soft sets [1]: Assume X is a universe set, $P(X)$ is the power set of X, and $\mathcal{P}$ is a set of parameters. A pair $(A,\mathcal{P}) = \{(e, A(e)): e \in \mathcal{P}, A(e) \in P(X)\}$ is known as a soft set over X, where $A: \mathcal{P} \to P(X)$ is a function. The family of all soft sets over the universal set X with the set of parameters $\mathcal{P}$ is indicated by $\tilde{S}S(X,\mathcal{P})$. In particular, $(X,\mathcal{P})$ is indicated by $\tilde{X}$. By Maji et al. [2], was defined a null soft set, indicated by $\tilde{\emptyset}$, if $A(e) = \emptyset, \forall\, e \in \mathcal{P}$ and an absolute soft set, indicated by $\tilde{X}$, if $A(e) = X, \forall\, e \in \mathcal{P}$ and the soft complement of a soft set $(A,\mathcal{P})$ is indicated by $\tilde{X}\,\tilde{\setminus}(A,\mathcal{P}) = (A^c,\mathcal{P})$ where $A^c: \mathcal{P} \to P(X)$ is a function defined as $A^c(e) = X\setminus A(e), \forall\, e \in \mathcal{P}$. The soft union of $(A_\vartheta,\mathcal{P}) \tilde{\in} \tilde{S}S(X,\mathcal{P})$, $\forall\, \vartheta \in \aleph$ is a soft set $(A,\mathcal{P}) \tilde{\in} \tilde{S}S(X,\mathcal{P})$, where $A(e) = \tilde{\cup}_{\vartheta\in\aleph} A_\vartheta(e), \forall\, e \in \mathcal{P}$, $\aleph$ is a random collection of index and the soft intersection of $(A_\vartheta,\mathcal{P}) \tilde{\in} \tilde{S}S(\tilde{X}), \forall\, \vartheta \in \aleph$ is a soft set $(A,\mathcal{P}) \tilde{\in} \tilde{S}S(X,\mathcal{P})$, where $A(e) = \tilde{\cap}_{\vartheta\in\aleph} A_\vartheta(e), \forall\, e \in \mathcal{P}$, were defined in [3]. A soft point [3] [4] $(A,\mathcal{P})$ is a soft set defined as $A(e) = \{x\}$ and $A(\acute{e}) = \emptyset$, $\forall\, \acute{e} \in \mathcal{P}\setminus\{e\}$, we indicated by $\tilde{e_x}$ such that $\tilde{e_x} = (e,\{x\})$, where $x \in X$ and $e \in \mathcal{P}$. $\tilde{e_x} \tilde{\in} (B,\mathcal{P})$, if $e \in \mathcal{P}$ and $\{x\} \subseteq B(e)$. The family of all soft points over X is indicated by $\tilde{S}P(\tilde{X})$.

The concept of soft topological space ($\tilde{S}TS$) over X was defined in [4] is $(\tilde{X},\tilde{\tau},\mathcal{P})$ (simply, $\tilde{X}$), where $\tilde{\tau} \tilde{\subseteq} \tilde{S}S(X,\mathcal{P})$ is known as soft topology on $\tilde{X}$, if $\tilde{\emptyset}, \tilde{X} \tilde{\in} \tilde{\tau}$, and $\tilde{\tau}$ is closed under finite soft intersection and arbitrary soft union. The members of $\tilde{\tau}$ are referred to as soft open sets. The soft complements of every soft open or members of $\tilde{\tau}^c$ are known as soft closed sets [5]. A soft set $(A,\mathcal{P})$ that is both soft open and soft closed is referred to as a soft clopen set. The family of all soft clopen sets in $\tilde{X}$ is indicated by $\tilde{S}CO(\tilde{X})$. The triple $(\tilde{Z},\tilde{\tau}_{\tilde{Z}},\mathcal{P})$ is a soft subspace of a $\tilde{S}TS$ $(\tilde{X},\tilde{\tau},\mathcal{P})$ where $Z \subseteq X$, $\tilde{\tau}_Z = \{(A_Z,\mathcal{P}) = \tilde{Z} \tilde{\cap} (A,\mathcal{P}); (A,\mathcal{P}) \tilde{\in} \tilde{\tau}\}$ is known as the soft relative topology on $\tilde{Z}$, and $A_Z(e) = \tilde{Z} \tilde{\cap} A(e)$, for all $e \tilde{\in} \mathcal{P}$ [4].

In the context of soft classes, Athar Kharal and B. Ahmad [6] defined a soft mapping and investigated some characteristics of soft set images and inverse images. The references ([7], [8] [9], [10], [11], [12], [13]) introduced and studied various types of soft functions, such as: soft irresolute, soft semi-open (closed), soft open (closed), soft α-open (closed), soft pre-open (closed), soft b-open (closed), soft β-open (closed), and soft β_c-open.

However, The structure of this paper is as follows:

In Section 2, we define and introduce soft S_p-irresolute functions via soft S_p-(respectively, open [14] and closed [15]) sets. Some of its basic properties and relationships with some other types of soft functions are given, and we study the behavior of soft S_p-irresolute functions under soft composition. In addition to these, we introduce the notions of soft S_p-Hausdorff space, and study their topological properties.

In Section 3, we use soft S_p-open [14] (respectively, soft S_p-closed [15]) sets to define and study new types of soft functions known as soft S_p-open (respectively, soft S_p-closed) as a strong form of soft semi-open (respectively, soft semi-closed) function. Some of its basic properties and relationships with some other types of soft functions are given, and we study the behavior of soft S_p-open (respectively, soft S_p-closed) functions under soft composition.

Throughout the paper, $(\tilde{X},\tilde{\tau},\mathcal{P})$ and $(\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ or simply $\tilde{X}$ and $\tilde{Y}$ denoted soft topological spaces on which no separation axioms are assumed unless mentioned. $\tilde{s}cl(A,\mathcal{P})$ (respectively, $\tilde{s}int(A,\mathcal{P})$) is soft closure (respectively, soft interior) of $(A,\mathcal{P})$.

Further important terms and results are pointed out in the coming sections.

Definition 1.1. A $(A,\mathcal{P}) \tilde{\subseteq} (\tilde{X},\tilde{\tau},\mathcal{P})$ is known as a soft semi- [7] (respectively, pre- [16], α- [9], b- [10], β- [17], and regular [16]) open set, if $(A,\mathcal{P}) \tilde{\subseteq} \tilde{s}cl(\tilde{s}int(A,\mathcal{P}))$ (respectively, $(A,\mathcal{P}) \tilde{\subseteq} \tilde{s}int(\tilde{s}cl(A,\mathcal{P}))$, $(A,\mathcal{P}) \tilde{\subseteq} \tilde{s}int(\tilde{s}cl(\tilde{s}int(A,\mathcal{P})))$, $(A,\mathcal{P}) \tilde{\subseteq} \tilde{s}cl(\tilde{s}int(A,\mathcal{P})) \tilde{\cup} \tilde{s}int(\tilde{s}cl(A,\mathcal{P}))$, $(A,\mathcal{P}) \tilde{\subseteq} \tilde{s}cl(\tilde{s}int(\tilde{s}cl(A,\mathcal{P})))$, and $(A,\mathcal{P}) = \tilde{s}int(\tilde{s}cl(A,\mathcal{P}))$).

The family of all soft semi- (respectively, pre-, α-, b-, β-, and regular) open sets in $\tilde{X}$ is indicated by $\tilde{S}SO(\tilde{X})$ (respectively, $\tilde{S}PO(\tilde{X})$, $\tilde{S}\alpha O(\tilde{X})$, $\tilde{S}bO(\tilde{X})$, $\tilde{S}\beta O(\tilde{X})$ and $\tilde{S}RO(\tilde{X})$).

Definition 1.2. The soft complement of a soft semi- (respectively, pre-, α-, b-, β-, and regular) open set is known as soft semi- [7] (respectively, pre- [16], α- [9], b- [10], β- [17], and regular [18]) closed. The family of all soft semi- (respectively, pre-, α-, b-, β-, and regular) closed sets in $\tilde{X}$ is indicated by $\tilde{S}SC(\tilde{X})$ (respectively, $\tilde{S}PC(\tilde{X})$, $\tilde{S}\alpha C(\tilde{X})$, $\tilde{S}bC(\tilde{X})$, $\tilde{S}\beta C(\tilde{X})$, and $\tilde{S}RC(\tilde{X})$).

Definition 1.3. A $\tilde{S}TS$ $(\tilde{X},\tilde{\tau},\mathcal{P})$ and $(A,\mathcal{P}) \tilde{\subseteq} (\tilde{X},\tilde{\tau},\mathcal{P})$ is known as a soft S_p-open [14] (respectively, $\tilde{S}S_c$-open [19], and soft β_c-open [13]) set, if $(A,\mathcal{P}) \tilde{\in} \tilde{S}SO(\tilde{X})$ (respectively, $\tilde{S}SO(\tilde{X})$, and $\tilde{S}\beta O(\tilde{X})$) and $\forall\, \tilde{e_x} \tilde{\in} (A,\mathcal{P})$, there is $(W,\mathcal{P}) \tilde{\in} \tilde{S}PC(\tilde{X})$ (respectively, $\tilde{\tau}^c$, and $\tilde{\tau}^c$) such that $\tilde{e_x} \tilde{\in} (W,\mathcal{P}) \tilde{\subseteq} (A,\mathcal{P})$. The family of all soft S_p- (respectively, $\tilde{S}S_c$-, and soft β_c-) open subsets of $\tilde{X}$ is indicated by $\tilde{S}S_pO(\tilde{X})$ (respectively, $\tilde{S}S_cO(\tilde{X})$, and $\tilde{S}\beta_cO(\tilde{X})$).

Definition 1.4. The soft complement of a soft S_p-open set is known as soft S_p-closed [15]. The family of all soft S_p-closed sets in $\tilde{X}$ is indicated by $\tilde{S}S_pC(\tilde{X})$.

Definition 1.5. Let $(A,\mathcal{P}) \tilde{\subseteq} (\tilde{X},\tilde{\tau},\mathcal{P})$. Then:

(1) $\tilde{s}S_pcl(A,\mathcal{P}) = \tilde{\cap}\{(C,\mathcal{P}):(C,\mathcal{P}) \tilde{\in} \tilde{S}S_pC(\tilde{X}), (A,\mathcal{P}) \tilde{\subseteq} (C,\mathcal{P})\}$. Clearly, $\tilde{s}S_pcl(A,\mathcal{P})$ is the smallest soft S_p-closed set contains $(A,\mathcal{P})$ [15].

(2) $\tilde{s}S_pint(A,\mathcal{P}) = \tilde{\cup}\{(O,\mathcal{P}):(O,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X}), (O,\mathcal{P}) \tilde{\subseteq} (A,\mathcal{P})\}$. Clearly, $\tilde{s}S_pint(A,\mathcal{P})$ is the largest soft S_p-open set contained in $(A,\mathcal{P})$ [15].

(3) $\tilde{s}S_pBd(A,\mathcal{P}) = \tilde{s}S_pcl(A,\mathcal{P}) \tilde{\cap} \tilde{s}S_pcl(\tilde{X}\tilde{\setminus}(A,\mathcal{P}))$ [15].

(4) $\tilde{s}Bd(A,\mathcal{P}) = \tilde{s}cl(A,\mathcal{P}) \tilde{\cap} \tilde{s}cl(\tilde{X}\tilde{\setminus}(A,\mathcal{P}))$ [5].

Definition 1.6. A $\tilde{S}TS$ $(\tilde{X},\tilde{\tau},\mathcal{P})$ is known as:

(1) Soft extremally disconnected [20], if $\tilde{s}cl(A,\mathcal{P}) \tilde{\in} \tilde{\tau}$, $\forall\, (A,\mathcal{P}) \tilde{\in} \tilde{\tau}$. Or, $\tilde{s}int(A,\mathcal{P}) \tilde{\in} \tilde{\tau}^c$, $\forall\, (A,\mathcal{P}) \tilde{\in} \tilde{\tau}^c$.

(2) Soft locally indiscrete [21], if $(A,\mathcal{P}) \tilde{\in} \tilde{\tau}^c$, $\forall\, (A,\mathcal{P}) \tilde{\in} \tilde{\tau}$. Or, $(A,\mathcal{P}) \tilde{\in} \tilde{\tau}$, $\forall\, (A,\mathcal{P}) \tilde{\in} \tilde{\tau}^c$.

(3) Soft submaximal [16], if each soft dense subset of $\tilde{X}$ is soft open set.

(4) Soft T_1-space [22], if $\tilde{e_x}, \tilde{e_y} \tilde{\in} \tilde{S}P(\tilde{X})$ such that $\tilde{e_x} \neq \tilde{e_y}$, there are $(A_1,\mathcal{P}),(A_2,\mathcal{P}) \tilde{\in} \tilde{\tau}$ such that $\tilde{e_x} \tilde{\in} (A_1,\mathcal{P})$, $\tilde{e_y} \tilde{\notin} (A_1,\mathcal{P})$ and $\tilde{e_y} \tilde{\in} (A_2,\mathcal{P})$, $\tilde{e_x} \tilde{\notin} (A_2,\mathcal{P})$.

(5) Soft T_2-space or soft Hausdorff space [22], if $\tilde{e_x}, \tilde{e_y} \tilde{\in} \tilde{S}P(\tilde{X})$ such that $\tilde{e_x} \neq \tilde{e_y}$, there are $(A_1,\mathcal{P}),(A_2,\mathcal{P}) \tilde{\in} \tilde{\tau}$ such that $\tilde{e_x} \tilde{\in} (A_1,\mathcal{P})$, $\tilde{e_y} \tilde{\in} (A_2,\mathcal{P})$, and $(A_1,\mathcal{P}) \tilde{\cap} (A_2,\mathcal{P}) = \tilde{\emptyset}$.

(6) Soft semi-T_2-space or soft semi-Hausdorff space [23], if $\tilde{e_x}, \tilde{e_y} \tilde{\in} \tilde{S}P(\tilde{X})$ such that $\tilde{e_x} \neq \tilde{e_y}$, there are $(A_1,\mathcal{P}),(A_2,\mathcal{P}) \tilde{\in} \tilde{S}SO(\tilde{X})$ such that $\tilde{e_x} \tilde{\in} (A_1,\mathcal{P})$, $\tilde{e_y} \tilde{\in} (A_2,\mathcal{P})$, and $(A_1,\mathcal{P}) \tilde{\cap} (A_2,\mathcal{P}) = \tilde{\emptyset}$.

(7) Soft regular space [22], if $(C,\mathcal{P}) \tilde{\in} \tilde{\tau}^c$ and $\tilde{e_x} \tilde{\in} \tilde{S}P(\tilde{X})$ such that $\tilde{e_x} \tilde{\notin} (C,\mathcal{P})$, there exist $(A_1,\mathcal{P}),(A_2,\mathcal{P}) \tilde{\in} \tilde{\tau}$ such that $\tilde{e_x} \tilde{\in} (A_1,\mathcal{P})$, $(C,\mathcal{P}) \tilde{\subseteq} (A_2,\mathcal{P})$ and $(A_1,\mathcal{P}) \tilde{\cap} (A_2,\mathcal{P}) = \tilde{\emptyset}$.

(8) Soft semi-regular space [24], if $\forall\, (A,\mathcal{P}) \tilde{\in} \tilde{\tau}$ and $\forall\, \tilde{e_x} \tilde{\in} (A,\mathcal{P})$, there is $(O,\mathcal{P}) \tilde{\in} \tilde{S}RO(\tilde{X})$ such that $\tilde{e_x} \tilde{\in} (O,\mathcal{P}) \tilde{\subseteq} (A,\mathcal{P})$.

Definition 1.7. Let $\tilde{S}S(X,\mathcal{P})$, $\tilde{S}S(Y,\acute{\mathcal{P}})$ be the families of all soft sets, $u: X \to Y$ and $p: \mathcal{P} \to \acute{\mathcal{P}}$ be functions. Then, a soft function $\tilde{f}_{pu}: \tilde{S}S(X,\mathcal{P}) \to \tilde{S}S(Y,\acute{\mathcal{P}})$ is defined as:

(1) If $(A,\mathcal{P}) \tilde{\in} \tilde{S}S(X,\mathcal{P})$, the soft image of $(A,\mathcal{P})$ under $\tilde{f}_{pu}$, written as $\tilde{f}_{pu}(A,\mathcal{P}) = (\tilde{f}_{pu}(A), p(\mathcal{P})) \tilde{\in} \tilde{S}S(Y,\acute{\mathcal{P}})$, $\forall\, \beta \in \acute{\mathcal{P}}$ defined as:

$\tilde{f}_{pu}(A)(\beta) = \begin{cases} u(\bigcup_{\alpha \in p^{-1}(\beta) \cap \mathcal{P}} A(\alpha)), & if\ p^{-1}(\beta) \cap \mathcal{P} \neq \emptyset \\ \tilde{\emptyset}, & otherwise \end{cases}$ [6], so if $\tilde{e_x} \tilde{\in} \tilde{S}P(\tilde{X})$, then $\tilde{f}_{pu}(\tilde{e_x}) = p(e)_{u(x)}$ [25].

(2) If $(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S(Y,\acute{\mathcal{P}})$, the soft inverse image of $(B,\acute{\mathcal{P}})$ under f_{pu}, written as $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) = (\tilde{f}_{pu}^{-1}(B), p^{-1}(\acute{\mathcal{P}})) \tilde{\in} \tilde{S}S(X,\mathcal{P})$, $\forall\, \alpha \in \mathcal{P}$ defined as:

$\tilde{f}_{pu}^{-1}(B)(\alpha) = \begin{cases} u^{-1}\left(B\big(p(\alpha)\big)\right), & p(\alpha) \in \acute{\mathcal{P}} \\ \tilde{\emptyset}, & otherwise \end{cases}$ [6], so if $\tilde{e_y} \tilde{\in} \tilde{S}P(\tilde{Y})$ and $\tilde{f}_{pu}$ is soft bijective, then $\tilde{f}_{pu}^{-1}(\tilde{e_y}) = p^{-1}(\acute{e})_{u^{-1}(y)}$ [25].

The soft function $\tilde{f}_{pu}: \tilde{S}S(X,\mathcal{P}) \to \tilde{S}S(Y,\acute{\mathcal{P}})$ is known as soft injective (respectively, soft surjective, soft bijective) if u, p are both injective (respectively, surjective, bijective) functions [26].

Theorem 1.8. ([6] [3] [26]) Let $\tilde{f}_{pu}: \tilde{S}S(X,\mathcal{P}) \to \tilde{S}S(Y,\acute{\mathcal{P}})$ be a soft function, the following are true:

(1) $\tilde{f}_{pu}((A_1,\mathcal{P}) \tilde{\cap} (A_2,\mathcal{P})) \tilde{\subseteq} \tilde{f}_{pu}(A_1,\mathcal{P}) \tilde{\cap} \tilde{f}_{pu}(A_2,\mathcal{P})$, $\forall\, (A_1,\mathcal{P}),(A_2,\mathcal{P}) \tilde{\in} \tilde{S}S(X,\mathcal{P})$, the equality holds if $\tilde{f}_{pu}$ is soft injective.

(2) $\tilde{Y} \tilde{\setminus} \tilde{f}_{pu}(A,\mathcal{P}) \tilde{\subseteq} \tilde{f}_{pu}(\tilde{X} \tilde{\setminus} (A,\mathcal{P}))$, $\forall\, (A,\mathcal{P}) \tilde{\in} \tilde{S}S(X,\mathcal{P})$, the equality holds if $\tilde{f}_{pu}$ is soft surjective.

(3) $\tilde{f}_{pu}^{-1}(\tilde{Y} \tilde{\setminus} (B,\acute{\mathcal{P}})) = \tilde{X} \tilde{\setminus} \tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})$, $\forall\, (B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S(\tilde{Y},\acute{\mathcal{P}})$.

(4) $\tilde{f}_{pu}(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})) \tilde{\subseteq} (B,\acute{\mathcal{P}})$, $\forall\, (B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S(\tilde{Y},\acute{\mathcal{P}})$, the equality holds if $\tilde{f}_{pu}$ is soft surjective.

(5) $(A,\mathcal{P}) \tilde{\subseteq} \tilde{f}_{pu}^{-1}(\tilde{f}_{pu}(A,\mathcal{P}))$, $\forall\, (A,\mathcal{P}) \tilde{\in} \tilde{S}S(X,\mathcal{P})$, the equality holds if $\tilde{f}_{pu}$ is soft injective.

Definition 1.9. A soft function $\tilde{f}_{pu}: (\tilde{X},\tilde{\tau},\mathcal{P}) \to (\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ is known as soft continuous [3] (respectively, soft semi-continuous [7], soft pre-continuous [9], soft α-continuous [9], soft β-continuous [12], soft b-continuous [10], soft perfectly continuous [27], soft RC-continuous [27], $\tilde{S}S_c$-continuous [19], and soft S_p-continuous [28]), if $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{\tau}$ (respectively, $\tilde{S}SO(\tilde{X})$, $\tilde{S}PO(\tilde{X})$, $\tilde{S}\alpha O(\tilde{X})$, $\tilde{S}\beta O(\tilde{X})$, $\tilde{S}bO(\tilde{X})$, $\tilde{S}CO(\tilde{X})$, $\tilde{S}RC(\tilde{X})$, $\tilde{S}S_cO(\tilde{X})$, $\tilde{S}S_pO(\tilde{X})$), $\forall\, (B,\acute{\mathcal{P}}) \tilde{\in} \tilde{\sigma}$.

Definition 1.10. A soft function $\tilde{f}_{pu}: (\tilde{X},\tilde{\tau},\mathcal{P}) \to (\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ is known as:

(1) Soft homeomorphism [8] if $\tilde{f}_{pu}$ is soft bijective and $\tilde{f}_{pu}, \tilde{f}_{pu}^{-1}$ are soft continuous.

(2) Soft irresolute [7] if $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}SO(\tilde{X})$, $\forall (B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}SO(\tilde{Y})$.

Definition 1.11. A soft function $\tilde{f}_{pu}: (\tilde{X},\tilde{\tau},\mathcal{P}) \to (\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ is known as soft open [8] (respectively, soft semi-open [7], soft α-open [9], soft pre-open [9], soft b-open [10], soft β-open [11], and soft β_c-open [13]), if $\tilde{f}_{pu}(A,\mathcal{P}) \tilde{\in} \tilde{\sigma}$ (respectively, $\tilde{S}SO(\tilde{Y})$, $\tilde{S}\alpha O(\tilde{Y})$, $\tilde{S}PO(\tilde{Y})$, $\tilde{S}bO(\tilde{Y})$, $\tilde{S}\beta O(\tilde{Y})$, and $\tilde{S}\beta_c O(\tilde{Y})$), $\forall\, (A,\mathcal{P}) \tilde{\in} \tilde{\tau}$.

Definition 1.12. A soft function $\tilde{f}_{pu}: (\tilde{X},\tilde{\tau},\mathcal{P}) \to (\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ is known as soft closed [8] (respectively, soft semi-closed [7], soft α-closed [9], soft pre-closed [9], soft b-closed [10], and soft β-closed [11]), if $\tilde{f}_{pu}(A,\mathcal{P}) \tilde{\in} \tilde{\sigma}^c$ (respectively, $\tilde{S}SC(\tilde{Y})$, $\tilde{S}\alpha C(\tilde{Y})$, $\tilde{S}PC(\tilde{Y})$, $\tilde{S}bC(\tilde{Y})$, and $\tilde{S}\beta C(\tilde{Y})$), $\forall\, (A,\mathcal{P}) \tilde{\in} \tilde{\tau}^c$.

Definition 1.13. A soft function $\tilde{f}_{pu}:(\tilde{X},\tilde{\tau},\mathcal{P}) \to (\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ is known as soft irresolute open [29] (respectively, soft irresolute closed [29]), if $\tilde{f}_{pu}(A,\mathcal{P}) \tilde{\in} \tilde{S}SO(\tilde{Y})$ (respectively, $\tilde{S}SC(\tilde{Y})$), $\forall$ $(A,\mathcal{P}) \tilde{\in} \tilde{S}SO(\tilde{X})$ (respectively, $\tilde{S}SC(\tilde{X})$).

Definition 1.14. A soft function $\tilde{f}_{pu}:(\tilde{X},\tilde{\tau},\mathcal{P}) \to (\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ is known as soft almost open [30] (respectively, soft almost semi-open [24], soft almost α-open [31], soft almost pre-open [32], soft almost b-open [33], and soft almost β-open [34]) if $\tilde{f}_{pu}(A,\mathcal{P}) \tilde{\in} \tilde{\sigma}$ (respectively, $\tilde{S}SO(\tilde{Y})$, $\tilde{S}\alpha O(\tilde{Y})$, $\tilde{S}PO(\tilde{Y})$, $\tilde{S}bO(\tilde{Y})$, and $\tilde{S}\beta O(\tilde{Y})$), $\forall$ $(A,\mathcal{P}) \tilde{\in} \tilde{S}RO(\tilde{X})$.

Definition 1.15. [26] Let $\tilde{S}S(X,\mathcal{P})$, $\tilde{S}S(Y,\acute{\mathcal{P}})$, and $\tilde{S}S(W,\ddot{\mathcal{P}})$ be the families of all soft sets, $\tilde{f}_{pu}:\tilde{S}S(X,\mathcal{P}) \to \tilde{S}S(Y,\acute{\mathcal{P}})$ and $\tilde{g}_{qv}:\tilde{S}S(Y,\acute{\mathcal{P}}) \to \tilde{S}S(W,\ddot{\mathcal{P}})$ be two soft functions. Then,

(1) soft composition is a soft function $\tilde{g}_{qv} \tilde{\circ} \tilde{f}_{pu}:\tilde{S}S(X,\mathcal{P}) \to \tilde{S}S(W,\ddot{\mathcal{P}})$ is defined as $(\tilde{g}_{qv} \tilde{\circ} \tilde{f}_{pu})(A,\mathcal{P}) = \tilde{g}_{qv}(\tilde{f}_{pu}(A,\mathcal{P}))$, $\forall$ $(A,\mathcal{P}) \tilde{\in} \tilde{S}S(X,\mathcal{P})$ where $u: X \to Y$, $p:\mathcal{P} \to \acute{\mathcal{P}}$, $v: Y \to W$, and $q:\acute{\mathcal{P}} \to \ddot{\mathcal{P}}$.

(2) $(\tilde{g}_{qv} \tilde{\circ} \tilde{f}_{pu})^{-1}(A,\ddot{\mathcal{P}}) = \tilde{f}_{pu}^{-1}(\tilde{g}_{qv}^{-1}(A,\ddot{\mathcal{P}}))$, $\forall$ $(A,\ddot{\mathcal{P}}) \tilde{\in} \tilde{S}S(W,\ddot{\mathcal{P}})$, if $\tilde{g}_{qv} \tilde{\circ} \tilde{f}_{pu}:\tilde{S}S(X,\mathcal{P}) \to \tilde{S}S(W,\ddot{\mathcal{P}})$ is a soft composition function.

Proposition 1.16. [23] Let $(A,\mathcal{P}) \tilde{\subseteq} (\tilde{X},\tilde{\tau},\mathcal{P})$. Then, $(A,\mathcal{P}) \tilde{\in} \tilde{S}SO(\tilde{X})$ iff there is $(O,\mathcal{P}) \tilde{\in} \tilde{\tau}$ such that $(O,\mathcal{P}) \tilde{\subseteq} (A,\mathcal{P}) \tilde{\subseteq} \tilde{s}cl(O,\mathcal{P})$.

Proposition 1.17. Let $\tilde{f}_{pu}:(\tilde{X},\tilde{\tau},\mathcal{P}) \to (\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ be a soft function. Then:

(1) $\tilde{f}_{pu}$ is soft continuous iff $\tilde{f}_{pu}(\tilde{s}cl(A,\mathcal{P})) \tilde{\subseteq} \tilde{s}cl(\tilde{f}_{pu}(A,\mathcal{P}))$, $\forall$ $(A,\mathcal{P}) \tilde{\subseteq} \tilde{X}$ [35].

(2) $\tilde{f}_{pu}$ is soft homeomorphism iff $\tilde{f}_{pu}$ is soft bijective, soft continuous and soft open [8].

(3) $\tilde{f}_{pu}$ is soft homeomorphism iff $\tilde{f}_{pu}(\tilde{s}cl(A,\mathcal{P})) = \tilde{s}cl(\tilde{f}_{pu}(A,\mathcal{P}))$, $\forall$ $(A,\mathcal{P}) \tilde{\subseteq} \tilde{X}$ [8].

Proposition 1.18. [14] Let $(A,\mathcal{P})$,$(B,\mathcal{P}) \tilde{\subseteq} (\tilde{X},\tilde{\tau},\mathcal{P})$. Then:

(1) $(A,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$ iff $(A,\mathcal{P}) = \tilde{\cup}\ (B_\vartheta,\mathcal{P})$, where $(A,\mathcal{P}) \tilde{\in} \tilde{S}SO(\tilde{X})$, and $(B_\vartheta,\mathcal{P}) \tilde{\in} \tilde{S}PC(\tilde{X})$, $\forall$ $\vartheta \in \aleph$.

(2) $(A,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$ iff $\forall$ $\tilde{e_x} \tilde{\in} (A,\mathcal{P})$, there is $(B,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$ such that $\tilde{e_x} \tilde{\in} (B,\mathcal{P}) \tilde{\subseteq} (A,\mathcal{P})$.

Proposition 1.19. Let $(\tilde{X},\tilde{\tau},\mathcal{P})$ be a $\tilde{S}TS$. Then:

(1) $\tilde{S}S_pO(\tilde{X}) \tilde{\subseteq} \tilde{S}SO(\tilde{X})$ [14].

(2) $\tilde{S}S_pC(\tilde{X}) \tilde{\subseteq} \tilde{S}SC(\tilde{X})$ [15].

(3) $\tilde{S}RC(\tilde{X}) \tilde{\subseteq} \tilde{S}S_pO(\tilde{X})$ [14].

(4) $\tilde{S}CO(\tilde{X}) \tilde{\subseteq} \tilde{S}S_pO(\tilde{X})$ [14].

(5) $\tilde{S}S_pO(\tilde{X}) \tilde{\subseteq} \tilde{S}\beta C(\tilde{X})$ (respectively, $\tilde{S}bC(\tilde{X})$) [14].

Proposition 1.20. [14] If a $\tilde{S}TS$ $(\tilde{X},\tilde{\tau},\mathcal{P})$ is soft locally indiscrete, then

(1) $\tilde{S}SO(\tilde{X}) = \tilde{S}S_cO(\tilde{X}) = \tilde{S}S_pO(\tilde{X})$.

(2) $\tilde{\tau} = \tilde{S}S_pO(\tilde{X}) = \tilde{S}SO(\tilde{X})$.

(3) $\tilde{S}\alpha O(\tilde{X}) = \tilde{S}S_pO(\tilde{X})$.

(4) $\tilde{S}S_pO(\tilde{X}) \tilde{\subseteq} \tilde{S}PO(\tilde{X})$.

(5) $\tilde{S}S_pO(\tilde{X}) \tilde{\subseteq} \tilde{S}\beta_cO(\tilde{X})$ (respectively, $\tilde{S}b_cO(\tilde{X})$).

Proposition 1.21. If a $\tilde{S}TS$ $(\tilde{X},\tilde{\tau},\mathcal{P})$ is a soft T_1-space, then

(1) $\tilde{S}SO(\tilde{X}) = \tilde{S}S_cO(\tilde{X}) = \tilde{S}S_pO(\tilde{X})$ [14].

(2) $\tilde{S}SC(\tilde{X}) = \tilde{S}S_cC(\tilde{X}) = \tilde{S}S_pC(\tilde{X})$ [15].

(3) $\tilde{\tau} \tilde{\subseteq} \tilde{S}S_pO(\tilde{X})$ [14].

(4) $\tilde{S}\alpha O(\tilde{X}) \tilde{\subseteq} \tilde{S}S_pO(\tilde{X})$ [14].

Proposition 1.22. [15] If a $\tilde{S}TS$ $(\tilde{X},\tilde{\tau},\mathcal{P})$ is soft locally indiscrete, then:

(1) $\tilde{S}SC(\tilde{X}) = \tilde{S}S_cC(\tilde{X}) = \tilde{S}S_pC(\tilde{X})$.

(2) $\tilde{S}S_pC(\tilde{X}) = \tilde{\tau}^c$.

(3) $\tilde{S}S_pC(\tilde{X}) = \tilde{S}\alpha C(\tilde{X})$.

(4) $\tilde{S}S_pC(\tilde{X}) \tilde{\subseteq} \tilde{S}PC(\tilde{X})$.

Proposition 1.23. If $(\tilde{X}, \tilde{\tau}, \mathcal{P})$ is a soft regular space, then:

(1) $\tilde{\tau} \tilde{\subseteq} \tilde{S}S_pO(\tilde{X})$ [14].

(2) $\tilde{\tau}^c \tilde{\subseteq} \tilde{S}S_pC(\tilde{X})$ [15].

Proposition 1.24. A $\tilde{S}TS$ $(\tilde{X}, \tilde{\tau}, \mathcal{P})$ is soft extremally disconnected iff

(1) $\tilde{S}S_pO(\tilde{X}) \tilde{\subseteq} \tilde{S}PO(\tilde{X})$ (respectively, $\tilde{S}\alpha O(\tilde{X})$) [14].

(2) $\tilde{S}S_pC(\tilde{X}) \tilde{\subseteq} \tilde{S}PC(\tilde{X})$ (respectively, $\tilde{S}\alpha C(\tilde{X})$) [15].

Proposition 1.25. [14] If $(\tilde{X}, \tilde{\tau}, \mathcal{P})$ is a soft submaximal space, then $\tilde{S}S_pO(\tilde{X}) \tilde{\subseteq} \tilde{S}\beta_cO(\tilde{X})$.

Corollary 1.26. [14] If a $\tilde{S}TS$, $(\tilde{X}, \tilde{\tau}, \mathcal{P})$ is soft extremally disconnected and soft T_1-space, then $\tilde{S}\alpha O(\tilde{X}) = \tilde{S}S_pO(\tilde{X})$.

Proposition 1.27. [28] Let $(\tilde{Z}, \tilde{\tau}_{\tilde{Z}}, \mathcal{P})$ be a soft subspace of a $\tilde{S}TS$ $(\tilde{X}, \tilde{\tau}, \mathcal{P})$ and $\tilde{Z} \tilde{\in} \tilde{\tau}$ (respectively, $\tilde{S}CO(\tilde{X})$). If $(A, \mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$, then $(A, \mathcal{P}) \tilde{\cap} \tilde{Z} \tilde{\in} \tilde{S}S_pO(\tilde{Z})$.

Theorem 1.28. [15] For any $(A, \mathcal{P}) \tilde{\subseteq} (\tilde{X}, \tilde{\tau}, \mathcal{P})$, we have

(1) $(A, \mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$ iff $(A, \mathcal{P}) = \tilde{s}S_pint(A, \mathcal{P})$.

(2) $(A, \mathcal{P}) \tilde{\in} \tilde{S}S_pC(\tilde{X})$ iff $(A, \mathcal{P}) = \tilde{s}S_pcl(A, \mathcal{P})$.

(3) $\tilde{s}S_pcl(A, \mathcal{P}) = (A, \mathcal{P}) \tilde{\cup} \tilde{s}S_pBd(A, \mathcal{P})$.

Theorem 1.29. [28] A soft function $\tilde{f}_{pu}: (\tilde{X}, \tilde{\tau}, \mathcal{P}) \to (\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ is soft S_p-continuous iff

(1) $\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$, $\forall$ $(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{\sigma}$.

(2) $\tilde{f}_{pu}^{-1}(C, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pC(\tilde{X})$, $\forall (C, \acute{\mathcal{P}}) \tilde{\in} \tilde{\sigma}^c$.

Proposition 1.30. [28] Let $\tilde{f}_{pu}: (\tilde{X}, \tilde{\tau}, \mathcal{P}) \to (\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ be a soft continuous, and soft open function. If $(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$, then $\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$.

Proposition 1.31. Let $\tilde{f}_{pu}: (\tilde{X}, \tilde{\tau}, \mathcal{P}) \to (\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ be a soft continuous and soft open function. If $(A, \mathcal{P}) \tilde{\in} \tilde{S}SO(\tilde{X})$, then $\tilde{f}_{pu}(A, \mathcal{P}) \tilde{\in} \tilde{S}SO(\tilde{Y})$.

Proof. Since $(A, \mathcal{P}) \tilde{\in} \tilde{S}SO(\tilde{X})$, then by Proposition 1.16, there exists $(O, \mathcal{P}) \tilde{\in} \tilde{\tau}$ such that $(O, \mathcal{P}) \tilde{\subseteq} (A, \mathcal{P}) \tilde{\subseteq} \tilde{s}cl(O, \mathcal{P})$. So, $\tilde{f}_{pu}(O, \mathcal{P}) \tilde{\subseteq} \tilde{f}_{pu}(A, \mathcal{P}) \tilde{\subseteq} \tilde{f}_{pu}(\tilde{s}cl(O, \mathcal{P}))$. Since $\tilde{f}_{pu}$ is soft open, then $\tilde{f}_{pu}(O, \mathcal{P}) \tilde{\in} \tilde{\sigma}$. By the soft continuity of $\tilde{f}_{pu}$ and Proposition 1.17(1), then $\tilde{f}_{pu}(\tilde{s}cl(O, \mathcal{P})) \tilde{\subseteq} \tilde{s}cl(\tilde{f}_{pu}(O, \mathcal{P}))$. Hence, we obtain that $\tilde{f}_{pu}(O, \mathcal{P}) \tilde{\subseteq} \tilde{f}_{pu}(A, \mathcal{P}) \tilde{\subseteq} \tilde{s}cl(\tilde{f}_{pu}(O, \mathcal{P}))$. Therefore, by Proposition 1.16, $\tilde{f}_{pu}(A, \mathcal{P}) \tilde{\in} \tilde{S}SO(\tilde{Y})$.

Proposition 1.32. Let $\tilde{f}_{pu}: (\tilde{X}, \tilde{\tau}, \mathcal{P}) \to (\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ be soft homeomorphism. If $(C, \mathcal{P}) \tilde{\in} \tilde{S}PC(\tilde{X})$, then $\tilde{f}_{pu}(C, \mathcal{P}) \tilde{\in} \tilde{S}PC(\tilde{Y})$.

Proof. Since $(C, \mathcal{P}) \tilde{\in} \tilde{S}PC(\tilde{X})$, then $\tilde{s}cl(\tilde{s}int(C, \mathcal{P})) \tilde{\subseteq} (C, \mathcal{P})$ and $\tilde{f}_{pu}(\tilde{s}cl(\tilde{s}int(C, \mathcal{P}))) \tilde{\subseteq} \tilde{f}_{pu}(C, \mathcal{P})$. Since $\tilde{f}_{pu}$ is soft homeomorphism, so Proposition 1.17(3) $\tilde{s}cl(\tilde{s}int(\tilde{f}_{pu}(C, \mathcal{P}))) = \tilde{f}_{pu}(\tilde{s}cl(\tilde{s}int(C, \mathcal{P}))) \tilde{\subseteq} \tilde{f}_{pu}(C, \mathcal{P})$. Therefore, $\tilde{f}_{pu}(C, \mathcal{P}) \tilde{\in} \tilde{S}PC(\tilde{Y})$.

2. Soft S_p-Irresolute Functions

Definition 2.1. Let $(\tilde{X}, \tilde{\tau}, \mathcal{P})$ and $(\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ be two $\tilde{S}TS$ and $u: X \to Y$, $p: \mathcal{P} \to \acute{\mathcal{P}}$ be functions. A soft function $\tilde{f}_{pu}: (\tilde{X}, \tilde{\tau}, \mathcal{P}) \to (\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ is known as soft S_p-irresolute at a soft point $\tilde{e_x} \tilde{\in} \tilde{S}P(\tilde{X})$, if $\forall$ $(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$ in $\tilde{Y}$ containing $\tilde{f}_{pu}(\tilde{e_x})$, there exists $\tilde{e_x} \tilde{\in} (A, \mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$ such that $\tilde{f}_{pu}(A, \mathcal{P}) \tilde{\subseteq} (B, \acute{\mathcal{P}})$. If $\tilde{f}_{pu}$ is soft S_p-irresolute at every soft point $\tilde{e_x} \tilde{\in} \tilde{S}P(\tilde{X})$, then it is called a soft S_p-irresolute function.

Theorem 2.2. A soft function $\tilde{f}_{pu}: (\tilde{X}, \tilde{\tau}, \mathcal{P}) \to (\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ is soft S_p-irresolute iff $\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$, $\forall$ $(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$.

Proof. Let $\tilde{f}_{pu}$ be soft S_p-irresolute and $(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$. To prove that $\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$. If $\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}}) = \tilde{\emptyset}$, then $\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$. If not, let $\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}}) \neq \tilde{\emptyset}$ and $\tilde{e_x} \tilde{\in} \tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}})$, we have $\tilde{f}_{pu}(\tilde{e_x}) \tilde{\in} (B, \acute{\mathcal{P}})$. Since $\tilde{f}_{pu}$ is soft S_p-

irresolute, there is $\tilde{e_x} \tilde{\in} (A,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$ such that $\tilde{f}_{pu}(A,\mathcal{P}) \tilde{\subseteq} (B,\acute{\mathcal{P}})$. Hence, $\tilde{e_x} \tilde{\in} (A,\mathcal{P}) \tilde{\subseteq} \tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})$ and therefore, by Proposition 1.18(2), $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$.

Conversely, let $\tilde{e_x} \tilde{\in} \tilde{S}P(\tilde{X})$ and $(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$ containing $\tilde{f}_{pu}(\tilde{e_x})$. Then, $\tilde{e_x} \tilde{\in} \tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$ and $(A,\mathcal{P}) = \tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})$ such that $\tilde{f}_{pu}(A,\mathcal{P}) = \tilde{f}_{pu}(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})) \tilde{\subseteq} (B,\acute{\mathcal{P}})$. Therefore, $\tilde{f}_{pu}$ is soft S_p-irresolute.

Proposition 2.3. A soft function $\tilde{f}_{pu}: (\tilde{X},\tilde{\tau},\mathcal{P}) \to (\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ is soft S_p-irresolute iff $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pC(\tilde{X})$, $\forall\, (B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pC(\tilde{Y})$.

Proof. Obvious.

Proposition 2.4. A soft function $\tilde{f}_{pu}: (\tilde{X},\tilde{\tau},\mathcal{P}) \to (\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ is soft S_p-irresolute iff $\tilde{f}_{pu}(\tilde{s}S_pcl(A,\mathcal{P})) \tilde{\subseteq} \tilde{s}S_pcl(\tilde{f}_{pu}(A,\mathcal{P}))$, $\forall\, (A,\mathcal{P}) \tilde{\subseteq} \tilde{X}$.

Proof. Let $\tilde{f}_{pu}$ be soft S_p-irresolute and $(A,\mathcal{P}) \tilde{\subseteq} \tilde{X}$. Then, $\tilde{f}_{pu}(A,\mathcal{P}) \tilde{\subseteq} \tilde{Y}$. Since $\tilde{f}_{pu}(A,\mathcal{P}) \tilde{\subseteq} \tilde{s}S_pcl(\tilde{f}_{pu}(A,\mathcal{P}))$ and $\tilde{s}S_pcl(\tilde{f}_{pu}(A,\mathcal{P})) \tilde{\in} \tilde{S}S_pC(\tilde{Y})$. By hypothesis and Proposition 2.3, $\tilde{f}_{pu}^{-1}(\tilde{s}S_pcl(\tilde{f}_{pu}(A,\mathcal{P}))) \tilde{\in} \tilde{S}S_pC(\tilde{X})$ and so $\tilde{s}S_pcl(A,\mathcal{P})) \tilde{\subseteq} \tilde{f}_{pu}^{-1}(\tilde{s}S_pcl(\tilde{f}_{pu}(A,\mathcal{P})))$. Hence, $\tilde{f}_{pu}(\tilde{s}S_pcl(A,\mathcal{P})) \tilde{\subseteq} \tilde{s}S_pcl(\tilde{f}_{pu}(A,\mathcal{P}))$.

Conversely, let $(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pC(\tilde{Y})$. Then, $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\subseteq} \tilde{X}$. By hypothesis, $\tilde{f}_{pu}(\tilde{s}S_pcl(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}))) \tilde{\subseteq} \tilde{s}S_pcl(\tilde{f}_{pu}(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}))) \tilde{\subseteq} \tilde{s}S_pcl(B,\acute{\mathcal{P}}) = (B,\acute{\mathcal{P}})$. Hence, $\tilde{s}S_pcl(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})) \tilde{\subseteq} \tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})$ so that $\tilde{s}S_pcl(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})) = \tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})$. By Theorem 1.28(2), $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pC(\tilde{X})$. Thus by Proposition 2.3, $\tilde{f}_{pu}$ is soft S_p-irresolute.

Proposition 2.5. A soft function $\tilde{f}_{pu}: (\tilde{X},\tilde{\tau},\mathcal{P}) \to (\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ is soft S_p-irresolute iff $\tilde{s}S_pcl(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})) \tilde{\subseteq} \tilde{f}_{pu}^{-1}(\tilde{s}S_pcl\,(B,\acute{\mathcal{P}}))$, $\forall\, (B,\acute{\mathcal{P}}) \tilde{\subseteq} \tilde{Y}$.

Proof. Let $\tilde{f}_{pu}$ be soft S_p-irresolute and $(B,\acute{\mathcal{P}}) \tilde{\subseteq} \tilde{Y}$. Then, $\tilde{s}S_pcl(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pC(\tilde{Y})$, so that $\tilde{f}_{pu}^{-1}(\tilde{s}S_pcl(B,\acute{\mathcal{P}})) \tilde{\in} \tilde{S}S_pC(\tilde{X})$, and so $\tilde{s}S_pcl(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})) \tilde{\subseteq} \tilde{f}_{pu}^{-1}(\tilde{s}S_pcl(B,\acute{\mathcal{P}}))$.

Conversely, let $(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pC(\tilde{Y})$. Then, $\tilde{s}S_pcl(B,\acute{\mathcal{P}}) = (B,\acute{\mathcal{P}})$. By hypothesis, $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\subseteq} \tilde{s}S_pcl(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})) \tilde{\subseteq} \tilde{f}_{pu}^{-1}(\tilde{s}S_pcl(B,\acute{\mathcal{P}})) = \tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})$, and so $\tilde{s}S_pcl(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})) = \tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})$. Hence by Theorem 1.28(2), $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pC(\tilde{X})$. Thus by Proposition 2.3, $\tilde{f}_{pu}$ is soft S_p-irresolute.

Proposition 2.6. A soft bijective function $\tilde{f}_{pu}: (\tilde{X},\tilde{\tau},\mathcal{P}) \to (\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ is soft S_p-irresolute iff $\tilde{s}S_pint(\tilde{f}_{pu}(A,\mathcal{P})) \tilde{\subseteq} \tilde{f}_{pu}(\tilde{s}S_pint(A,\mathcal{P}))$, $\forall\, (A,\mathcal{P}) \tilde{\subseteq} \tilde{X}$.

Proof. Let $\tilde{f}_{pu}$ be soft S_p-irresolute and $(A,\mathcal{P}) \tilde{\subseteq} \tilde{X}$. Then, $\tilde{f}_{pu}(A,\mathcal{P}) \tilde{\subseteq} \tilde{Y}$. Since $\tilde{s}S_pint(\tilde{f}_{pu}(A,\mathcal{P})) \tilde{\subseteq} \tilde{f}_{pu}(A,\mathcal{P})$ and $\tilde{s}S_pint(\tilde{f}_{pu}(A,\mathcal{P})) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$. By hypothesis and Theorem 2.2, $\tilde{f}_{pu}^{-1}(\tilde{s}S_pint(\tilde{f}_{pu}(A,\mathcal{P}))) \tilde{\in} \tilde{S}S_pO(\tilde{X})$ and $\tilde{f}_{pu}^{-1}(\tilde{s}S_pint(\tilde{f}_{pu}(A,\mathcal{P}))) \tilde{\subseteq} \tilde{f}_{pu}^{-1}(\tilde{f}_{pu}(A,\mathcal{P}))$. Since $\tilde{f}_{pu}$ is a soft bijective function, so $\tilde{f}_{pu}^{-1}(\tilde{s}S_pint(\tilde{f}_{pu}(A,\mathcal{P}))) \tilde{\subseteq} (A,\mathcal{P})$, and so $\tilde{f}_{pu}^{-1}(\tilde{s}S_pint(\tilde{f}_{pu}(A,\mathcal{P}))) \tilde{\subseteq} \tilde{s}S_pint(A,\mathcal{P})$. Also, since $\tilde{f}_{pu}$ is a soft bijective function, so $\tilde{s}S_pint(\tilde{f}_{pu}(A,\mathcal{P})) \tilde{\subseteq} \tilde{f}_{pu}(\tilde{s}S_pint(A,\mathcal{P}))$.

Conversely, let $(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$. Then, $\tilde{s}S_pint(B,\acute{\mathcal{P}}) = (B,\acute{\mathcal{P}})$ and $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\subseteq} \tilde{X}$. By hypothesis and $\tilde{f}_{pu}$ is a soft bijective function, $(B,\acute{\mathcal{P}}) = \tilde{s}S_pint(\tilde{f}_{pu}(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}))) \tilde{\subseteq} \tilde{f}_{pu}(\tilde{s}S_pint(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})))$, and so $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) = \tilde{s}S_pint(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}))$. Hence by Theorem 1.28(1), $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$. Thus by Theorem 2.2, $\tilde{f}_{pu}$ is soft S_p-irresolute.

Proposition 2.7. A soft function $\tilde{f}_{pu}: (\tilde{X},\tilde{\tau},\mathcal{P}) \to (\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ is soft S_p-irresolute iff $\tilde{f}_{pu}^{-1}(\tilde{s}S_pint(B,\acute{\mathcal{P}})) \tilde{\subseteq} \tilde{s}S_pint(\tilde{f}_{pu}^{-1}\,(B,\acute{\mathcal{P}}))$, $\forall\, (B,\acute{\mathcal{P}}) \tilde{\subseteq} \tilde{Y}$.

Proof. Let $\tilde{f}_{pu}$ be soft S_p-irresolute and $(B,\acute{\mathcal{P}}) \tilde{\subseteq} \tilde{Y}$. Then, $\tilde{s}S_pint(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$ so that $\tilde{f}_{pu}^{-1}(\tilde{s}S_pint(B,\acute{\mathcal{P}})) \tilde{\in} \tilde{S}S_pO(\tilde{X})$ and $\tilde{f}_{pu}^{-1}(\tilde{s}S_pint(B,\acute{\mathcal{P}})) \tilde{\subseteq} \tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})$. But $\tilde{s}S_pint(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}))$ is the largest soft S_p-open set contained in $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})$, so $\tilde{f}_{pu}^{-1}(\tilde{s}S_pint(B,\acute{\mathcal{P}})) \tilde{\subseteq} \tilde{s}S_pint(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}))$.

Conversely, let $(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$. Then, $\tilde{s}S_pint(B,\acute{\mathcal{P}}) = (B,\acute{\mathcal{P}})$. By hypothesis, $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) = \tilde{f}_{pu}^{-1}(\tilde{s}S_pint(B,\acute{\mathcal{P}})) \tilde{\subseteq} \tilde{s}S_pint(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}))$, and so $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) = \tilde{s}S_pint(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}))$. Hence by Theorem 1.28(1), $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$. Thus by Theorem 2.2, $\tilde{f}_{pu}$ is soft S_p-irresolute.

Proposition 2.8. Let $\tilde{f}_{pu}: (\tilde{X},\tilde{\tau},\mathcal{P}) \to (\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ be soft continuous and soft open. Then, $\tilde{f}_{pu}$ is soft S_p-irresolute.

Proof. Let $(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$. Then by Proposition 1.30, $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$. Thus by Theorem 2.2, $\tilde{f}_{pu}$ is a soft S_p-irresolute.

Remark 2.9. Soft S_p-irresolute functions are independent of soft irresolute and soft S_p-continuous functions, as shown in the following examples.

Example 2.10. Let $X = \{x_1, x_2\}$, $Y = \{y_1, y_2\}$, $\mathcal{P} = \{e_1, e_2\}$, and $\acute{\mathcal{P}} = \{\acute{e}_1, \acute{e}_2\}$. Let $\tilde{\tau} = \{\widetilde{\emptyset}_X, \tilde{X}, (A_1,\mathcal{P}), (A_2,\mathcal{P}), (A_3,\mathcal{P}), (A_4,\mathcal{P}), (A_5,\mathcal{P}), (A_6,\mathcal{P}), (A_7,\mathcal{P})\}$ and $\tilde{\sigma} = \{\widetilde{\emptyset}_Y, \tilde{Y}, (B,\acute{\mathcal{P}})\}$ be soft topology on $\tilde{X}$ and $\tilde{Y}$ respectively, where $\widetilde{\emptyset}_X = \{(e_1,\emptyset),(e_2,\emptyset)\}$, $\tilde{X} = \{(e_1,X),(e_2,X)\}$, $(A_1,\mathcal{P}) = \{(e_1,\{x_1\}),(e_2,\emptyset)\}$, $(A_2,\mathcal{P}) = \{(e_1,\{x_2\}),(e_2,\emptyset)\}$, $(A_3,\mathcal{P}) = \{(e_1,X),(e_2,\emptyset)\}$, $(A_4,\mathcal{P}) = \{(e_1,\emptyset),(e_2,\{x_2\})\}$, $(A_5,\mathcal{P}) = \{(e_1,\{x_1\}),(e_2,\{x_2\})\}$, $(A_6,\mathcal{P}) = \{(e_1,\{x_2\}),(e_2,\{x_2\})\}$, $(A_7,\mathcal{P}) = \{(e_1,X),(e_2,\{x_2\})\}$, $\widetilde{\emptyset}_Y = \{(\acute{e}_1,\emptyset),(\acute{e}_2,\emptyset)\}$, $\tilde{Y} = \{(\acute{e}_1,Y),(\acute{e}_2,Y)\}$, and $(B,\acute{\mathcal{P}}) = \{(\acute{e}_1,\{y_1\}),(\acute{e}_2,\{y_2\})\}$. Thus, $(\tilde{X},\tilde{\tau},\mathcal{P})$ and $(\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ are $\tilde{S}TS$ over X and Y, respectively. Now define the soft function $\tilde{f}_{pu}: (\tilde{X},\tilde{\tau},\mathcal{P}) \to (\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$, where $p: \mathcal{P} \to \acute{\mathcal{P}}$ is a function defined by $p(e_1) = \acute{e}_1$, $p(e_2) = \acute{e}_2$ and $u: X \to Y$ is a function defined by $u(x_1) = \{y_1\}$, $u(x_2) = \{y_2\}$. The soft function $\tilde{f}_{pu}$ is soft irresolute, but it is not soft S_p-irresolute, since $(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{Y}) \tilde{\subseteq} \tilde{S}SO(\tilde{Y})$, while $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) = \{(e_1,\{x_1\}),(e_2,\{x_2\})\} \tilde{\in} \tilde{S}SO(\tilde{X})$ but $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\notin} \tilde{S}S_pO(\tilde{X})$.

Example 2.11. Let $X = \{x_1, x_2\}$ and $\mathcal{P} = \{e_1, e_2\}$ with the soft topology $\tilde{\tau} = \{\widetilde{\emptyset}, \tilde{X}, (A_1,\mathcal{P}), (A_2,\mathcal{P}), (A_3,\mathcal{P}), (A_4,\mathcal{P}), (A_5,\mathcal{P}), (A_6,\mathcal{P}), (A_7,\mathcal{P})\}$ where $\widetilde{\emptyset} = \{(e_1,\emptyset),(e_2,\emptyset)\}$, $\tilde{X} = \{(e_1,X),(e_2,X)\}$, $(A_1,\mathcal{P}) = \{(e_1,\{x_1\}),(e_2,\emptyset)\}$ $(A_2,\mathcal{P}) = \{(e_1,\{x_2\}),(e_2,\emptyset)\}$, $(A_3,\mathcal{P}) = \{(e_1,X),(e_2,\emptyset)\}$, $(A_4,\mathcal{P}) = \{(e_1,\emptyset),(e_2,\{x_2\})\}$, $(A_5,\mathcal{P}) = \{(e_1,\{x_1\}),(e_2,\{x_2\})\}$, $(A_6,\mathcal{P}) = \{(e_1,\{x_2\}),(e_2,\{x_2\})\}$, $(A_7,\mathcal{P}) = \{(e_1,X),(e_2,\{x_2\})\}$. Thus, $(\tilde{X},\tilde{\tau},\mathcal{P})$ is a $\tilde{S}TS$ over X. Now define the soft function $\tilde{f}_{pu}: (\tilde{X},\tilde{\tau},\mathcal{P}) \to (\tilde{X},\tilde{\tau},\mathcal{P})$, where p and u are identity functions on $\mathcal{P}$ and X, respectively. The soft function $\tilde{f}_{pu}$ is soft S_p-irresolute, but it is not soft S_p-continuous. Since $(A_1,\mathcal{P}) \tilde{\in} \tilde{\tau}$, while $\tilde{f}_{pu}^{-1}(A_1,\mathcal{P}) = (A_1,\mathcal{P}) \tilde{\notin} \tilde{S}S_pO(\tilde{X})$.

Example 2.12. Let $X = \{x_1, x_2, x_3\}$, $\mathcal{P} = \{e_1, e_2\}$, $\tilde{\tau} = \{\widetilde{\emptyset}, \tilde{X}, (A_1,\mathcal{P}), (A_2,\mathcal{P}), (A_3,\mathcal{P})\}$, and $\tilde{\sigma} = \{\widetilde{\emptyset}, \tilde{X}, (A_3,\mathcal{P})\}$ be two soft topologies on $\tilde{X}$, where $\widetilde{\emptyset} = \{(e_1,\emptyset),(e_2,\emptyset)\}$, $\tilde{X} = \{(e_1,X),(e_2,X)\}$, $(A_1,\mathcal{P}) = \{(e_1,\{x_1\}),(e_2,\{x_3\})\}$, $(A_2,\mathcal{P}) = \{(e_1,\{x_3\}),(e_2,\{x_1\})\}$, and $(A_3,\mathcal{P}) = \{(e_1,\{x_1,x_3\}),(e_2,\{x_1,x_3\})\}$. Thus, $(\tilde{X},\tilde{\tau},\mathcal{P})$ and $(\tilde{X},\tilde{\sigma},\mathcal{P})$ are $\tilde{S}TS$ over X. Now define the soft function $\tilde{f}_{pu}: (\tilde{X},\tilde{\tau},\mathcal{P}) \to (\tilde{X},\tilde{\sigma},\mathcal{P})$, where p and u are identity functions on $\mathcal{P}$ and X, respectively. The soft function $\tilde{f}_{pu}$ is soft S_p-continuous, but it is neither soft irresolute nor soft S_p-irresolute. Since $(A_2,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X},\tilde{\sigma},\mathcal{P})$, while

$$
\begin{aligned}
\tilde{f}_{pu}^{-1}(A_2,\mathcal{P}) &= \{(e_1,u^{-1}(A_2(p(e_1)))),(e_2,u^{-1}(A_2(p(e_2))))\} \\
&= \{(e_1,u^{-1}(A_2(e_1))),(e_2,u^{-1}(A_2(e_2)))\} \\
&= \{(e_1,u^{-1}(\{x_3\})),(e_2,u^{-1}(\{x_1\}))\} = \{(e_1,\{x_3\}),(e_2,\{x_1\})\} \tilde{\notin} \tilde{S}S_pO(\tilde{X},\tilde{\tau},\mathcal{P}).
\end{aligned}
$$

Proposition 2.13. Let $\tilde{f}_{pu}: (\tilde{X},\tilde{\tau},\mathcal{P}) \to (\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ be a soft function from a $\tilde{S}TS$ $(\tilde{X},\tilde{\tau},\mathcal{P})$ to soft locally indiscrete $(\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$. Then,

(1) $\tilde{f}_{pu}$ is soft S_p-irresolute iff $\tilde{f}_{pu}$ is soft S_p-continuous.
(2) $\tilde{f}_{pu}$ is soft S_p-irresolute, if $\tilde{f}_{pu}$ is soft RC-continuous (respectively, soft perfectly continuous).
(3) $\tilde{f}_{pu}$ is soft β-continuous (respectively, soft b-continuous), if $\tilde{f}_{pu}$ is soft S_p-irresolute.

Proof. (1) Let $(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{\sigma}$. Since $(\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ is soft locally indiscrete, then by Proposition 1.20(2), $(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$. Since $\tilde{f}_{pu}$ is soft S_p-irresolute, then $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$. Thus by Theorem 1.29(1), $\tilde{f}_{pu}$ is soft S_p-continuous.

Conversely, let $(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$. Since $(\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ is soft locally indiscrete, then by Proposition 1.20(2), $(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{\sigma}$. Since $\tilde{f}_{pu}$ is soft S_p-continuous, then $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$. Thus by Theorem 2.2, $\tilde{f}_{pu}$ is soft S_p-irresolute.

(2) Let $(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$. Since $(\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ is soft locally indiscrete, then by Proposition 1.20(2), $(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{\sigma}$. Since $\tilde{f}_{pu}$ is soft RC-continuous (respectively, soft perfectly continuous), then $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}RC(\tilde{X})$ (respectively, $\tilde{S}CO(\tilde{X})$). So by Proposition 1.19(3, 4), $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$. Thus by Theorem 2.2, $\tilde{f}_{pu}$ is soft S_p-irresolute.

(3) Let $(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$. Since $(\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ is soft locally indiscrete, then by Proposition 1.20(2), $(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{\sigma}$. Since $\tilde{f}_{pu}$ is soft S_p-continuous, then $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$ and so by Proposition 1.19(5), $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}\beta O(\tilde{X})$ (respectively, $\tilde{S}bO(\tilde{X})$). Thus, $\tilde{f}_{pu}$ is soft β-continuous (respectively, soft b-continuous).

Proposition 2.14. Let $\tilde{f}_{pu}: (\tilde{X}, \tilde{\tau}, \mathcal{P}) \to (\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ be a soft function. If $(\tilde{X}, \tilde{\tau}, \mathcal{P})$ and $(\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ are soft locally indiscrete, then:

(1) $\tilde{f}_{pu}$ is soft S_p-irresolute iff $\tilde{f}_{pu}$ is soft irresolute.

(2) $\tilde{f}_{pu}$ is soft S_p-irresolute iff $\tilde{f}_{pu}$ is $\tilde{S}S_c$-continuous (respectively, soft α-continuous).

(3) $\tilde{f}_{pu}$ is soft S_p-irresolute iff $\tilde{f}_{pu}$ is soft semi-continuous.

(4) $\tilde{f}_{pu}$ is soft pre-continuous, if $\tilde{f}_{pu}$ is soft S_p-irresolute.

Proof. (1) Let $(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}SO(\tilde{Y})$. Since $(\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ is soft locally indiscrete, then by Proposition 1.20(1), $(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$. Since $\tilde{f}_{pu}$ is soft S_p-irresolute, then $\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$. So by Proposition 1.19(1), $\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}SO(\tilde{X})$. Thus, $\tilde{f}_{pu}$ is soft irresolute.

Conversely, let $(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$. Then, $(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}SO(\tilde{Y})$. Since $\tilde{f}_{pu}$ is soft irresolute, then $\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}SO(\tilde{X})$. Since $\tilde{X}$ is soft locally indiscrete, then by Proposition 1.20(1), $\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$. Thus by Theorem 2.2, $\tilde{f}_{pu}$ is soft S_p-irresolute.

(2) Let $(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{\sigma}$. Since $(\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ is soft locally indiscrete, then by Proposition 1.20(2), $(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$. Since $\tilde{f}_{pu}$ is soft S_p-irresolute, then $\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$. Since $\tilde{X}$ is soft locally indiscrete, then by Proposition 1.20(1) (respectively, Proposition 1.20(3)), $\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_cO(\tilde{X})$ (respectively, $\tilde{S}\alpha O(\tilde{X})$). Thus, $\tilde{f}_{pu}$ is $\tilde{S}S_c$- continuous (respectively, soft α-continuous).

Conversely, let $(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$. Since $(\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ is soft locally indiscrete, then by Proposition 1.20(2), $(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{\sigma}$. Since $\tilde{f}_{pu}$ is $\tilde{S}S_c$- continuous (respectively, soft α-continuous), then $\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_cO(\tilde{X})$ (respectively, $\tilde{S}\alpha O(\tilde{X})$). Since $\tilde{X}$ is soft locally indiscrete, then by Proposition 1.20(1) (respectively, Proposition 1.20(3)), $\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$. Thus by Theorem 2.2, $\tilde{f}_{pu}$ is soft S_p-irresolute.

(3) Let $(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{\sigma}$. Since $(\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ is soft locally indiscrete, then by Proposition 1.20(2), $(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$. Since $\tilde{f}_{pu}$ is soft S_p-irresolute, then $\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$. So by Proposition 1.19(1), $\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}SO(\tilde{X})$. Thus, $\tilde{f}_{pu}$ is soft semi-continuous.

Conversely, let $(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$. Since $(\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ is soft locally indiscrete, then by Proposition 1.20(2), $(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{\sigma}$. Since $\tilde{f}_{pu}$ is soft semi-continuous, then $\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}SO(\tilde{X})$. Since $\tilde{X}$ is soft locally indiscrete, then by Proposition 1.20(1), $\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$. Thus by Theorem 2.2, $\tilde{f}_{pu}$ is soft S_p-irresolute.

(4) Let $(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{\sigma}$. Since $(\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ is soft locally indiscrete, then by Proposition 1.20(2), $(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$. Since $\tilde{f}_{pu}$ is soft S_p-irresolute, then $\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$. Since $\tilde{X}$ is soft locally indiscrete, then by Proposition 1.20(4), $\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}PO(\tilde{X})$. Thus, $\tilde{f}_{pu}$ is soft pre-continuous.

Proposition 2.15. Let $\tilde{f}_{pu}: (\tilde{X}, \tilde{\tau}, \mathcal{P}) \to (\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ be a soft function. If $(\tilde{X}, \tilde{\tau}, \mathcal{P})$ and $(\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ are soft T_1-spaces, then:

(1) $\tilde{f}_{pu}$ is soft S_p-irresolute iff $\tilde{f}_{pu}$ is soft irresolute.

(2) $\tilde{f}_{pu}$ is $\tilde{S}S_c$-continuous (respectively, soft semi-continuous) if $\tilde{f}_{pu}$ is soft S_p-irresolute.

Proof. (1) Let $(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}SO(\tilde{Y})$. Since $(\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ is a soft T_1-space, then by Proposition 1.21(1), $(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$. Since $\tilde{f}_{pu}$ is soft S_p-irresolute, then $\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$. So by Proposition 1.19(1), $\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}SO(\tilde{X})$. Thus, $\tilde{f}_{pu}$ is soft irresolute.

Conversely, let $(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$. Then, $(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}SO(\tilde{Y})$. Since $\tilde{f}_{pu}$ is soft irresolute, then $\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}SO(\tilde{X})$. Since $\tilde{X}$ is a soft T_1-space, then by Proposition 1.21(1), $\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$. Thus by Theorem 2.2, $\tilde{f}_{pu}$ is soft S_p-irresolute.

(2) Let $(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{\sigma}$. Since $(\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ is a soft T_1-space, then by Proposition 1.21(3), $(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$. Since $\tilde{f}_{pu}$ is soft S_p-irresolute, then $\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$. Since $\tilde{X}$ is a soft T_1-space, then by Proposition 1.21(1), $\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_cO(\tilde{X})$ (respectively, $\tilde{S}SO(\tilde{X})$). Thus, $\tilde{f}_{pu}$ is $\tilde{S}S_c$-continuous (respectively, soft semi-continuous).

Proposition 2.16. Let $\tilde{f}_{pu}: (\tilde{X}, \tilde{\tau}, \mathcal{P}) \to (\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ and $\tilde{g}_{qv}: (\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}}) \to (\tilde{W}, \tilde{\mu}, \ddot{\mathcal{P}})$ be two soft functions. Then,

(1) $\tilde{g}_{qv} \tilde{\circ} \tilde{f}_{pu}: (\tilde{X}, \tilde{\tau}, \mathcal{P}) \to (\tilde{W}, \tilde{\mu}, \ddot{\mathcal{P}})$ is soft S_p-continuous, if $\tilde{f}_{pu}$ is soft S_p-irresolute and $\tilde{g}_{qv}$ is soft S_p-continuous.

(2) $\tilde{g}_{qv} \tilde{\circ} \tilde{f}_{pu}: (\tilde{X}, \tilde{\tau}, \mathcal{P}) \to (\tilde{W}, \tilde{\mu}, \ddot{\mathcal{P}})$ is soft S_p-irresolute, if $\tilde{f}_{pu}$ and $\tilde{g}_{qv}$ are both soft S_p-irresolute functions.

Proof. (1) Let $(C, \ddot{\mathcal{P}}) \tilde{\in} \tilde{\mu}$. Since $\tilde{g}_{qv}$ is soft S_p-continuous, then $\tilde{g}_{qv}^{-1}(C, \ddot{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$. Since $\tilde{f}_{pu}$ is soft S_p-irresolute, then by Theorem 2.2, $(\tilde{g}_{qv} \tilde{\circ} \tilde{f}_{pu})^{-1}(C, \ddot{\mathcal{P}}) = \tilde{f}_{pu}^{-1}(\tilde{g}_{qv}^{-1}(C, \ddot{\mathcal{P}})) \tilde{\in} \tilde{S}S_pO(\tilde{X})$. Therefore, by Theorem 1.29(1), $\tilde{g}_{qv} \tilde{\circ} \tilde{f}_{pu}$ is soft S_p-continuous.

(2) Let $(C, \ddot{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{W})$. Since $\tilde{g}_{qv}$ is a soft S_p-irresolute function, then by Theorem 2.2, $\tilde{g}_{qv}^{-1}(C, \ddot{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$. Since $\tilde{f}_{pu}$ is also soft S_p-irresolute, then by Theorem 2.2, $(\tilde{g}_{qv} \tilde{\circ} \tilde{f}_{pu})^{-1}(C, \ddot{\mathcal{P}}) = \tilde{f}_{pu}^{-1}(\tilde{g}_{qv}^{-1}(C, \ddot{\mathcal{P}})) \tilde{\in} \tilde{S}S_pO(\tilde{X})$. Therefore, by Theorem 2.2, $\tilde{g}_{qv} \tilde{\circ} \tilde{f}_{pu}$ is soft S_p-irresolute.

Definition 2.17. A $\tilde{S}TS$ $(\tilde{X}, \tilde{\tau}, \mathcal{P})$ is known as a soft S_p-Hausdorff space (or soft S_p-T_2-space) if whenever $\widetilde{e_x}$ and $\widetilde{e_y}$ are distinct soft points of $\tilde{X}$ there are disjoint soft S_p-open sets $(A_1, \mathcal{P})$ and $(A_2, \mathcal{P})$ with $\widetilde{e_x} \tilde{\in} (A_1, \mathcal{P})$ and $\widetilde{e_y} \tilde{\in} (A_2, \mathcal{P})$.

Remark 2.18. The definition indicates that every soft S_p-Hausdorff space is soft semi-Hausdorff. The following example shows that the converse is not true in general:

In the Example 2.11, $\tilde{S}SO(\tilde{X}) = \{\tilde{\emptyset}, \tilde{X}, (A_1, \mathcal{P}), (A_2, \mathcal{P}), (A_3, \mathcal{P}), (A_4, \mathcal{P}), (A_5, \mathcal{P}), (A_6, \mathcal{P}), (A_7, \mathcal{P}), (A_8, \mathcal{P}), (A_9, \mathcal{P}), (A_{10}, \mathcal{P}), (A_{11}, \mathcal{P}), (A_{12}, \mathcal{P}), (A_{13}, \mathcal{P})\}$ is soft semi-Hausdorff but $\tilde{S}S_pO(\tilde{X}) = \{\tilde{\emptyset}, \tilde{X}, (A_8, \mathcal{P}), (A_9, \mathcal{P}), (A_{10}, \mathcal{P}), (A_{11}, \mathcal{P}), (A_{12}, \mathcal{P}), (A_{13}, \mathcal{P})\}$ is not a soft S_p-Hausdorff space, where $(A_8, \mathcal{P}) = \{(e_1, \{x_2\}), (e_2, X)\}$, $(A_9, \mathcal{P}) = \{(e_1, \{x_2\}), (e_2, \{x_1\})\}$, $(A_{10}, \mathcal{P}) = \{(e_1, \{x_1\}), (e_2, \{x_1\})\}$, $(A_{11}, \mathcal{P}) = \{(e_1, X), (e_2, \{x_1\})\}$, $(A_{12}, \mathcal{P}) = \{(e_1, \{x_1\}), (e_2, X)\}$, $(A_{13}, \mathcal{P}) = \{(e_1, \emptyset), (e_2, X)\}$.

Proposition 2.19. Let $(\tilde{Z}, \tilde{\tau}_{\tilde{Z}}, \mathcal{P})$ be a soft subspace of a soft S_p-Hausdorff space $(\tilde{X}, \tilde{\tau}, \mathcal{P})$ and $\tilde{Z} \tilde{\in} \tilde{S}CO(\tilde{X})$. Then, $(\tilde{Z}, \tilde{\tau}_{\tilde{Z}}, \mathcal{P})$ is soft S_p-Hausdorff.

Proof. Let $\widetilde{e_x}, \widetilde{e_y} \tilde{\in} \tilde{S}P(\tilde{Z})$ and $\widetilde{e_x} \neq \widetilde{e_y}$. Then $\widetilde{e_x}, \widetilde{e_y} \tilde{\in} \tilde{S}P(\tilde{X})$ such that $\widetilde{e_x} \neq \widetilde{e_y}$. Since $\tilde{X}$ is soft S_p-Hausdorff, there exist disjoint soft S_p-open sets $(A_1, \mathcal{P})$ and $(A_2, \mathcal{P})$ with $\widetilde{e_x} \tilde{\in} (A_1, \mathcal{P})$ and $\widetilde{e_y} \tilde{\in} (A_2, \mathcal{P})$. Then by Proposition 1.27, $\widetilde{e_x} \tilde{\in} (A_1, \mathcal{P}) \tilde{\cap} \tilde{Z} \tilde{\in} \tilde{S}S_pO(\tilde{Z})$ and $\widetilde{e_y} \tilde{\in} (A_2, \mathcal{P}) \tilde{\cap} \tilde{Z} \tilde{\in} \tilde{S}S_pO(\tilde{Z})$. Since $(A_1, \mathcal{P}) \tilde{\cap} (A_2, \mathcal{P}) = \tilde{\emptyset}$, we have $((A_1, \mathcal{P}) \tilde{\cap} \tilde{Z}) \tilde{\cap} ((A_2, \mathcal{P}) \tilde{\cap} \tilde{Z}) = ((A_1, \mathcal{P}) \tilde{\cap} (A_2, \mathcal{P})) \tilde{\cap} \tilde{Z} = \tilde{\emptyset} \tilde{\cap} \tilde{Z} = \tilde{\emptyset}$. Thus, $(A_1, \mathcal{P}) \tilde{\cap} \tilde{Z}$ and $(A_2, \mathcal{P}) \tilde{\cap} \tilde{Z}$ are disjoint soft S_p-open sets in $\tilde{Z}$ containing $\widetilde{e_x}$ and $\widetilde{e_y}$, respectively. Hence, $(\tilde{Z}, \tilde{\tau}_{\tilde{Z}}, \mathcal{P})$ is soft S_p-Hausdorff.

Proposition 2.20. Let $(\tilde{X}, \tilde{\tau}, \mathcal{P})$ be a $\tilde{S}TS$ and $(\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ be a soft Hausdorff (respectively, a soft S_p-Hausdorff) space. If $\tilde{f}_{pu}: (\tilde{X}, \tilde{\tau}, \mathcal{P}) \to (\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ is a soft injective and a soft S_p-continuous (respectively, soft S_p-irresolute) function, then $(\tilde{X}, \tilde{\tau}, \mathcal{P})$ is soft S_p-Hausdorff.

Proof. Let $\widetilde{e_x}, \widetilde{e_y} \tilde{\in} \tilde{S}P(\tilde{X})$ and $\widetilde{e_x} \neq \widetilde{e_y}$. Then, $\tilde{f}_{pu}(\widetilde{e_x}), \tilde{f}_{pu}(\widetilde{e_y}) \tilde{\in} \tilde{S}P(\tilde{Y})$. Since $\tilde{f}_{pu}$ is soft injective, then $\tilde{f}_{pu}(\widetilde{e_x}) \neq \tilde{f}_{pu}(\widetilde{e_y})$. Since $(\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ is soft Hausdorff (respectively, soft S_p-Hausdorff), there are disjoint soft open (respectively, soft S_p-open) sets $(A_1, \acute{\mathcal{P}})$ and $(A_2, \acute{\mathcal{P}})$ in $\tilde{Y}$ with $\tilde{f}_{pu}(\widetilde{e_x}) \tilde{\in} (A_1, \acute{\mathcal{P}})$ and $\tilde{f}_{pu}(\widetilde{e_y}) \tilde{\in} (A_2, \acute{\mathcal{P}})$. Since $\tilde{f}_{pu}$ is soft S_p-continuous (respectively, soft S_p-irresolute) and $(A_1, \acute{\mathcal{P}}) \tilde{\cap} (A_2, \acute{\mathcal{P}}) = \tilde{\emptyset}$, we have $\tilde{f}_{pu}^{-1}(A_1, \acute{\mathcal{P}})$ and $\tilde{f}_{pu}^{-1}(A_2, \acute{\mathcal{P}})$ are disjoint soft S_p-open sets in $\tilde{X}$ such that $\widetilde{e_x} \tilde{\in} \tilde{f}_{pu}^{-1}(A_1, \acute{\mathcal{P}})$ and $\widetilde{e_y} \tilde{\in} \tilde{f}_{pu}^{-1}(A_2, \acute{\mathcal{P}})$. Hence, $(\tilde{X}, \tilde{\tau}, \mathcal{P})$ is soft S_p-Hausdorff.

3. Soft S_p-Open and Soft S_p-Closed Functions

Definition 3.1. Let $(\tilde{X}, \tilde{\tau}, \mathcal{P})$ and $(\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ be two $\tilde{S}TS$ and $u: X \to Y$, $p: \mathcal{P} \to \acute{\mathcal{P}}$ be functions. A soft function $\tilde{f}_{pu}: (\tilde{X}, \tilde{\tau}, \mathcal{P}) \to (\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ is known as

(1) **soft S_p-open,** if $\tilde{f}_{pu}(A, \mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$, $\forall$ $(A, \mathcal{P}) \tilde{\in} \tilde{\tau}$.

(2) **soft S_p-closed,** if $\tilde{f}_{pu}(C, \mathcal{P}) \tilde{\in} \tilde{S}S_pC(\tilde{Y})$, $\forall$ $(C, \mathcal{P}) \tilde{\in} \tilde{\tau}^c$.

Proposition 3.2. A soft function $\tilde{f}_{pu}: (\tilde{X}, \tilde{\tau}, \mathcal{P}) \to (\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ is soft S_p-open iff $\forall$ $\widetilde{e_x} \tilde{\in} \tilde{S}P(\tilde{X})$, $\forall$ $(A, \mathcal{P}) \tilde{\in} \tilde{\tau}$ containing $\widetilde{e_x}$, there exists $(B, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$ containing $\tilde{f}_{pu}(\widetilde{e_x})$ such that $(B, \acute{\mathcal{P}}) \tilde{\subseteq} \tilde{f}_{pu}(A, \mathcal{P})$.

Proof. Let $\tilde{f}_{pu}$ be a soft S_p-open function, $\widetilde{e_x} \tilde{\in} \tilde{S}P(\tilde{X})$ and $\widetilde{e_x} \tilde{\in} (A, \mathcal{P}) \tilde{\in} \tilde{\tau}$. Then, $\tilde{f}_{pu}(\widetilde{e_x}) \tilde{\in} \tilde{f}_{pu}(A, \mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$ and take $\tilde{f}_{pu}(A, \mathcal{P}) = (B, \acute{\mathcal{P}})$. Hence, the proof is complete.

Conversely, to show that $\tilde{f}_{pu}$ is a soft S_p-open function. Let $(A,\mathcal{P}) \tilde{\in} \tilde{\tau}$. Then, by hypothesis $\forall\ \tilde{e_x} \tilde{\in} (A,\mathcal{P})$, there is $\tilde{f}_{pu}(\tilde{e_x}) \tilde{\in} (B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$ such that $(B,\acute{\mathcal{P}}) \tilde{\subseteq} \tilde{f}_{pu}(A,\mathcal{P})$. Therefore, by Proposition 1.18(2), $\tilde{f}_{pu}(A,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$. Thus, $\tilde{f}_{pu}$ is soft S_p-open.

Proposition 3.3. For a soft surjective function $\tilde{f}_{pu}: (\tilde{X},\tilde{\tau},\mathcal{P}) \to (\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$, the following sentences are equivalent:

(1) $\tilde{f}_{pu}$ is soft S_p-closed.

(2) $\forall\ \tilde{e_y} \tilde{\in} \tilde{S}P(\tilde{Y})$, and $\tilde{f}_{pu}^{-1}(\tilde{e_y}) \tilde{\in} (A,\mathcal{P}) \tilde{\in} \tilde{\tau}$, there is $\tilde{e_y} \tilde{\in} (B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$ such that $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\subseteq} (A,\mathcal{P})$.

(3) $\forall\ \tilde{e_y} \tilde{\in} \tilde{S}P(\tilde{Y})$, and $(C,\mathcal{P}) \tilde{\in} \tilde{\tau}^c$ such that $\tilde{f}_{pu}^{-1}(\tilde{e_y}) \tilde{\cap} (C,\mathcal{P}) = \tilde{\emptyset}$, there is $(D,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pC(\tilde{Y})$ such that $\tilde{e_y} \tilde{\notin} (D,\acute{\mathcal{P}})$ and $(C,\mathcal{P}) \tilde{\subseteq} \tilde{f}_{pu}^{-1}(D,\acute{\mathcal{P}})$.

Proof. (1) → (2). Let $\tilde{e_y} \tilde{\in} \tilde{S}P(\tilde{Y})$, and $\tilde{f}_{pu}^{-1}(\tilde{e_y}) \tilde{\in} (A,\mathcal{P}) \tilde{\in} \tilde{\tau}$. Since $\tilde{f}_{pu}$ is soft surjective, then there exists a soft point $\tilde{e_x} \tilde{\in} (A,\mathcal{P})$ such that $\tilde{e_y} = \tilde{f}_{pu}(\tilde{e_x})$. Since $\tilde{f}_{pu}$ is soft S_p-closed, then $(B,\acute{\mathcal{P}}) = \tilde{Y}\tilde{\setminus}\tilde{f}_{pu}(\tilde{X}\tilde{\setminus}(A,\mathcal{P})) \tilde{\subseteq} \tilde{f}_{pu}(\tilde{X}\tilde{\setminus}\tilde{X}\tilde{\setminus}(A,\mathcal{P})) = \tilde{f}_{pu}(A,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$.

(2) → (3) and (3) →(1). Obvious.

Proposition 3.4. Let $\tilde{f}_{pu}: (\tilde{X},\tilde{\tau},\mathcal{P}) \to (\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ be a soft function. Then,:

(1) $\tilde{f}_{pu}$ is soft semi-open (respectively, soft β-open and soft b-open), if $\tilde{f}_{pu}$ is soft S_p-open.

(2) $\tilde{f}_{pu}$ is soft almost β-open (respectively, soft almost b-open), if $\tilde{f}_{pu}$ is soft S_p-open.

(3) $\tilde{f}_{pu}$ is soft semi-closed (respectively, soft β-closed, and soft b-closed), if $\tilde{f}_{pu}$ is soft S_p-closed.

Proof. (1) and (3) Obvious.

(2) Let $(A,\mathcal{P}) \tilde{\in} \tilde{S}RO(\tilde{X})$. Then, $(A,\mathcal{P}) \tilde{\in} \tilde{\tau}$. Since $\tilde{f}_{pu}$ is soft S_p-open, then $\tilde{f}_{pu}(A,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$. By Proposition 1.19(5), $\tilde{f}_{pu}(A,\mathcal{P}) \tilde{\in} \tilde{S}\beta O(\tilde{Y})$ (respectively, $\tilde{S}bO(\tilde{Y})$). Thus, $\tilde{f}_{pu}$ is soft almost β-open (respectively, soft almost b-open).

As illustrated in the following example, the converse of Proposition 3.4 is not true in general:

Example 3.5. In Example 2.11, now define the soft function $\tilde{f}_{pu}: (\tilde{X},\tilde{\tau},\mathcal{P}) \to (\tilde{X},\tilde{\tau},\mathcal{P})$, where p and u are identity functions on $\mathcal{P}$ and X, respectively.

(1) The soft function $\tilde{f}_{pu}$ is soft semi-open (respectively, soft β-open, soft almost β-open, soft b-open, and soft almost b-open), but it is not soft S_p-open. Since $(A_1,\mathcal{P}) \tilde{\in} \tilde{S}RO(\tilde{X}) \tilde{\in} \tilde{\tau}$, while:

$\tilde{f}_{pu}(A_1,\mathcal{P}) = \{(e_1, u(\tilde{\cup}_{\alpha_1 \tilde{\in} p^{-1}(e_1)\cap\mathcal{P}}\ (A_1(\alpha_1)))), (e_2, u(\tilde{\cup}_{\alpha_2 \tilde{\in} p^{-1}(e_2)\cap\mathcal{P}}\ (A_1(\alpha_2))))\}$

$= \{(e_1, u(A_1(e_1))), (e_2, u(A_1(e_2)))\} = \{(e_1, u(\{x_1\})), (e_2, u(\emptyset))\} = (A_1,\mathcal{P}) \tilde{\notin} \tilde{S}S_pO(\tilde{X})$, where $p^{-1}(e_1) \cap \mathcal{P} = \{e_1\}$.

(2) The soft function $\tilde{f}_{pu}$ is soft semi-closed (respectively, soft β-closed, and soft b-closed), but it is not soft S_p-closed. Since $(A_8,\mathcal{P}) = \{(e_1,\{x_2\}), (e_2, X)\} \tilde{\in} \tilde{\tau}^c$, while

$\tilde{f}_{pu}(A_8,\mathcal{P}) = \{(e_1, u(\tilde{\cup}_{\alpha_1 \tilde{\in} p^{-1}(e_1)\cap\mathcal{P}}\ (A_8(\alpha_1)))), (e_2, u(\tilde{\cup}_{\alpha_2 \tilde{\in} p^{-1}(e_2)\cap\mathcal{P}}\ (A_8(\alpha_2))))\}$

$= \{(e_1, u(A_8(e_1))), (e_2, u(A_8(e_2)))\} = \{(e_1, u(\{x_2\})), (e_2, u(X))\} = (A_8,\mathcal{P}) \tilde{\notin} \tilde{S}S_pC(\tilde{X})$, where $p^{-1}(e_1) \cap \mathcal{P} = \{e_1\}$.

Corollary 3.6. Let $\tilde{f}_{pu}: (\tilde{X},\tilde{\tau},\mathcal{P}) \to (\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ be a soft function and $(\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ is soft locally indiscrete (respectively, a soft T_1-space). Then,

(1) $\tilde{f}_{pu}$ is soft S_p-open iff $\tilde{f}_{pu}$ is soft semi-open.

(2) $\tilde{f}_{pu}$ is soft S_p-closed iff $\tilde{f}_{pu}$ is soft semi-closed.

Proof. (1) This follows from Definition 3.1(1) and Proposition 1.20(1) (respectively, Proposition 1.21(1)).

(2) This follows from Definition 3.1(2) and Proposition 1.22(1) (respectively, Proposition 1.21(2)).

Corollary 3.7. Let $\tilde{f}_{pu}: (\tilde{X},\tilde{\tau},\mathcal{P}) \to (\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ be a soft function and $(\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ is soft locally indiscrete. Then,:

(1) $\tilde{f}_{pu}$ is soft S_p-open iff $\tilde{f}_{pu}$ is soft open.

(2) $\tilde{f}_{pu}$ is soft S_p-open iff $\tilde{f}_{pu}$ is soft α-open.

(3) $\tilde{f}_{pu}$ is soft pre-open, if $\tilde{f}_{pu}$ is soft S_p-open.

(4) $\tilde{f}_{pu}$ is soft β_c-open, if $\tilde{f}_{pu}$ is soft S_p-open.

***Proof*.** By Definition 3.1(1), the proofs (1,2, and 3) are followed by Corollary 2.1.22. While the proof (4) follows from Proposition 1.20(5).

Corollary 3.8. Let $\tilde{f}_{pu}: (\tilde{X}, \tilde{\tau}, \mathcal{P}) \to (\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ be a soft function and $(\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ is soft locally indiscrete. Then,:

(1) $\tilde{f}_{pu}$ is soft S_p-closed iff $\tilde{f}_{pu}$ is soft closed.

(2) $\tilde{f}_{pu}$ is soft S_p-closed iff $\tilde{f}_{pu}$ is soft α-closed.

(3) $\tilde{f}_{pu}$ is soft pre-closed, if $\tilde{f}_{pu}$ is soft S_p-closed.

***Proof*.** Definition 3.1(2) and Proposition 1.22(2-4) provide the proof.

Corollary 3.9. Let $\tilde{f}_{pu}: (\tilde{X}, \tilde{\tau}, \mathcal{P}) \to (\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ be a soft function and $(\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ be a soft T_1-space. Then, $\tilde{f}_{pu}$ is soft S_p-open if $\tilde{f}_{pu}$ is soft open (respectively, soft α-open).

***Proof*.** Definition 3.1(1) and Proposition 1.21(3) (respectively, Proposition 1.21(4)) provide the proof.

Corollary 3.10. If $\tilde{f}_{pu}: (\tilde{X}, \tilde{\tau}, \mathcal{P}) \to (\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ is a soft open (respectively, soft closed) function and $(\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ is a soft regular space, then $\tilde{f}_{pu}$ is soft S_p-open (respectively, soft S_p-closed).

***Proof*.** Definition 3.1(1) (respectively, Definition 3.1(2)) and Proposition 1.23(1) (respectively, Proposition 1.23(2)) provide the proof.

Corollary 3.11. If $\tilde{f}_{pu}: (\tilde{X}, \tilde{\tau}, \mathcal{P}) \to (\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ is a soft S_p-open function and $(\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ is a soft submaximal space, then $\tilde{f}_{pu}$ is soft β_c-open.

***Proof*.** Definition 3.1(1) and Proposition 1.25 provide the proof.

Corollary 3.12. Let $\tilde{f}_{pu}: (\tilde{X}, \tilde{\tau}, \mathcal{P}) \to (\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ be a soft function and $(\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ is soft extremally disconnected. Then,

(1) $\tilde{f}_{pu}$ is soft pre-open (respectively, soft α-open) if $\tilde{f}_{pu}$ is soft S_p-open.

(2) $\tilde{f}_{pu}$ is soft pre-closed (respectively, soft α-closed) if $\tilde{f}_{pu}$ is soft S_p-closed.

***Proof*.** (1) Definition 3.1(1) and Proposition 1.24(1) provide the proof.

(2) Definition 3.1(2) and Proposition 1.24(2) provide the proof.

Corollary 3.13. Let $\tilde{f}_{pu}: (\tilde{X}, \tilde{\tau}, \mathcal{P}) \to (\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ be a soft function and $(\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ be soft extremally disconnected and a soft T_1-space. Then, $\tilde{f}_{pu}$ is soft S_p-open iff it is a soft α-open function.

***Proof*.** Definition 3.1(1) and Corollary 1.26 provide the proof.

Proposition 3.14. Let $\tilde{f}_{pu}: (\tilde{X}, \tilde{\tau}, \mathcal{P}) \to (\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ be a soft function, $(\tilde{X}, \tilde{\tau}, \mathcal{P})$ and $(\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ are soft locally indiscrete. Then,:

(1) $\tilde{f}_{pu}$ is soft S_p-open iff $\tilde{f}_{pu}$ is soft irresolute open.

(2) $\tilde{f}_{pu}$ is soft S_p-closed iff $\tilde{f}_{pu}$ is soft irresolute closed.

***Proof*.** (1) Let $(A, \mathcal{P}) \tilde{\in} \tilde{S}SO(\tilde{X})$. Since $\tilde{X}$ is soft locally indiscrete, then by Proposition 1.20(2), $(A, \mathcal{P}) \tilde{\in} \tilde{\tau}$. Since $\tilde{f}_{pu}$ is soft S_p-open, then $\tilde{f}_{pu}(A, \mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$. So by Proposition 1.19(1), $\tilde{f}_{pu}(A, \mathcal{P}) \tilde{\in} \tilde{S}SO(\tilde{Y})$. Thus, $\tilde{f}_{pu}$ is soft irresolute open.

Conversely, let $(A, \mathcal{P}) \tilde{\in} \tilde{\tau}$. Then, $(A, \mathcal{P}) \tilde{\in} \tilde{S}SO(\tilde{X})$. Since $\tilde{f}_{pu}$ is soft irresolute open, so $\tilde{f}_{pu}(A, \mathcal{P}) \tilde{\in} \tilde{S}SO(\tilde{Y})$. Since $\tilde{Y}$ is soft locally indiscrete, then by Proposition 1.20(1), $\tilde{f}_{pu}(A, \mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$. Thus, $\tilde{f}_{pu}$ is soft S_p-open.

(2) Using Proposition 1.19(2) and Proposition 1.22(1) in place of Proposition 1.19(1) and Proposition 1.20(1), respectively, the proof is similar to (1).

Proposition 3.15. Let $\tilde{f}_{pu}: (\tilde{X}, \tilde{\tau}, \mathcal{P}) \to (\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ be a soft function from soft semi-regular $(\tilde{X}, \tilde{\tau}, \mathcal{P})$ to soft locally indiscrete $(\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$. Then, $\tilde{f}_{pu}$ is soft S_p-open iff $\tilde{f}_{pu}$ is soft almost open (respectively, soft almost semi-open, and soft almost α-open).

***Proof*.** Let $(A, \mathcal{P}) \tilde{\in} \tilde{S}RO(\tilde{X})$. Then, $(A, \mathcal{P}) \tilde{\in} \tilde{\tau}$. Since $\tilde{f}_{pu}$ is soft S_p-open, then $\tilde{f}_{pu}(A, \mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$. Since $\tilde{Y}$ is soft locally indiscrete, then by Proposition 1.20(2) (respectively, Proposition 1.19(1), and Proposition 1.20(3)), $\tilde{f}_{pu}(A, \mathcal{P}) \tilde{\in} \tilde{\sigma}$ (respectively, $\tilde{S}SO(\tilde{Y})$, and $\tilde{S}\alpha O(\tilde{Y})$). Thus, $\tilde{f}_{pu}$ is soft almost open (respectively, soft almost semi-open, and soft almost α-open).

Conversely, let $(A,\mathcal{P}) \tilde{\in} \tilde{\tau}$ and $\tilde{f}_{pu}(\tilde{e_x}) \tilde{\in} \tilde{f}_{pu}(A,\mathcal{P})$, we have $\tilde{e_x} \tilde{\in} (A,\mathcal{P})$. By the soft semi-regularity of $\tilde{X}$, there is $(O,\mathcal{P}) \tilde{\in} \tilde{S}RO(\tilde{X})$ such that $\tilde{e_x} \tilde{\in} (O,\mathcal{P}) \tilde{\subseteq} (A,\mathcal{P})$. Since $\tilde{f}_{pu}$ is soft almost open (respectively, soft almost semi-open, and soft almost α-open), then $\tilde{f}_{pu}(O,\mathcal{P}) \tilde{\in} \tilde{\sigma}$ (respectively, $\tilde{S}SO(\tilde{Y})$, and $\tilde{S}\alpha O(\tilde{Y})$), and $\tilde{f}_{pu}(\tilde{e_x}) \tilde{\in} \tilde{f}_{pu}(O,\mathcal{P}) \tilde{\subseteq} \tilde{f}_{pu}(A,\mathcal{P})$. Since $\tilde{Y}$ is soft locally indiscrete, then by Proposition 1.20(2) (respectively, Proposition 1.20(1), and Proposition 1.20(3)), $\tilde{f}_{pu}(O,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$. Therefore, by Proposition 1.18(2), $\tilde{f}_{pu}(A,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$. Thus Definition 3.1(1), $\tilde{f}_{pu}$ is soft S_p-open.

Proposition 3.16. Let $\tilde{f}_{pu}: (\tilde{X},\tilde{\tau},\mathcal{P}) \to (\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ be a soft function and $(\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ is soft extremally disconnected. Then, $\tilde{f}_{pu}$ is soft almost pre-open (respectively, soft almost α-open), if $\tilde{f}_{pu}$ is soft S_p-open.

Proof. Let $(A,\mathcal{P}) \tilde{\in} \tilde{S}RO(\tilde{X})$. Then, $(A,\mathcal{P}) \tilde{\in} \tilde{\tau}$. Since $\tilde{f}_{pu}$ is soft S_p-open, then $\tilde{f}_{pu}(A,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$. Since $\tilde{Y}$ is soft extremally disconnected, then by Proposition 1.24(1), $\tilde{f}_{pu}(A,\mathcal{P}) \tilde{\in} \tilde{S}PO(\tilde{Y})$ (respectively, $\tilde{S}\alpha O(\tilde{Y})$). Thus, $\tilde{f}_{pu}$ is soft almost pre-open (respectively, soft almost α-open).

Proposition 3.17. Let $\tilde{f}_{pu}: (\tilde{X},\tilde{\tau},\mathcal{P}) \to (\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ be a soft homeomorphism function. Then, $(A,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$ iff $\tilde{f}_{pu}(A,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$.

Proof. Let $(A,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$. Then by Proposition 1.18(1), $(A,\mathcal{P}) \tilde{\in} \tilde{S}SO(\tilde{X})$ and $(A,\mathcal{P}) = \tilde{\bigcup} (B_\vartheta,\mathcal{P})$, where $(B_\vartheta,\mathcal{P}) \tilde{\in} \tilde{S}PC(\tilde{X})$, $\forall\, \vartheta \in \aleph$. So, $\tilde{f}_{pu}(A,\mathcal{P}) = \tilde{f}_{pu}(\tilde{\bigcup} (B_\vartheta,\mathcal{P})) = \tilde{\bigcup}\, \tilde{f}_{pu}(B_\vartheta,\mathcal{P})$. By Proposition 1.17(2), $\tilde{f}_{pu}$ is soft continuous and soft open, so by Proposition 1.31, $\tilde{f}_{pu}(A,\mathcal{P}) \tilde{\in} \tilde{S}SO(\tilde{Y})$. Also, $\tilde{f}_{pu}$ is soft homeomorphism, so by Proposition 1.32, $\tilde{f}_{pu}(B_\vartheta,\mathcal{P}) \tilde{\in} \tilde{S}PC(\tilde{Y})$, $\forall\, \vartheta \in \aleph$. Hence by Proposition 1.18(1), $\tilde{f}_{pu}(A,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$.

Conversely, this follows from Proposition 1.17(2) and Proposition 1.30.

Corollary 3.18. Let $\tilde{f}_{pu}: (\tilde{X},\tilde{\tau},\mathcal{P}) \to (\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ be a soft homeomorphism function. Then, $(C,\mathcal{P}) \tilde{\in} \tilde{S}S_pC(\tilde{X})$ iff $\tilde{f}_{pu}(C,\mathcal{P}) \tilde{\in} \tilde{S}S_pC(\tilde{Y})$.

Proof. Applying Proposition 3.17 and Definition 1.4.

Proposition 3.19. Let $\tilde{f}_{pu}: (\tilde{X},\tilde{\tau},\mathcal{P}) \to (\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ be soft irresolute open. If $(\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ is soft locally indiscrete and $(A,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$, then $\tilde{f}_{pu}(A,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$.

Proof. Since $(A,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$, then $(A,\mathcal{P}) \tilde{\in} \tilde{S}SO(\tilde{X})$. Since $\tilde{f}_{pu}$ is soft irresolute open, then $\tilde{f}_{pu}(A,\mathcal{P}) \tilde{\in} \tilde{S}SO(\tilde{Y})$. Also since $\tilde{Y}$ is soft locally indiscrete, then by Proposition 1.20(1), $\tilde{f}_{pu}(A,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$.

Theorem 3.20. Let $\tilde{f}_{pu}: (\tilde{X},\tilde{\tau},\mathcal{P}) \to (\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ be a soft function. Then, the following sentences are equivalent:

(1) $\tilde{f}_{pu}$ is soft S_p-open.

(2) $\tilde{f}_{pu}(\tilde{s}int(A,\mathcal{P})) \tilde{\subseteq} \tilde{s}S_pint(\tilde{f}_{pu}(A,\mathcal{P}))$, $\forall\, (A,\mathcal{P}) \tilde{\subseteq} \tilde{X}$.

(3) $\tilde{s}int(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})) \tilde{\subseteq} \tilde{f}_{pu}^{-1}(\tilde{s}S_pint(B,\acute{\mathcal{P}}))$, $\forall\, (B,\acute{\mathcal{P}}) \tilde{\subseteq} \tilde{Y}$.

(4) $\tilde{f}_{pu}^{-1}(\tilde{s}S_pcl(B,\acute{\mathcal{P}})) \tilde{\subseteq} \tilde{s}cl(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}))$, $\forall\, (B,\acute{\mathcal{P}}) \tilde{\subseteq} \tilde{Y}$.

(5) $\tilde{f}_{pu}^{-1}(\tilde{s}S_pBd(B,\acute{\mathcal{P}})) \tilde{\subseteq} \tilde{s}Bd(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}))$, $\forall\, (B,\acute{\mathcal{P}}) \tilde{\subseteq} \tilde{Y}$.

Proof. (1) $\longrightarrow$ (2). Let $(A,\mathcal{P}) \tilde{\subseteq} \tilde{X}$. Then, $\tilde{s}int(A,\mathcal{P}) \tilde{\in} \tilde{\tau}$. Since $\tilde{f}_{pu}$ is soft S_p-open, then $\tilde{f}_{pu}(\tilde{s}int(A,\mathcal{P})) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$, also since $\tilde{s}int(A,\mathcal{P}) \tilde{\subseteq} (A,\mathcal{P})$ implies that $\tilde{f}_{pu}(\tilde{s}int(A,\mathcal{P})) \tilde{\subseteq} \tilde{f}_{pu}(A,\mathcal{P})$. Therefore, $\tilde{f}_{pu}(\tilde{s}int(A,\mathcal{P})) \tilde{\subseteq} \tilde{s}S_pint(\tilde{f}_{pu}(A,\mathcal{P}))$.

(2) $\longrightarrow$ (3). Let $(B,\acute{\mathcal{P}}) \tilde{\subseteq} \tilde{Y}$. Then, $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\subseteq} \tilde{X}$. By (2), we have $\tilde{f}_{pu}(\tilde{s}int(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}))) \tilde{\subseteq} \tilde{s}S_pint(\tilde{f}_{pu}(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})))$. So, $\tilde{f}_{pu}(\tilde{s}int(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}))) \tilde{\subseteq} \tilde{s}S_pint(B,\acute{\mathcal{P}})$. Hence, $\tilde{s}int(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})) \tilde{\subseteq} \tilde{f}_{pu}^{-1}(\tilde{s}S_pint(B,\acute{\mathcal{P}}))$.

(3) $\longrightarrow$ (1). Let $(A,\mathcal{P}) \tilde{\in} \tilde{\tau}$. Then, $\tilde{f}_{pu}(A,\mathcal{P}) \tilde{\subseteq} \tilde{Y}$. So by (3), $\tilde{s}int(A,\mathcal{P}) \tilde{\subseteq} \tilde{s}int(\tilde{f}_{pu}^{-1}(\tilde{f}_{pu}(A,\mathcal{P}))) \tilde{\subseteq} \tilde{f}_{pu}^{-1}(\tilde{s}S_pint(\tilde{f}_{pu}(A,\mathcal{P})))$. Since $\tilde{s}int(A,\mathcal{P}) = (A,\mathcal{P})$, then $(A,\mathcal{P}) \tilde{\subseteq} \tilde{f}_{pu}^{-1}(\tilde{s}S_pint(\tilde{f}_{pu}(A,\mathcal{P})))$ and so $\tilde{f}_{pu}(A,\mathcal{P}) \tilde{\subseteq} \tilde{s}S_pint(\tilde{f}_{pu}(A,\mathcal{P}))$. Hence, $\tilde{f}_{pu}(A,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$. Thus, $\tilde{f}_{pu}$ is soft S_p-open.

(3) $\leftrightarrow$ (4). Let $(B,\acute{\mathcal{P}}) \tilde{\subseteq} \tilde{Y}$. Then, $\tilde{Y} \tilde{\setminus} (B,\acute{\mathcal{P}}) \tilde{\subseteq} \tilde{Y}$ and $\tilde{s}int(\tilde{f}_{pu}^{-1}(\tilde{Y} \tilde{\setminus} (B,\acute{\mathcal{P}}))) \tilde{\subseteq} \tilde{f}_{pu}^{-1}(\tilde{s}S_pint(\tilde{Y} \tilde{\setminus} (B,\acute{\mathcal{P}})))$ $\leftrightarrow$ $\tilde{s}int(\tilde{X} \tilde{\setminus} \tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})) \tilde{\subseteq} \tilde{f}_{pu}^{-1}(\tilde{Y} \tilde{\setminus} (\tilde{s}S_pcl(B,\acute{\mathcal{P}})))$ $\leftrightarrow$ $\tilde{X} \tilde{\setminus} \tilde{s}cl(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})) \tilde{\subseteq} \tilde{X} \tilde{\setminus} \tilde{f}_{pu}^{-1}(\tilde{s}S_pcl(B,\acute{\mathcal{P}}))$ $\leftrightarrow$ $\tilde{f}_{pu}^{-1}(\tilde{s}S_pcl(B,\acute{\mathcal{P}})) \tilde{\subseteq} \tilde{s}cl(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}))$.

(4) ⟶ (5). Let $(B,\acute{\mathcal{P}}) \tilde{\subseteq} \tilde{Y}$. Then by Definition 1.5(3) and (4), $\tilde{f}_{pu}^{-1}(\tilde{s}S_pBd(B,\acute{\mathcal{P}})) = \tilde{f}_{pu}^{-1}[\tilde{s}S_pcl(B,\acute{\mathcal{P}}) \tilde{\cap} \tilde{s}S_pcl(\tilde{Y}\tilde{\setminus}(B,\acute{\mathcal{P}}))] = \tilde{f}_{pu}^{-1}(\tilde{s}S_pcl(B,\acute{\mathcal{P}})) \tilde{\cap} \tilde{f}_{pu}^{-1}(\tilde{s}S_pcl(\tilde{Y}\tilde{\setminus}(B,\acute{\mathcal{P}}))) \tilde{\subseteq} \tilde{s}cl(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})) \tilde{\cap} \tilde{s}cl(\tilde{f}_{pu}^{-1}(\tilde{Y}\tilde{\setminus}(B,\acute{\mathcal{P}}))) = \tilde{s}Bd(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}))$. So, $\tilde{f}_{pu}^{-1}(\tilde{s}S_pBd(B,\acute{\mathcal{P}})) \tilde{\subseteq} \tilde{s}Bd(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}))$.

(5) ⟶ (4). Let $(B,\acute{\mathcal{P}}) \tilde{\subseteq} \tilde{Y}$. Then by (5) and Theorem 1.28(3), $\tilde{f}_{pu}^{-1}(\tilde{s}S_pcl(B,\acute{\mathcal{P}})) = \tilde{f}_{pu}^{-1}((B,\acute{\mathcal{P}}) \tilde{\cup} \tilde{s}S_pBd(B,\acute{\mathcal{P}})) = \tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\cup} \tilde{f}_{pu}^{-1}(\tilde{s}S_pBd(B,\acute{\mathcal{P}})) \tilde{\subseteq} \tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\cup} \tilde{s}Bd(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})) = \tilde{s}cl(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}))$. Therefore, $\tilde{f}_{pu}^{-1}(\tilde{s}S_pcl(B,\acute{\mathcal{P}})) \tilde{\subseteq} \tilde{s}cl(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}))$.

Proposition 3.21. Let $\tilde{f}_{pu}: (\tilde{X},\tilde{\tau},\mathcal{P}) \to (\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ be a soft surjective function. Then, the following sentences are equivalent:

(1) $\tilde{f}_{pu}$ is soft S_p-open.

(2) $\forall\, (A,\mathcal{P}) \tilde{\subseteq} \tilde{X}$, $\tilde{f}_{pu}(\tilde{s}int(A,\mathcal{P})) \tilde{\subseteq} \tilde{s}cl\tilde{s}int\tilde{f}_{pu}(A,\mathcal{P})$, and $\tilde{f}_{pu}(\tilde{s}int(A,\mathcal{P})) = \tilde{\cup}_{\vartheta \in \aleph}(C_\vartheta,\acute{\mathcal{P}})$ where $(C_\vartheta,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}PC(\tilde{Y})$, $\forall\, \vartheta \in \aleph$.

(3) $\forall\, (B,\acute{\mathcal{P}}) \tilde{\subseteq} \tilde{Y}$, $\tilde{s}int(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})) \tilde{\subseteq} \tilde{f}_{pu}^{-1}(\tilde{s}cl\tilde{s}int(B,\acute{\mathcal{P}}))$, and $\tilde{f}_{pu}(\tilde{s}int(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}))) = \tilde{\cup}_{\vartheta \in \aleph}(C_\vartheta,\acute{\mathcal{P}})$ where $(C_\vartheta,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}PC(\tilde{Y})$, $\forall\, \vartheta \in \aleph$.

Proof. (1) ⟶ (2). Let $(A,\mathcal{P}) \tilde{\subseteq} \tilde{X}$. Then, $\tilde{s}int(A,\mathcal{P}) \tilde{\in} \tilde{\tau}$ and by (1), $\tilde{f}_{pu}(\tilde{s}int(A,\mathcal{P})) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$. So by Proposition 1.19(1), $\tilde{f}_{pu}(\tilde{s}int(A,\mathcal{P})) \tilde{\in} \tilde{S}SO(\tilde{Y})$ and $\tilde{f}_{pu}(\tilde{s}int(A,\mathcal{P})) = \tilde{\cup}_{\vartheta \in \aleph}(C_\vartheta,\acute{\mathcal{P}})$ where $(C_\vartheta,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}PC(\tilde{Y})$, $\forall\, \vartheta \in \aleph$. Thus, $\tilde{f}_{pu}(\tilde{s}int(A,\mathcal{P})) \tilde{\subseteq} \tilde{s}cl\tilde{s}int\tilde{f}_{pu}(A,\mathcal{P})$, and $\tilde{f}_{pu}(\tilde{s}int(A,\mathcal{P})) = \tilde{\cup}_{\vartheta \in \aleph}(C_\vartheta,\acute{\mathcal{P}})$, $(C_\vartheta,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}PC(\tilde{Y})$, $\forall\, \vartheta \in \aleph$.

(2) ⟶ (1). Let $(A,\mathcal{P}) \tilde{\in} \tilde{\tau}$. Then, $\tilde{s}int(A,\mathcal{P}) = (A,\mathcal{P})$ and by (2), $\tilde{f}_{pu}(A,\mathcal{P}) = \tilde{f}_{pu}(\tilde{s}int(A,\mathcal{P})) \tilde{\subseteq} \tilde{s}cl\tilde{s}int\tilde{f}_{pu}(A,\mathcal{P})$ and $\tilde{f}_{pu}(A,\mathcal{P}) = \tilde{\cup}_{\vartheta \in \aleph}(C_\vartheta,\acute{\mathcal{P}})$ where $(C_\vartheta,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}PC(\tilde{Y})$, $\forall\, \vartheta \in \aleph$. So, $\tilde{f}_{pu}(A,\mathcal{P}) \tilde{\in} \tilde{S}SO(\tilde{Y})$ and $\tilde{f}_{pu}(A,\mathcal{P}) = \tilde{\cup}_{\vartheta \in \aleph}(C_\vartheta,\acute{\mathcal{P}})$ where $(C_\vartheta,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}PC(\tilde{Y})$, $\forall\, \vartheta \in \aleph$. Therefore, by Proposition 1.18(1), $\tilde{f}_{pu}(A,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$. Thus, $\tilde{f}_{pu}$ is soft S_p-open.

(2) ⟶ (3). Let $(B,\acute{\mathcal{P}}) \tilde{\subseteq} \tilde{Y}$. Then, $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\subseteq} \tilde{X}$ and by (2), $\tilde{f}_{pu}(\tilde{s}int(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}))) \tilde{\subseteq} \tilde{s}cl\tilde{s}int\tilde{f}_{pu}(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})) \subseteq \tilde{s}cl\tilde{s}int(B,\acute{\mathcal{P}})$ and $\tilde{f}_{pu}(\tilde{s}int(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}))) = \tilde{\cup}_{\vartheta \in \aleph}(C_\vartheta,\acute{\mathcal{P}})$ where $(C_\vartheta,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}PC(\tilde{Y})$, $\forall\, \vartheta \in \aleph$. So, $\tilde{s}int(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})) \tilde{\subseteq} \tilde{f}_{pu}^{-1}(\tilde{s}cl\tilde{s}int(B,\acute{\mathcal{P}}))$ and $\tilde{f}_{pu}(\tilde{s}int(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}))) = \tilde{\cup}_{\vartheta \in \aleph}(C_\vartheta,\acute{\mathcal{P}})$ where $(C_\vartheta,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}PC(\tilde{Y})$, $\forall\, \vartheta \in \aleph$.

(3) ⟶ (2). Let $(A,\mathcal{P}) \tilde{\subseteq} \tilde{X}$. Then, $\tilde{f}_{pu}(A,\mathcal{P}) \tilde{\subseteq} \tilde{Y}$ and by (3), $\tilde{s}int(A,\mathcal{P}) \tilde{\subseteq} \tilde{s}int(\tilde{f}_{pu}^{-1}(\tilde{f}_{pu}(A,\mathcal{P}))) \tilde{\subseteq} \tilde{f}_{pu}^{-1}(\tilde{s}cl\tilde{s}int(\tilde{f}_{pu}(A,\mathcal{P})))$, and $\tilde{f}_{pu}(\tilde{s}int(A,\mathcal{P})) \tilde{\subseteq} \tilde{f}_{pu}(\tilde{s}int(\tilde{f}_{pu}^{-1}(\tilde{f}_{pu}(A,\mathcal{P})))) = \tilde{\cup}_{\vartheta \in \aleph}(C_\vartheta,\acute{\mathcal{P}})$ where $(C_\vartheta,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}PC(\tilde{Y})$, $\forall\, \vartheta \in \aleph$. Therefore, $\tilde{s}int(A,\mathcal{P}) \tilde{\subseteq} \tilde{f}_{pu}^{-1}(\tilde{s}cl\tilde{s}int(\tilde{f}_{pu}(A,\mathcal{P})))$ and $\tilde{f}_{pu}(\tilde{s}int(A,\mathcal{P})) = \tilde{\cup}_{\vartheta \in \aleph}(C_\vartheta,\acute{\mathcal{P}})$. Thus, $\tilde{f}_{pu}(\tilde{s}int(A,\mathcal{P})) \tilde{\subseteq} \tilde{s}cl\tilde{s}int(\tilde{f}_{pu}(A,\mathcal{P}))$ and $\tilde{f}_{pu}(\tilde{s}int(A,\mathcal{P})) = \tilde{\cup}_{\vartheta \in \aleph}(C_\vartheta,\acute{\mathcal{P}})$.

Proposition 3.22. Let $\tilde{f}_{pu}: (\tilde{X},\tilde{\tau},\mathcal{P}) \to (\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ is a soft bijective and soft S_p-open function. If $(\tilde{X},\tilde{\tau},\mathcal{P})$ is a soft Hausdorff space, then $(\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ is soft S_p-Hausdorff.

Proof. Let $\tilde{e_x},\widetilde{e_y} \tilde{\in} \tilde{S}P(\tilde{Y})$ such that $\tilde{e_x} \neq \widetilde{e_y}$. Then, $\tilde{f}_{pu}^{-1}(\tilde{e_x}),\tilde{f}_{pu}^{-1}(\widetilde{e_y}) \tilde{\in} \tilde{S}P(\tilde{X})$. Since $\tilde{f}_{pu}$ is soft bijective, then $\tilde{f}_{pu}^{-1}(\tilde{e_x}) \neq \tilde{f}_{pu}^{-1}(\widetilde{e_y})$. Since $(\tilde{X},\tilde{\tau},\mathcal{P})$ is soft Hausdorff, there are disjoint soft open sets $(A_1,\mathcal{P})$ and $(A_2,\mathcal{P})$ in $\tilde{X}$ with $\tilde{f}_{pu}^{-1}(\tilde{e_x}) \tilde{\in} (A_1,\mathcal{P})$ and $\tilde{f}_{pu}^{-1}(\widetilde{e_y}) \tilde{\in} (A_2,\mathcal{P})$. Since $\tilde{f}_{pu}$ is soft S_p-open, then $\tilde{f}_{pu}(A_1,\mathcal{P}),\tilde{f}_{pu}(A_2,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$. Also, since $\tilde{f}_{pu}$ is soft bijective and $(A_1,\mathcal{P}) \tilde{\cap} (A_2,\mathcal{P}) = \tilde{\emptyset}$, we have $\tilde{f}_{pu}(A_1,\mathcal{P}) \tilde{\cap} \tilde{f}_{pu}(A_2,\mathcal{P}) = \tilde{f}_{pu}((A_1,\mathcal{P}) \tilde{\cap} (A_2,\mathcal{P})) = \tilde{\emptyset}$ and $\tilde{e_x} \tilde{\in} \tilde{f}_{pu}(A_1,\mathcal{P})$, $\widetilde{e_y} \tilde{\in} \tilde{f}_{pu}(A_2,\mathcal{P})$. Hence, $(\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ is soft S_p-Hausdorff.

Proposition 3.23. Let $\tilde{f}_{pu}: (\tilde{X},\tilde{\tau},\mathcal{P}) \to (\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ be a soft function. Then, $\tilde{f}_{pu}$ is soft S_p-closed iff $\tilde{s}S_pcl(\tilde{f}_{pu}(A,\mathcal{P})) \tilde{\subseteq} \tilde{f}_{pu}(\tilde{s}cl(A,\mathcal{P}))$, $\forall\, (A,\mathcal{P}) \tilde{\subseteq} \tilde{X}$.

Proof. Let $(A,\mathcal{P}) \tilde{\subseteq} \tilde{X}$. Then, $\tilde{s}cl(A,\mathcal{P}) \tilde{\in} \tilde{\tau}^c$. Since $\tilde{f}_{pu}$ is soft S_p-closed, then $\tilde{f}_{pu}\big(\tilde{s}cl(A,\mathcal{P})\big) \tilde{\in} \tilde{S}S_pC(\tilde{Y})$. Also, since $(A,\mathcal{P}) \tilde{\subseteq} \tilde{s}cl(A,\mathcal{P})$ implies that $\tilde{f}_{pu}(A,\mathcal{P}) \tilde{\subseteq} \tilde{f}_{pu}(\tilde{s}cl(A,\mathcal{P}))$, then $\tilde{s}S_pcl(\tilde{f}_{pu}(A,\mathcal{P})) \tilde{\subseteq} \tilde{s}S_pcl(\tilde{f}_{pu}(\tilde{s}cl(A,\mathcal{P}))) = \tilde{f}_{pu}(\tilde{s}cl(A,\mathcal{P}))$. So, $\tilde{s}S_pcl(\tilde{f}_{pu}(A,\mathcal{P})) \tilde{\subseteq} \tilde{f}_{pu}(\tilde{s}cl(A,\mathcal{P}))$.

Conversely, let $(A,\mathcal{P}) \tilde{\in} \tilde{\tau}^c$. Then, $(A,\mathcal{P}) = \tilde{s}cl(A,\mathcal{P})$. By hypothesis, we get $\tilde{s}S_pcl(\tilde{f}_{pu}(A,\mathcal{P})) \tilde{\subseteq} \tilde{f}_{pu}(\tilde{s}cl(A,\mathcal{P})) = \tilde{f}_{pu}(A,\mathcal{P})$. So, $\tilde{s}S_pcl(\tilde{f}_{pu}(A,\mathcal{P})) \tilde{\subseteq} \tilde{f}_{pu}(A,\mathcal{P})$. Hence, $\tilde{f}_{pu}(A,\mathcal{P}) \tilde{\in} \tilde{S}S_pC(\tilde{Y})$. Thus by Definition 3.1(2), $\tilde{f}_{pu}$ is soft S_p-closed.

Proposition 3.24. Let $\tilde{f}_{pu}: (\tilde{X}, \tilde{\tau}, \mathcal{P}) \to (\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ be a soft bijective function. Then, $\tilde{f}_{pu}$ is soft S_p-closed iff $\tilde{f}_{pu}^{-1}(\tilde{s}S_p cl(B, \acute{\mathcal{P}})) \tilde{\subseteq} \tilde{s}cl(\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}}))$, $\forall\ (B, \acute{\mathcal{P}}) \tilde{\subseteq} \tilde{Y}$.

Proof. Let $(B, \acute{\mathcal{P}}) \tilde{\subseteq} \tilde{Y}$. Then, $\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}}) \tilde{\subseteq} \tilde{X}$, $\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}}) \tilde{\subseteq} \tilde{s}cl(\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}}))$ and so $\tilde{s}cl(\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}})) \tilde{\in} \tilde{\tau}^c$. By soft S_p-closedness of $\tilde{f}_{pu}$, then $\tilde{f}_{pu}(\tilde{s}cl(\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}}))) \tilde{\in} \tilde{S}S_pC(\tilde{Y})$ and $\tilde{f}_{pu}(\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}})) \tilde{\subseteq} \tilde{f}_{pu}(\tilde{s}cl(\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}})))$. Since $\tilde{f}_{pu}$ is a soft bijective function, so $(B, \acute{\mathcal{P}}) \tilde{\subseteq} \tilde{f}_{pu}(\tilde{s}cl(\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}})))$ and hence, $\tilde{s}S_p cl(B, \acute{\mathcal{P}}) \tilde{\subseteq} \tilde{s}S_p cl(\tilde{f}_{pu}(\tilde{s}cl(\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}})))) = \tilde{f}_{pu}(\tilde{s}cl(\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}})))$. Also, since $\tilde{f}_{pu}$ is a soft bijective function, so $\tilde{f}_{pu}^{-1}(\tilde{s}S_p cl(B, \acute{\mathcal{P}})) \tilde{\subseteq} \tilde{s}cl(\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}}))$.

Conversely, let $(C, \mathcal{P}) \tilde{\in} \tilde{\tau}^c$. Then, $(C, \mathcal{P}) = \tilde{s}cl(C, \mathcal{P})$ and $\tilde{f}_{pu}(C, \mathcal{P}) \tilde{\subseteq} \tilde{Y}$. By hypothesis, we get $\tilde{f}_{pu}^{-1}(\tilde{s}S_p cl(\tilde{f}_{pu}(C, \mathcal{P}))) \tilde{\subseteq} \tilde{s}cl(\tilde{f}_{pu}^{-1}(\tilde{f}_{pu}(C, \mathcal{P})))$. Since $\tilde{f}_{pu}$ is a soft bijective function, so $\tilde{f}_{pu}^{-1}(\tilde{s}S_p cl(\tilde{f}_{pu}(C, \mathcal{P}))) \tilde{\subseteq} \tilde{s}cl(C, \mathcal{P}) = (C, \mathcal{P})$. Hence, $\tilde{s}S_p cl(\tilde{f}_{pu}(C, \mathcal{P})) \tilde{\subseteq} \tilde{f}_{pu}(C, \mathcal{P})$. Thus, $\tilde{f}_{pu}(C, \mathcal{P}) \tilde{\in} \tilde{S}S_pC(\tilde{Y})$. Therefore, by Definition 3.1(2), $\tilde{f}_{pu}$ is soft S_p-closed.

Proposition 3.25. Let $\tilde{f}_{pu}: (\tilde{X}, \tilde{\tau}, \mathcal{P}) \to (\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ be a soft bijective function. Then, the following sentences are equivalent:

(1) $\tilde{f}_{pu}$ is soft S_p-open.
(2) $\tilde{f}_{pu}$ is soft S_p-closed.
(3) $\tilde{f}_{pu}^{-1}: (\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}}) \to (\tilde{X}, \tilde{\tau}, \mathcal{P})$ is soft S_p-continuous.

Proof. (1) ⟶ (2). Obvious.

(2) ⟶ (3). Let $(C, \mathcal{P}) \tilde{\in} \tilde{\tau}^c$. By (2), we get $\tilde{f}_{pu}(C, \mathcal{P}) \tilde{\in} \tilde{S}S_pC(\tilde{Y})$. But $\tilde{f}_{pu}(C, \mathcal{P}) = (\tilde{f}_{pu}^{-1})^{-1}(C, \mathcal{P})$ and therefore, by Theorem 1.29(2), $\tilde{f}_{pu}^{-1}$ is soft S_p-continuous.

(3) ⟶ (1). Let $(A, \mathcal{P}) \tilde{\in} \tilde{\tau}$. By (3), we get $(\tilde{f}_{pu}^{-1})^{-1}(A, \mathcal{P}) = \tilde{f}_{pu}(A, \mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$ and so by Definition 3.1(1), $\tilde{f}_{pu}$ is soft S_p-open.

Proposition 3.26. A soft surjective function $\tilde{f}_{pu}: (\tilde{X}, \tilde{\tau}, \mathcal{P}) \to (\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ is soft S_p-closed iff $\forall\ (B, \acute{\mathcal{P}}) \tilde{\subseteq} \tilde{Y}$ and $(A, \mathcal{P}) \tilde{\in} \tilde{\tau}$ such that $\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}}) \tilde{\subseteq} (A, \mathcal{P})$, there exists $(Q, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$ such that $(B, \acute{\mathcal{P}}) \tilde{\subseteq} (Q, \acute{\mathcal{P}})$ and $\tilde{f}_{pu}^{-1}(Q, \acute{\mathcal{P}}) \tilde{\subseteq} (A, \mathcal{P})$.

Proof. Let $(B, \acute{\mathcal{P}}) \tilde{\subseteq} \tilde{Y}$ and $(A, \mathcal{P}) \tilde{\in} \tilde{\tau}$ such that $\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}}) \tilde{\subseteq} (A, \mathcal{P})$. Then, $\tilde{X} \tilde{\setminus} (A, \mathcal{P}) \tilde{\in} \tilde{\tau}^c$. Since $\tilde{f}_{pu}$ is soft S_p-closed, then $\tilde{f}_{pu}(\tilde{X} \tilde{\setminus} (A, \mathcal{P})) \tilde{\in} \tilde{S}S_pC(\tilde{Y})$ and so $(Q, \acute{\mathcal{P}}) = \tilde{Y} \tilde{\setminus} \tilde{f}_{pu}(\tilde{X} \tilde{\setminus} (A, \mathcal{P})) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$. Since $\tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}}) \tilde{\subseteq} (A, \mathcal{P})$, then $\tilde{X} \tilde{\setminus} (A, \mathcal{P}) \tilde{\subseteq} \tilde{X} \tilde{\setminus} \tilde{f}_{pu}^{-1}(B, \acute{\mathcal{P}}) = \tilde{f}_{pu}^{-1}(\tilde{Y} \tilde{\setminus} (B, \acute{\mathcal{P}}))$, so $\tilde{X} \tilde{\setminus} (A, \mathcal{P}) \tilde{\subseteq} \tilde{f}_{pu}^{-1}(\tilde{Y} \tilde{\setminus} (B, \acute{\mathcal{P}}))$. Since $\tilde{f}_{pu}$ is soft surjective, so $\tilde{f}_{pu}(\tilde{X} \tilde{\setminus} (A, \mathcal{P})) \tilde{\subseteq} \tilde{f}_{pu}(\tilde{f}_{pu}^{-1}(\tilde{Y} \tilde{\setminus} (B, \acute{\mathcal{P}}))) = \tilde{Y} \tilde{\setminus} (B, \acute{\mathcal{P}})$. This implies that $(B, \acute{\mathcal{P}}) \tilde{\subseteq} \tilde{Y} \tilde{\setminus} \tilde{f}_{pu}(\tilde{X} \tilde{\setminus} (A, \mathcal{P})) = (Q, \acute{\mathcal{P}})$, so $(B, \acute{\mathcal{P}}) \tilde{\subseteq} (Q, \acute{\mathcal{P}})$ and $\tilde{f}_{pu}^{-1}(Q, \acute{\mathcal{P}}) = \tilde{f}_{pu}^{-1}(\tilde{Y} \tilde{\setminus} \tilde{f}_{pu}(\tilde{X} \tilde{\setminus} (A, \mathcal{P}))) = \tilde{X} \tilde{\setminus} \tilde{f}_{pu}^{-1}(\tilde{f}_{pu}(\tilde{X} \tilde{\setminus} (A, \mathcal{P}))) \tilde{\subseteq} \tilde{X} \tilde{\setminus} \tilde{X} \tilde{\setminus} (A, \mathcal{P}) = (A, \mathcal{P})$. Thus, $\tilde{f}_{pu}^{-1}(Q, \acute{\mathcal{P}}) \tilde{\subseteq} (A, \mathcal{P})$.

Conversely, let $(C, \mathcal{P}) \tilde{\in} \tilde{\tau}^c$ and $\widetilde{e_y} \tilde{\in} \tilde{Y} \tilde{\setminus} \tilde{f}_{pu}(C, \mathcal{P})$. Then, $\tilde{X} \tilde{\setminus} (C, \mathcal{P}) \tilde{\in} \tilde{\tau}$ and $\tilde{Y} \tilde{\setminus} \tilde{f}_{pu}(C, \mathcal{P}) \tilde{\subseteq} \tilde{Y}$ such that $\tilde{f}_{pu}^{-1}(\widetilde{e_y}) \tilde{\in} \tilde{f}_{pu}^{-1}(\tilde{Y} \tilde{\setminus} \tilde{f}_{pu}(C, \mathcal{P})) \tilde{\subseteq} \tilde{X} \tilde{\setminus} (C, \mathcal{P})$. By hypothesis, there exists $(Q, \acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$ such that $\widetilde{e_y} \tilde{\in} \tilde{Y} \tilde{\setminus} \tilde{f}_{pu}(C, \mathcal{P}) \tilde{\subseteq} (Q, \acute{\mathcal{P}})$ and $\tilde{f}_{pu}^{-1}(Q, \acute{\mathcal{P}}) \tilde{\subseteq} \tilde{X} \tilde{\setminus} (C, \mathcal{P})$, and so $(C, \mathcal{P}) \tilde{\subseteq} \tilde{X} \tilde{\setminus} \tilde{f}_{pu}^{-1}(Q, \acute{\mathcal{P}})$. That is $(C, \mathcal{P}) \tilde{\subseteq} \tilde{f}_{pu}^{-1}(\tilde{Y} \tilde{\setminus} (Q, \acute{\mathcal{P}}))$ implies that $\tilde{f}_{pu}(C, \mathcal{P}) \tilde{\subseteq} \tilde{Y} \tilde{\setminus} (Q, \acute{\mathcal{P}})$, so $\widetilde{e_y} \tilde{\in} (Q, \acute{\mathcal{P}}) \tilde{\subseteq} \tilde{Y} \tilde{\setminus} \tilde{f}_{pu}(C, \mathcal{P})$. Thus Proposition 1.18(2), $\tilde{Y} \tilde{\setminus} \tilde{f}_{pu}(C, \mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$ and so $\tilde{f}_{pu}(C, \mathcal{P}) \tilde{\in} \tilde{S}S_pC(\tilde{Y})$. Therefore, $\tilde{f}_{pu}$ is soft S_p-closed.

Proposition 3.27. Let $\tilde{f}_{pu}: (\tilde{X}, \tilde{\tau}, \mathcal{P}) \to (\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ and $\tilde{g}_{qv}: (\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}}) \to (\tilde{W}, \tilde{\mu}, \ddot{\mathcal{P}})$ be two soft functions. Then, $\tilde{g}_{qv} \tilde{\circ} \tilde{f}_{pu}: (\tilde{X}, \tilde{\tau}, \mathcal{P}) \to (\tilde{W}, \tilde{\mu}, \ddot{\mathcal{P}})$ is soft S_p-open (respectively, soft S_p-closed), if $\tilde{f}_{pu}$ is soft open (respectively, soft closed) and $\tilde{g}_{qv}$ is soft S_p-open (respectively, soft S_p-closed).

Proof. Let $(A, \mathcal{P}) \tilde{\in} \tilde{\tau}$ (respectively, $\tilde{\tau}^c$). Since $\tilde{f}_{pu}$ is soft open (respectively, soft closed), then $\tilde{f}_{pu}(A, \mathcal{P}) \tilde{\in} \tilde{\sigma}$ (respectively, $\tilde{\sigma}^c$). Since $\tilde{g}_{qv}$ is soft S_p-open (respectively, soft S_p-closed), then $\tilde{g}_{qv}(\tilde{f}_{pu}(A, \mathcal{P})) = (\tilde{g}_{qv} \tilde{\circ} \tilde{f}_{pu})(A, \mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{W})$ (respectively, $\tilde{S}S_pC(\tilde{W})$). Therefore, by Definition 3.1(1) (respectively, Definition 3.1(2)), $\tilde{g}_{qv} \tilde{\circ} \tilde{f}_{pu}$ is soft S_p-open (respectively, soft S_p-closed).

Proposition 3.28. If $\tilde{f}_{pu}: (\tilde{X}, \tilde{\tau}, \mathcal{P}) \to (\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ and $\tilde{g}_{qv}: (\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}}) \to (\tilde{W}, \tilde{\mu}, \ddot{\mathcal{P}})$ are soft S_p-open (respectively, soft S_p-closed) functions and $(\tilde{Y}, \tilde{\sigma}, \acute{\mathcal{P}})$ is soft locally indiscrete, then, $\tilde{g}_{qv} \tilde{\circ} \tilde{f}_{pu}: (\tilde{X}, \tilde{\tau}, \mathcal{P}) \to (\tilde{W}, \tilde{\mu}, \ddot{\mathcal{P}})$ is soft S_p-open (respectively, soft S_p-closed).

Proof. Let $(A,\mathcal{P}) \tilde{\in} \tilde{\tau}$ (respectively, $\tilde{\tau}^c$). Since $\tilde{f}_{pu}$ is soft S_p-open (respectively, soft S_p-closed), then $\tilde{f}_{pu}(A,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$ (respectively, $\tilde{S}S_pC(\tilde{Y})$). Since $\tilde{Y}$ is soft locally indiscrete, then by Proposition 1.20(2) (respectively, Proposition 1.22(2)), then $\tilde{f}_{pu}(A,\mathcal{P}) \tilde{\in} \tilde{\sigma}$ (respectively, $\tilde{\sigma}^c$) and so as in Proposition 3.27, $\tilde{g}_{qv} \tilde{\circ} \tilde{f}_{pu}$ is soft S_p-open (respectively, soft S_p-closed).

Theorem 3.29. Let $\tilde{f}_{pu}: (\tilde{X},\tilde{\tau},\mathcal{P}) \to (\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ and $\tilde{g}_{qv}: (\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}}) \to (\widetilde{W},\tilde{\mu},\ddot{\mathcal{P}})$ be two soft functions such that $\tilde{g}_{qv} \tilde{\circ} \tilde{f}_{pu}: (\tilde{X},\tilde{\tau},\mathcal{P}) \to (\widetilde{W},\tilde{\mu},\ddot{\mathcal{P}})$ is a soft S_p-open function. Then,:

(1) $\tilde{g}_{qv}$ is soft S_p-open, if $\tilde{f}_{pu}$ is soft continuous and soft surjective.

(2) $\tilde{g}_{qv}$ is soft S_p-open, if $\tilde{f}_{pu}$ is soft S_p-continuous, soft surjective and $(\tilde{X},\tilde{\tau},\mathcal{P})$ is soft locally indiscrete.

(3) $\tilde{f}_{pu}$ is soft S_p-open, if $\tilde{g}_{qv}$ is soft S_p-irresolute and soft injective.

Proof. (1) Let $(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{\sigma}$. Since $\tilde{f}_{pu}$ is soft continuous, then $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{\tau}$. Since $\tilde{g}_{qv} \tilde{\circ} \tilde{f}_{pu}$ is soft S_p-open, then $(\tilde{g}_{qv} \tilde{\circ} \tilde{f}_{pu})\left(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}})\right) \tilde{\in} \tilde{S}S_pO(\widetilde{W})$. Since $\tilde{f}_{pu}$ is soft surjective, then $\tilde{g}_{qv}\left(\tilde{f}_{pu}(\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}))\right) = \tilde{g}_{qv}(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\widetilde{W})$. Therefore, by Definition 3.1(1), $\tilde{g}_{qv}$ is soft S_p-open.

(2) Let $(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{\sigma}$. Since $\tilde{f}_{pu}$ is soft S_p-continuous, then $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$. Since $\tilde{X}$ is soft locally indiscrete, then by Proposition 1.20(2), $\tilde{f}_{pu}^{-1}(B,\acute{\mathcal{P}}) \tilde{\in} \tilde{\tau}$ and so in a similar way as we have done in (1), we get $\tilde{g}_{qv}$ is soft S_p-open.

(3) Let $(A,\mathcal{P}) \tilde{\in} \tilde{\tau}$. Since $\tilde{g}_{qv} \tilde{\circ} \tilde{f}_{pu}$ is soft S_p-open, then $(\tilde{g}_{qv} \tilde{\circ} \tilde{f}_{pu})(A,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\widetilde{W})$, and so $\tilde{g}_{qv}^{-1}(\tilde{g}_{qv} \tilde{\circ} \tilde{f}_{pu})(A,\mathcal{P}) = \tilde{f}_{pu}(A,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{Y})$ as $\tilde{g}_{qv}$ is soft S_p-irresolute and soft injective. Therefore, by Definition 3.1(1), $\tilde{f}_{pu}$ is soft S_p-open.

Theorem 3.30. Let $\tilde{f}_{pu}: (\tilde{X},\tilde{\tau},\mathcal{P}) \to (\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}})$ and $\tilde{g}_{qv}: (\tilde{Y},\tilde{\sigma},\acute{\mathcal{P}}) \to (\widetilde{W},\tilde{\mu},\ddot{\mathcal{P}})$ be two soft functions such that $\tilde{g}_{qv} \tilde{\circ} \tilde{f}_{pu}: (\tilde{X},\tilde{\tau},\mathcal{P}) \to (\widetilde{W},\tilde{\mu},\ddot{\mathcal{P}})$ is a soft S_p-closed function. Then,:

(1) $\tilde{g}_{qv}$ is soft S_p-closed, if $\tilde{f}_{pu}$ is soft continuous and soft surjective.

(2) $\tilde{g}_{qv}$ is soft S_p-closed, if $\tilde{f}_{pu}$ is soft S_p-continuous, soft surjective and $(\tilde{X},\tilde{\tau},\mathcal{P})$ is soft locally indiscrete.

(3) $\tilde{f}_{pu}$ is soft S_p-closed, if $\tilde{g}_{qv}$ is soft S_p-irresolute and soft injective.

Proof. Using Definition 3.1(2) and Proposition 1.22(2) in place of Definition 3.1(1) and Proposition 1.20(2), respectively, the proof is similar to Proposition 3.29.

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