Applications of the SUM Integral Transform in Science and Technology

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Abstract—In this paper, a new integral application named SUM transform is introduced and some of its properties are studied. In addition, the application of the SUM integral transform are nuclear physics, population growth, electric circuits, pharmacokinetic, beam deflection, Newton's law of cooling and other fields of mechanics.

Keywords: SUM transform, LODEs, Pharmacokinetic, beam deflection, Newton law of cooling, Kirchhoff's law, mechanics, Hooke's law.

1 Introduction

Integral transforms have been applied in various fields such as automatic missile control, mechanical engineering, population growth and decay, electrical engineering, nuclear physics, pharmacokinetics, beam deflection, cryptography, etc. [1-6]. As recently as 2022, a new integral transformation called the SUM integral transform was introduced in [7] in the following equation:

$$S_a\{f(t)\}_s = \frac{1}{s^n}\int_{t=0}^{\infty} f(t)a^{-st}dt = F_a(s), \quad (1)$$

where $t \geq 0, n \in Z, a > 0, n_1 \leq s \leq n_2, n_1, n_2 > 0$ and $f(t)$ is sectionally continuous and exponential order. Some properties of this integral transform such linearity, change scale, first shifting and second shifting properties have also been established in [7]

Lemma 1. [7] If $S_a\{f(t)\}_s = F_a(s)$, then
$S_a\left\{f^{(m)}(t)\right\}_s$

$$= -\frac{f^{(m-1)}(0)}{s^r} - [slog(a)]\frac{f^{(m-2)}(0)}{s^r} - \cdots + [slog(a)]^m F_a(s). \qquad (2)$$

The following SUM transform of some elementary function was also given in [7]:

Table 1: The SUM integral transform of some elementary functions

S/N	$f(t)$	$S_a\{f(t)\}_s$
1	k	$\frac{k}{s^n[slog(a)]}$
2	$t^r, r \in N_0$	$\frac{r!}{s^n[slog(a)]^{r+1}}$
3	$e^{\delta t}$, δ is a constant	$\frac{1}{s^n[slog(a) - \delta]}$
4	$sin\, sin\,(\delta t)$, δ is a constant	$\frac{\delta}{s^n\left[[slog(a)]^2 + \delta^2\right]}$
5	$cos\, cos\,(\delta t)$, δ is a constant	$\frac{[slog(a)]}{s^n\left[[slog(a)]^2 + \delta^2\right]}$
6	$sin\, sin\, h(\delta t)$, δ is a constant	$\frac{\delta}{s^n\left[[slog(a)]^2 - \delta^2\right]}$
7	$cos\, cos\, h(\delta t)$, δ is a constant	$\frac{[slog(a)]}{s^n\left[[slog(a)]^2 - \delta^2\right]}$

2. Main result

Following the application of the SUM integral transforms above are discussed

2.1 The SUM transform in nuclear physics

The relation representing radioactive decay is [8]

$$\frac{dN(t)}{dt} = -\rho N(t) \text{ with } N(0) = N_0, \qquad (3)$$

where $N(t)$ denote the number of un-decayed atoms left in a sample of radioactive isotope and ρ represent the decay constant.

Example 1. If a radioactive material is known to decay at a rate proportional to the amount present. If initially there is 100 milligrams of the radioactive material present and after 3 hours. It has been observed that the radioactive material has lost 20 per cent of its original mass [3].

The half-life of a radioactive substance can be estimated using the SUM transform.

By applying the SUM transform to both sides of equation (3), we have

$$-\frac{N(0)}{s^n}+[slog(a)]F(s)+\rho F_a(s)=0.$$

Simplification leads to the following:

$$F_a(s)=\frac{N_0}{s^n[slog(a)+\rho]}.$$

Taking the inverse of the SUM transform, gives

$$N(t)=N_0e^{-\rho t}. \quad (4)$$

Applying the condition $t=0$, then $N_0=100$ to equation (4), implies that

$$N(t)=100e^{-\rho t}. \quad (5)$$

At $t=3$ then $N(t)=100-20=80$ and so equation (5), produce

$$80=100e^{-3\rho}.$$

Simplifying

$$\rho=-\frac{1}{3}In\left(\frac{4}{5}\right). \quad (6)$$

Since the time require when $N(t)=\frac{N_0}{2}=50$, we have

$$50=100e^{-\rho t}. \quad (7)$$

Putting equation (6) into (7), we have

$$50=100e^{\left[\frac{t}{3}In\left(\frac{4}{5}\right)\right]}. \quad (8)$$

Simplifying for t from equation (8), give

$$t=9.32 \text{ hours}.$$

2.2 The SUM transform in solving population growth

In 1798, T.R. Malthusian proposed a model of exponential growth assuming that rate of over time is proportional to the current population [9]:

$$\frac{dP(t)}{dt}=\mu P(t), \qquad \text{with } P(0)=P_0, \quad (9)$$

where $P(t)$ denote the number of people living in a city at any time t and μ represent population growth rate.

Example 2. If the population of a city growth at a rate proportional to the number of people currently living in the city. Assuming, after 2 years the population has double, and after 3 years the population is 20 000, it is possible to use the SUM transform to estimate the number of people initially lived in the city [3].

Applying the SUM transform to both sides of equation (9), we have

$$-\frac{N(0)}{s^n}+[slog(a)]F_a(s)-\mu F_a(s)=0.$$

Simplifying, lead to the following

$$F_a(s)=\frac{N_0}{s^n[slog(a)-\mu]}.$$

Taking the SUM inverse transform, gives

$$P(t)=P_0e^{\mu t}. \tag{10}$$

At $t=2$ then $P(2)=2P_0$, so that from equation (10), we have

$$2P_0=P_0e^{2\mu}.$$

Simplifying

$$\mu=\frac{1}{2}In(2). \tag{11}$$

Using the condition at $t=3$ then $P(3)=20\ 000$ and so

$$P_0=20\ 000P_0e^{-3\mu}. \tag{12}$$

Substituting equation (11) into (12), yields

$$P_0=20\ 000P_0e^{\frac{3}{2}In(2)}$$
$$=71\ 071.07 \text{ people}.$$

2.3 The SUM transform in analyzing electric circuit

A simple electric circuit consists of a resistance R, an inductor L, a capacitor with capacitance C and a power source with potential E are connected in series as follows [10]:

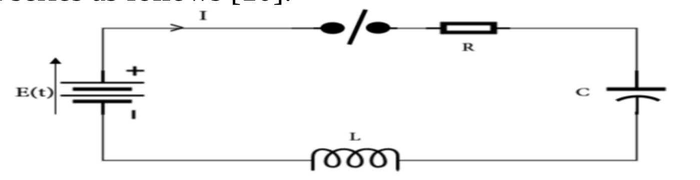

Fig. 1. simple electric circuit (adapted from [10])

Applying Kirchhoff's law to the circuit gives us

$$L\frac{dI}{dt}+RI+\frac{Q}{C}=E,$$

can be simplified by using $I=\frac{dQ}{dt}$ in the form

$$L\frac{d^2Q}{dt^2}+R\frac{dQ}{dt}+\frac{Q}{C}=E. \tag{13}$$

Example 3. Suppose the inductance value is 2 Henrys, resistor value is 16 Ohms, capacitor value 0.02 Farad and e.m.f. value is 300 volts. At $t=0$ the charge in the circuit's capacitor and the current are zero, the charges and the current are charged accordingly at $t > 0$. Q is the prompt charge and I is the current charge at the time t [11].

From equation (13), we can obtain

$$2\frac{d^2Q}{dt^2} + 16\frac{dQ}{dt} + 50Q = 300,$$

can be reduced

$$\frac{d^2Q}{dt^2} + 8\frac{dQ}{dt} + 25Q = 150. \tag{14}$$

Applying the SUM transform to equation (14), give

$$-\frac{Q'(0)}{s^n} - [slog(a)]\frac{Q(0)}{s^n} + [slog(a)]^2F_a(s) + 8\left\{-\frac{Q(0)}{s^n} + [slog(a)]F_a(s)\right\} + 25F_a(s) = \frac{150}{s^n[slog(a)]}.$$

Simplifying

$$F_a(s) = \frac{150}{s^n[slog(a)]\{[slog(a)]^2+8[slog(a)]+25\}}.$$

Applying partial fraction, yields

$$F_a(s) = \frac{6}{s^n[slog(a)]} - \frac{6[slog(a)+4]}{s^n\{[slog(a)+4]^2+9\}} - \frac{24}{s^n\{[slog(a)+4]^2+9\}}.$$

Taking the inverse SUM transform, yields

$$Q(t) = 6 - 6e^{-4t}\, cos\, cos\, (3t) - 8e^{-4t}\, sin\, sin\, (3t). \tag{15}$$

Differentiating equation (15) with respect to t, I is obtained as follows:

$$I = \frac{dQ}{dt} = 50e^{-4t}\, sin\, sin\, (3t).$$

2.4 The SUM transform in pharmacokinetic

Pharmacokinetics is the study of the drug absorption, distribution, metabolism, and excretion of drugs [12]. Such a problem can sometimes be described by the following differential equation [13]:

$$\frac{dM(t)}{dt} + \gamma M(t) = \frac{\beta}{v}, \quad t > 0 \text{ with } M(0) = 0, \tag{16}$$

where $M(t)$ is the medication concentration in the blood an any time t, γ is the β represent the propotion of the infusion in milligram per minute and v denote the volume in which drug is distributed.

Applying the SUM transform to both sides equation (16), we obtain

$$-\frac{M(0)}{s^n} + [slog(a)]F_a(s) + \gamma F_a(s) = \frac{\beta}{v}\frac{1}{s^n[slog(a)]}.$$

Simplifying

$$F_a(s) = \frac{\beta}{v}\frac{1}{s^n[slog(a)][slog(a)+\gamma]}.$$

Considering partial fraction

$$F_a(s) = \frac{\beta}{\gamma v}\left[\frac{1}{s^n[slog(a)]} - \frac{1}{s^n[slog(a)+\gamma]}\right].$$

Taking the inverse SUM transform, gives

$$M(t) = \frac{\beta}{\gamma v}[1 - e^{-\gamma t}].$$

2.5 Analyzing beam deflection problems

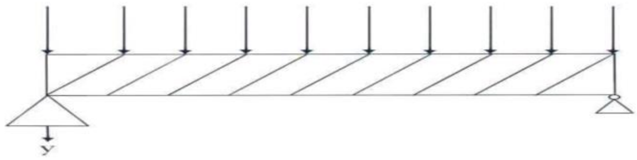

Fig. 2. hanged beam under uniform load (adapted from [14])

The beam depletion problem hinged at its end $x = 0$ and $x = L$ that supports a uniform load ω_0 (per unit length) in figure 2 can be expressed using the following 4th order ordinary differential equation with boundary conditions [14]:

$$\frac{d^4y}{dx^4} = \frac{\omega_0}{\mathcal{E}I}, \qquad 0 < x < 1, \qquad (17)$$

with the conditions $y(0) = y''(0) = 0$ and $y(L) = y''(0) = 0$, where $\mathcal{E}$ donate the Young modulus, I represent the normal bending plain moment of inertia for a cross-section about an axis and $\mathcal{E}I$ is the beam's flexural rigidity.

Taking the SUM transform of equation (17), leads to

$$-\frac{y'''(0)}{s^n} - [slog(a)]\frac{y''(0)}{s^n} - [slog(a)]^2\frac{y'(0)}{s^n} -$$
$$[slog(a)]^3\frac{y(0)}{s^n}$$
$$+[slog(a)]^4 F_a(s) = \frac{\omega_0}{\mathcal{E}I}\frac{1}{s^n[slog(a)]}.$$

Considering the condition $y(0) = y''(0) = 0,\ y'(0) = A, y'''(0) = B$ and simplifying

$$F_a(s) = A\frac{1}{s^n[slog(a)]^2} + B\frac{1}{s^n[slog(a)]^4} + \frac{\omega_0}{\mathcal{E}I}\frac{1}{s^n[slog(a)]^5}.$$

Taking the inverse SUM transform

$$y(x) = Ax + B\frac{x^3}{6} + \frac{\omega_0}{\mathcal{E}I}\frac{x^4}{24}. \qquad (18)$$

Using $y(l) = 0$ and $y''(l) = 0$ we obtain

$$A = \frac{\omega_0}{\mathcal{E}I}\frac{l^3}{24} \text{ and } B = -\frac{\omega_0}{\mathcal{E}I}\frac{l}{2},$$

hence

$$y(x) = \frac{\omega_0}{\mathcal{E}I}\frac{l^3}{24}x - \frac{\omega_0}{\mathcal{E}I}\frac{l}{2}\frac{x^3}{6} + \frac{\omega_0}{\mathcal{E}I}\frac{x^4}{24}.$$

Which can be simplify to the form

$$y(x) = \frac{\omega_0}{\mathcal{E}I}x(l-x)(l^2+lx-x^2).$$

2.6 Solving the Newton law of cooling problems

The Newton's law of cooling states that the temperature of an object that is surrounded by a temperature different environment changes relative to the temperature different between the object and the temperature, which is expressed in the following equation [15][16]:

$$\frac{dT(t)}{dt} = -\varrho[T(t) - T_0(t)], \tag{19}$$

where T represent the body temperature at any time t, $T_0(t)$ donate the surrounding environment temperature and ϱ is the proportionality constant ($\varrho > 0$).

Example 4. [14] If the surrounding air temperature is 40^0 C and the body temperature drops from 80^0 C to 60^0 C in 20 min.

(i) What would be the temperature of the body be after 40 min?
(ii) When the temperature of the body reaches 55^0 C?

From equation (19), we have

$$\frac{dT(t)}{dt} + \varrho T(t) = 40\varrho. \tag{20}$$

On taking the inverse SUM transform gives

$$-\frac{T(0)}{s^n} + [slog(a)]F_a(s) + \varrho F_a(s) = \frac{40\varrho}{s^n[slog(a)]}.$$

By applying the condition $T(0) = 80$ and simplifying

$$F_a(s) = \frac{80}{s^n[slog(a)+\varrho]} + \frac{40\varrho}{s^n[slog(a)][slog(a)+\varrho]}.$$

Applying partial fraction, gives

$$F_a(s) = \frac{40}{s^n[slog(a)]} + \frac{40}{s^n[slog(a)+\varrho]}.$$

Taking inverse transform, leads to

$$T(t) = 40[1 + e^{-\varrho t}]. \tag{21}$$

At $t = 20$ then $T(20) = 60$ and so

$$\varrho = -\frac{1}{20}In\left(\frac{1}{2}\right), \tag{22}$$

and the fact that equation (21) can be re-expressed using (22) as

$$T(t) = 40\left[1 + e^{\frac{t}{20}In\left(\frac{1}{2}\right)}\right]$$
$$= 40\left[1 + \left(\frac{1}{2}\right)^{\frac{t}{20}}\right]. \qquad (23)$$

(i) Putting $t = 20$ into equation (23), then

$$T(t) = 40\left[1 + \left(\frac{1}{2}\right)^{\frac{1}{2}}\right]$$
$$= 50^0 C.$$

(ii) Setting $T(t) = 55$, then

$$55 = 40\left[1 + \left(\frac{1}{2}\right)^{\frac{t}{20}}\right].$$

This can be rewritten as

$$t = 20\left[\frac{In\left(\frac{3}{5}\right)}{In\left(\frac{1}{2}\right)}\right]$$
$$= 28.30 \text{ min.}$$

2.7 Other mechanical problems

Other problems that arise in mechanics such as oscillating mechanical systems and the motion of particles are discussed in this section.

Example 5. [17] A particle with mass 2 gram, which moves on X-axis and gets attracted towards the origin with a force aX. If q is initially at rest at $X = 10$ then find its position at any time with an assumption

(i) No other external force acts on it.

(ii) A damping forces equal to 8 times the instantaneous velocity act.

This problem can be solved by using the SUM transform

(i) The equation of motion of the particle from the Newton's law is

$$2\frac{d^2X}{dt^2} = -8X.$$

This can also be express by

$$\frac{d^2X}{dt^2} + 4X = 0 \quad \text{with } X(0) = 10 \ \text{ and } X'(0) = 0. \qquad (24)$$

Applying the SUM transform to equation (24), we have

$$-\frac{X'(0)}{s^n} - [slog(a)]\frac{X(0)}{s^n} + [slog(a)]^2 F_a(s) + 4F_a(s) = 0.$$

Simplifying

$$F_a(s) = \frac{10[slog(a)]}{s^n\{[slog(a)]^2+4\}}.$$

By taking the inverse SUM transform, we get the solution of equation (24)

$$X(t) = 10\, cos\, cos\,(2t).$$

(ii) The equation of the particle is given by

$$2\frac{d^2X}{dt^2} = -8X - 8\frac{dX}{dt}.$$

This can also be express by

$$\frac{d^2X}{dt^2} + 4\frac{dX}{dt} + 4X = 0 \text{ with } X(0) = 10 \text{ and } X'(0) = 0. \qquad (25)$$

Applying the SUM transform to equation (24), we have

$$-\frac{X'(0)}{s^n} - [slog(a)]\frac{X(0)}{s^n} + [slog(a)]^2 F_a(s)$$
$$+4\left\{-\frac{X(0)}{s^n} + [slog(a)]F_a(s)\right\} + 4F_a(s) = 0.$$

Applying the conditions $X(0) = 10$ and $X'(0) = 0$ gives

$$F_a(s) = \frac{10[slog(a)]}{s^n\{[slog(a)]+2\}^2} + \frac{40}{s^n\{[slog(a)]+2\}^2}.$$

Using partial fraction, leads to

$$F_a(s) = \frac{10}{s^n\{[slog(a)]+2\}} + \frac{20}{s^n\{[slog(a)]+2\}^2}.$$

Taking the inverse SUM transform the solution of equation (25) is obtained as

$$X(t) = 10e^{-2t} + 20te^{-2t}.$$

Example 6. [17] A body of mass M moves along the X-axis, it is under the influence of a force which is directly proportional to its instantaneous speed and in a direction opposite of motion. Assuming that at $t = 0$ the particle is located at $x = A$ and moving to the right with a speed ϑ_0, find the position where the mass comes to rest.

$$M\frac{d^2x}{dt^2} = -\omega\frac{dx}{dt} \quad x(0) = A \qquad \text{and } x'(0) = \vartheta_0.$$

Rewritten this equation results in the following equation:

$$M\frac{d^2x}{dt^2} + \omega\frac{dx}{dt} = 0 \text{ with } \; x(0) = A \text{ and } x'(0) = \vartheta_0. \qquad (26)$$

$$M\left\{-\frac{x'(0)}{s^n} - [slog(a)]\frac{x(0)}{s^n} + [slog(a)]^2 F_a(s)\right\}$$

$$+\omega\left\{-\frac{x(0)}{s^n}+[slog(a)]F_a(s)\right\}=0.$$

Applying the conditions $x(0)=A$ and $x'(0)=\vartheta_0$, gives

$$F_a(s)=\frac{(M\vartheta_0+A\omega)}{s^n[slog(a)]\{M[slog(a)]+\omega\}}+\frac{AM[slog(a)]}{s^n[slog(a)]\{M[slog(a)]+\omega\}}.$$

Applying partial fraction gives

$$F_a(s)=\left(\frac{M\vartheta_0}{\omega}+A\right)\frac{1}{s^n[slog(a)]}-\left(\frac{M\vartheta_0}{\omega}\right)\frac{1}{s^n[slog(a)]\left\{[slog(a)]+\left(\frac{\omega}{M}\right)\right\}}.$$

Taking the inverse of the SUM transform leads to the following solution of equation (26):

$$x(t)=\left(\frac{M\vartheta_0}{\omega}+A\right)-\left(\frac{M\vartheta_0}{\omega}\right)e^{-\left(\frac{\omega}{M}\right)t}. \quad (27)$$

If $x'(t)=0$ we can obtain

$$x'^{(t)}=\vartheta_0 e^{-\left(\frac{\omega}{M}\right)t}=0.$$

Therefore, equation (27) can now be written as

$$x(t)=\frac{M\vartheta_0}{\omega}+A.$$

Hence, the mass M come to rest at a distance $\frac{M\vartheta_0}{\omega}+A$ from the centre O O.

Example 7. Find the solution of suspended spring vibration that is free and un-dumped [17].

Suspended spring is used to indicate that the spring is hanging vertically with the upper end fixed and the lower end acting on an object of mass M, which is initially the spring is stretched by moving the mass at A, some positive distance below the equilibrium position. It is the released from rest, the initial condition are

$$x(0)=A \text{ and } \vartheta(0)=x'(0)=0. \quad (28)$$

Free vibration spring refers to the fact that there is no external $F(t)$, un-damped indicate that there is on air or any medium resistance force, that is, the only force acting on the system is the force of the spring which by the Hooke's law is given by κx.

Using the Newton's second law of motion, the differentiation equation in consideration is

$$M\frac{d^2x}{dx^2}=-\kappa x.$$

This can be express as

$$\frac{d^2x}{dx^2} + \left(\frac{\kappa}{M}\right) x = 0. \tag{29}$$

Applying the SUM transform to equation (29), gives

$$-\frac{x'(0)}{s^n} - [slog(a)]\frac{x(0)}{s^n} + [slog(a)]^2 F_a(s) + \left(\frac{\kappa}{M}\right) F_a(s) = 0.$$

Using the initial conditions in equation (28), yields

$$F_a(s) = A\frac{[slog(a)]}{[slog(a)]^2 + \left(\frac{\kappa}{M}\right)}.$$

Taking inverse of the SUM transform the solution of equation (29) is give by

$$x(t) = Acos\left[\sqrt{\left(\frac{\kappa}{M}\right)}\, t\right],$$

where A is the amplitude of the motion, $\sqrt{\left(\frac{\kappa}{M}\right)}$ is the circular frequency, the natural frequency is given by $f = \frac{1}{2\pi}\sqrt{\left(\frac{\kappa}{M}\right)}$ and the period is $T = \frac{1}{f} = 2\pi\sqrt{\left(\frac{M}{\kappa}\right)}$.

3. Conclusion

The SUM integral transform has been successfully applied to number scientific and technological problems such as physics, population growth, electrical circuits, pharmacokinetics, beam deflection, Newton's law of cooling and mechanics. The SUM integral is useful for finding differential equations in science and engineering.

References

[1] E.A. Kuffi, E.S. Abbas and S.O. Mebayet, Al-Tememe transform in solving develop missile automatic problem, International Journal of Engineering and Technology, vol. 7, no. 4, pp.6565-6567, 2019.

[2] E.A. Kuffi and E.S. Abbas, Applying Al-Zughair transform into ome engineering fields, 3rd International Scientific Conference of Alkafeel University, pp. 1-7, 2021.

[3] E.A. Monsour, E.A. Kuffi and S.A. Mehdi, Solving population and decay problems using complex SEE transform, 7th International Conference on Contemporary Information Technology and Mathematics, Mosul, Iraq, pp. 243-246, 2021.

[4] E.A. Monsour, E.A. Kuffi and S.A. Mehdi, Applying complex SEE transform in cryptography, MJPS, vol. 8, no. 2, pp. 99-104, 2021.

[5] E.A. Monsour, S.A. Mehdi and E.A. Kuffi, The new integral transform and its application, International Journal of Nonlinear analysis and Applications, vol. 12, no. 2, pp. 1477-1483, 2021.
[6] H.S., Kadhem, and S.M. Hassan, Solving some fractional partial differential equations by invariant subspace and double Sumudu transform methods, Ibn Al Haitham Journal for Pure and Applied Science, vol.33, no. 3, pp. 127-139, 2020.
[7] Q.H. Sameer, U.M. Abubakar, M.L. Kaurangini, The new integral transform ''SUM transform'' and its properties, accepted in the Third International Conference of Mathematics and its Applications (TICMA 2022).
[8] D.P. Patil, P.S. Kikam, S.D. Shirsath and A.J. Aher, ''Kushere transform'' for solving the problems on growth and decay, Journal of Emerging Technology and Innovative Research, vol. 9, no. 4, pp. 317-323, 2022.
[9] U.M. Abubakar and S. Usaini, Note on mathematical modeling for population projection and management: A case study of Niger state, IOSR Journal of Mathematics, vol. 15, no. 6, 8-11, 2019.
[10] A.S. Kokane and B.R. Wadkar, Applications of Laplace transform in science and technology, Journal of Physics: Conference Series 1913(2021)012127, pp. 1-10, 2021.
[11] L.S. Sawat, Application of Laplace transform in Engineering fields, International Journal of Engineering and Technology, vol.5, no. 5, pp.3100-3105, 2018.
[12] Ratain MJ, Plunkett WK Jr. Principles of Pharmacokinetics. In: Kufe DW, Pollock RE, Weichselbaum RR, et al., editors. Holland-Frei Cancer Medicine. 6th edition. Hamilton (ON): BC Decker; 2003. Available from: https://www.ncbi.nlm.nih.gov/books/NBK12815/
[13] E.A. Kuffi, F. Karaaslan, G.S. Sadhkhan, The complex EFG integral transform and its applications, International Journal of Health Sciences, vol. 6, no. S3, pp.537-547, 2022.
[14] A.H. Ali, E.A. Kuffi, The SEA integral transform and its application on differential equations, vol. 6, no. S3, pp. 613-622, 2022.
[15] R.S. Sanap, D.P. Patil, Kushare integral transform for Newton's law of cooling, International Journal of Advances in Engineering and Management, vol. 4, no. 1, pp. 166-170, 2022.
[16] L. Sahoo, Application of Laplace transform for solving problems on Newton's law of cooling, International Journal of Recent Scientific Research, vol. 11, no 12, pp. 40207-40209, 2020.
[17] U.R. Ramachandruni and Y. Namala, Analytical solutions of some mechanics problems by Elzaki transform, India Journal of Science and Technology, vol. 13, no. 47, pp. 4606-4618, 2020.

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